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Takagi-Sugeno Fuzzy Coordinated Control System with Original Plant Fuzzy State Observer for a Power Unit

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Abstract: The Takagi-Sugeno (T-S) fuzzy multiple-variable integral control system with fuzzy state observer is designed for the coordinated control of multi-input-multi-output (MIMO) nonlinear boiler-turbine system whose operating points vary within a wide range. Linear models are first derived from the original nonlinear model on several operating points. Next the fuzzy multi-variable integral controller and the fuzzy state observer are designed via using the parallel distributed compensation (PDC) scheme, and the T-S fuzzy control system is constructed with the drum pressure as the premise variable. The problem that part of state variables can not be measured is successfully solved by introducing fuzzy state observer. Last the stability analysis is given by means of linear matrix inequality (LMI) approach, and the control system is guaranteed to be stable within a large range. The simulation results demonstrate that the control system works well over a wide region of operation.

1. INTRODUCTION

Most of current control systems for fossil-fuel power plants consist of decentralized multi-loop configurations of single-input-single-output (SISO) feedback control loops evaluating conventional PI or PID algorithms. These configurations have been developed through various decades of research and practical experience. Their effectiveness to regulate the process under random load disturbances around a fixed operating point is daily proven all over the world. Normally, the parameters of the controller are tuned at some predefined operating point (i.e., base load) assuming nearly constant load conditions, and left fixed thereafter. However, current requirements demanding wide-range operation of fossil-fuel units of all sizes challenge this approach. The performance of the power unit may decrease due to the nonlinear and interactive dynamics of the process, that change with the operating point (Garduno-Ramirez *et al*., 2000a, b).

In order to make full use of the potential of the boiler-turbine unit, multi-variable control strategies should be taken. In fact, the need for simultaneous control of the strongly interacting variables of the boiler-turbine system makes the boiler-turbine control an ideal application for multivariable control (Kwon *et al*., 1989). Direct application of the multivariable control theories in the boiler-turbine system had been reported in some literatures, e.g., Cori *et al*. (1984) and Johansson *et al*. (1984). Nether less, these controller systems hold true in limited range, the performance of which will lapse if the range is gone beyond.

In Mamdani (1974) a controller based on fuzzy set theory is introduced for the first time. A fuzzy algorithm, that emulates the reasoning procedure of a human operator, is used to control a laboratory-built boiler steam engine. This work showed the feasibility to build effective real-time decision algorithms, inaugurating a new era in control engineering.

In everyday life, as well as in solving engineering problems, the standard approach to complex problem solving is the divide-and-conquer strategy: A complex problem is somehow partitioned into a number of simpler sub-problems that can be solved independently, and whose individual solutions yield the solution of the original complex problem (Murray-Smith *et al*., 1997). This is the thought of multi-model control, which is fit to deal with the instance that the load operating points change in a wide range.

The application of fuzzy control and multi-model control pave the way for the research on the coordinated control system of boiler-turbine.

Takagi and Sugeno (1985) proposed a fuzzy model to describe the complex systems. On the basis of the idea, some fuzzy models based fuzzy control system design methods have appeared in the fuzzy control field (Tanaka *et al*., 2001). These methods combine the thought of fuzzy control with that of multi-model control. First, the nonlinear plant is represented by a Takagi-Sugeno (T-S) type fuzzy model. In this type of fuzzy model, local dynamics in different state space regions are represented by linear models. The overall model of the system is obtained by fuzzy "blending" of these linear models through nonlinear fuzzy membership functions. Second, the control design is completed on the basis of the fuzzy model via the so-called parallel distributed compensation (PDC) scheme. The idea is that for each local linear model, a linear feedback control is designed. Finally, the resulting overall controller, which is nonlinear in general, is again a fuzzy "blending" of each individual linear controller (Ma *et al.*, 2000).

In the fore work of the author of this paper, the T-S fuzzy multi-variable integral control system was used to design the nonlinear coordinated control system for the power unit, in which the fuzzy observer was constructed to estimate the state in the system so as to resolve the problem that the fluid density ρ_f is difficult to measure usually (Luan *et al.*, 2006). In that work the fuzzy observer is based on the Augmentd Plant Observer, in which the estimation of the additional state $\boldsymbol{\eta}$ is unnecessary. In order to reduce the dimension of the control system, this paper proposes the T-S fuzzy multi-variable integral control system with the Original Plant Observer to construct the coordinated control system for the power unit.

2. THE MULTI-VARIABLE INTEGRAL CONTROL BASED ON T-S FUZZY MODEL

2.1 Description of Plant by The Fuzzy Dynamic Model

The fuzzy model proposed by Takagi and Sugeno is described by ***IF-THEN*** rules, which represent local linear input–output relations of a nonlinear system. The *i*th rule of the T-S fuzzy models are of the following forms

Plant Rule *i*

IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip},
THEN

$$\begin{aligned}\dot{\boldsymbol{x}}(t)&=\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{B}_i\boldsymbol{u}(t)\\ \boldsymbol{y}(t)&=\boldsymbol{C}_i\boldsymbol{x}(t)+\boldsymbol{D}_i\boldsymbol{u}(t)\\ i&=1,2,\ldots,r\end{aligned}\tag{1}$$

where M_{ij} is the fuzzy set and r is the number of ***IF-THEN*** rules. $\boldsymbol{x}(t)\in\mathbf{R}^n$ is the state vector, $\boldsymbol{u}(t)\in\mathbf{R}^m$ is the input vector, $\boldsymbol{y}(t)\in\mathbf{R}^q$ is the output vector, $\boldsymbol{A}_i\in\mathbf{R}^{n\times n}$, $\boldsymbol{B}_i\in\mathbf{R}^{n\times m}$, $\boldsymbol{C}_i\in\mathbf{R}^{q\times n}$ and $\boldsymbol{D}_i\in\mathbf{R}^{q\times m}$, and $z_1(t)$, …, $z_p(t)$ are the premise variables, which are measurable and denoted as $\boldsymbol{z}(t)=[z_1(t),\ldots,z_p(t)]$. It is assumed in this paper that the premise variables do not depend on the input variables.

2.2 Fuzzy Controller

The controller is designed to gain the control law $\boldsymbol{u}(t)$ so that the output vector $\boldsymbol{y}(t)$ can trace the reference input signal $\boldsymbol{r}(t)$. Integrating the tracing error $\boldsymbol{e}(t)=\boldsymbol{y}(t)-\boldsymbol{r}(t)$, we gain

$$\boldsymbol{\eta}(t)=\int_0^t\boldsymbol{e}(\tau)\mathrm{d}\tau=\int_0^t[\boldsymbol{y}(\tau)-\boldsymbol{r}(\tau)]\mathrm{d}\tau\tag{2}$$

where $\boldsymbol{r}(t)\in\mathbf{R}^q$, $\boldsymbol{r}(t)=\boldsymbol{r}_0 1(t)$, $1(t)$ denotes the unit step input.

Acting $\boldsymbol{\eta}$ as an additional state vector, we construct the augmented plant for the *i*th rules of the T-S fuzzy models as following

$$\begin{aligned}\dot{\boldsymbol{x}}(t)&=\boldsymbol{A}_i\boldsymbol{x}(t)+\boldsymbol{B}_i\boldsymbol{u}(t)\\ \dot{\boldsymbol{\eta}}(t)&=\boldsymbol{y}(t)-\boldsymbol{r}(t)=\boldsymbol{C}_i\boldsymbol{x}(t)+\boldsymbol{D}_i\boldsymbol{u}(t)-\boldsymbol{r}(t)\\ \boldsymbol{y}(t)&=\boldsymbol{C}_i\boldsymbol{x}(t)+\boldsymbol{D}_i\boldsymbol{u}(t)\\ i&=1,2,\ldots,r\end{aligned}\tag{3}$$

Then the augmented plant has the following form

Augmented Plant Rule *i*

IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip},
THEN

$$\begin{aligned}\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{\eta}}(t)\end{bmatrix}&=\begin{bmatrix}\boldsymbol{A}_i & \mathbf{0}\\ \boldsymbol{C}_i & \mathbf{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{\eta}(t)\end{bmatrix}+\begin{bmatrix}\boldsymbol{B}_i\\ \boldsymbol{D}_i\end{bmatrix}\boldsymbol{u}(t)-\begin{bmatrix}\mathbf{0}\\ \boldsymbol{r}(t)\end{bmatrix}\\ \boldsymbol{y}(t)&=\boldsymbol{C}_i\boldsymbol{x}(t)+\boldsymbol{D}_i\boldsymbol{u}(t)\\ i&=1,2,\ldots,r\end{aligned}\tag{4}$$

Given a pair of $\{\boldsymbol{x}(t),\boldsymbol{u}(t)\}$, the final outputs of the fuzzy systems are inferred as follows:

$$\begin{aligned}\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{\eta}}(t)\end{bmatrix}&=\frac{\sum_{i=1}^{r}w_i(\boldsymbol{z}(t))\left\{\begin{bmatrix}\boldsymbol{A}_i & \mathbf{0}\\ \boldsymbol{C}_i & \mathbf{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{\eta}(t)\end{bmatrix}+\begin{bmatrix}\boldsymbol{B}_i\\ \boldsymbol{D}_i\end{bmatrix}\boldsymbol{u}(t)-\begin{bmatrix}\mathbf{0}\\ \boldsymbol{r}(t)\end{bmatrix}\right\}}{\sum_{i=1}^{r}w_i(\boldsymbol{z}(t))}\\ &=\sum_{i=1}^{r}h_i(\boldsymbol{z}(t))\left\{\begin{bmatrix}\boldsymbol{A}_i & \mathbf{0}\\ \boldsymbol{C}_i & \mathbf{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{\eta}(t)\end{bmatrix}+\begin{bmatrix}\boldsymbol{B}_i\\ \boldsymbol{D}_i\end{bmatrix}\boldsymbol{u}(t)-\begin{bmatrix}\mathbf{0}\\ \boldsymbol{r}(t)\end{bmatrix}\right\}\end{aligned}\tag{5}$$

$$\begin{aligned}\boldsymbol{y}(t)&=\frac{\sum_{i=1}^{r}w_i(\boldsymbol{z}(t))\{\boldsymbol{C}_i\boldsymbol{x}(t)+\boldsymbol{D}_i\boldsymbol{u}(t)\}}{\sum_{i=1}^{r}w_i(\boldsymbol{z}(t))}\\ &=\sum_{i=1}^{r}h_i(\boldsymbol{z}(t))\{\boldsymbol{C}_i\boldsymbol{x}(t)+\boldsymbol{D}_i\boldsymbol{u}(t)\}\end{aligned}\tag{6}$$

where

$$\boldsymbol{z}(t)=[z_1(t),z_2(t),\cdots,z_p(t)],\tag{7}$$

$$w_i(\boldsymbol{z}(t))=\prod_{j=1}^{p}M_{ij}(z_j(t)),\tag{8}$$

$$h_i(\boldsymbol{z}(t))=\frac{w_i(\boldsymbol{z}(t))}{\sum_{j=1}^{r}w_j(\boldsymbol{z}(t))}\tag{9}$$

for all t. The term $M_{ij}(\bullet)$ is the grade of membership of $\boldsymbol{z}(t)$ in $M_{ij}(\bullet)$. Since

$$\begin{cases}\sum_{i=1}^{r}w_i(\boldsymbol{z}(t))>0,\\ w_i(\boldsymbol{z}(t))\geq 0,\qquad i=1,2,\ldots,r\end{cases}\tag{10}$$

we have

$$\begin{cases} \sum_{i=1}^{r} h_i(z(t)) = 1, \\ h_i(z(t)) \geq 0, \quad i = 1,2,\ldots,r \end{cases} \tag{11}$$

for all t.

If the original plant (1) is locally controllable, i.e., $(\boldsymbol{A}_i, \boldsymbol{B}_i)$ ($i = 1,2,\ldots,r$) is controllable and $\operatorname{rank}\begin{bmatrix} \boldsymbol{A}_i & \boldsymbol{B}_i \\ \boldsymbol{C}_i & \boldsymbol{D}_i \end{bmatrix} = n + q$, the fuzzy controller in the form of state feedback control law is constructed as

Control Rule *i*

IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip},

THEN $\boldsymbol{u}(t) = -[\boldsymbol{K}_i, \boldsymbol{K}_{1i}]\begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{\eta}(t) \end{bmatrix}$

$$i = 1,2,\ldots,r \tag{12}$$

by the means of PDC (Tanaka *et al*., 1992; Wang *et al*., 1995a; Wang *et al*., 1995b), where the premise variables have the same fuzzy sets as the fuzzy model.

The overall fuzzy regulator is represented by

$$\boldsymbol{u}(t) = -\sum_{i=1}^{r} h_i(z(t))[\boldsymbol{K}_i, \boldsymbol{K}_{1i}]\begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{\eta}(t) \end{bmatrix} \tag{13}$$

2.3 Fuzzy Observer

In the practice, sometimes the states of system are immeasurable or the cost of measuring the states is expensive. So it is necessary to construct the observer to reconstruct the needed states.

If the original plant (1) is locally observerable, i.e. $(\boldsymbol{A}_i, \boldsymbol{C}_i)$ is observerable, the fuzzy observer is obtained as

Original Plant Observer Rule *i*

IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip},

THEN

$$\begin{aligned} \dot{\hat{\boldsymbol{x}}}(t) &= \boldsymbol{A}_i\hat{\boldsymbol{x}}(t) + \boldsymbol{B}_i\boldsymbol{u}(t) + \boldsymbol{H}_i[\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)] \\ \boldsymbol{y}(t) &= \boldsymbol{C}_i\boldsymbol{x}(t) + \boldsymbol{D}_i\boldsymbol{u}(t) \\ & i = 1,2,\ldots,r \end{aligned} \tag{14}$$

by the means of PDC, which is same as the design of controller, where the premise variables have the same fuzzy sets as the fuzzy model.

The overall fuzzy observer is represented as follows:

$$\begin{aligned} \dot{\hat{\boldsymbol{x}}}(t) &= \frac{\sum_{i=1}^{r} w_i(z(t))\{\boldsymbol{A}_i\hat{\boldsymbol{x}}(t) + \boldsymbol{B}_i\boldsymbol{u}(t) + \boldsymbol{H}_i[\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)]\}}{\sum_{i=1}^{r} w_i(z(t))} \\ &= \sum_{i=1}^{r} h_i(z(t))\{\boldsymbol{A}_i\hat{\boldsymbol{x}}(t) + \boldsymbol{B}_i\boldsymbol{u}(t) + \boldsymbol{H}_i[\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t)]\} \end{aligned} \tag{15}$$

$$\begin{aligned} \hat{\boldsymbol{y}}(t) &= \frac{\sum_{i=1}^{r} w_i(z(t))\{\boldsymbol{C}_i\hat{\boldsymbol{x}}(t) + \boldsymbol{D}_i\boldsymbol{u}(t)\}}{\sum_{i=1}^{r} w_i(z(t))} \\ &= \sum_{i=1}^{r} h_i(z(t))\{\boldsymbol{C}_i\hat{\boldsymbol{x}}(t) + \boldsymbol{D}_i\boldsymbol{u}(t)\} \end{aligned} \tag{16}$$

In the presence of the fuzzy observer, the PDC fuzzy controller takes on the following form, instead of (13):

$$\begin{aligned} \boldsymbol{u}(t) &= -\frac{\sum_{i=1}^{r} w_i(z(t))[\boldsymbol{K}_i, \boldsymbol{K}_{1i}]\begin{bmatrix} \hat{\boldsymbol{x}}(t) \\ \boldsymbol{\eta}(t) \end{bmatrix}}{\sum_{i=1}^{r} w_i(z(t))} \\ &= -\sum_{i=1}^{r} h_i(z(t))[\boldsymbol{K}_i, \boldsymbol{K}_{1i}]\begin{bmatrix} \hat{\boldsymbol{x}}(t) \\ \boldsymbol{\eta}(t) \end{bmatrix} \end{aligned} \tag{17}$$

2.4 Closed-loop Control System

According to (5)(6)(15)(16)(17),

$$\begin{aligned} \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{\eta}}(t) \end{bmatrix} &= \sum_{i=1}^{r} h_i(z(t))\left\{\begin{bmatrix} \boldsymbol{A}_i & \boldsymbol{0} \\ \boldsymbol{C}_i & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{\eta}(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{B}_i \\ \boldsymbol{D}_i \end{bmatrix}\left\{-\sum_{j=1}^{r} h_j(z(t))[\boldsymbol{K}_j, \boldsymbol{K}_{1j}]\begin{bmatrix} \hat{\boldsymbol{x}}(t) \\ \boldsymbol{\eta}(t) \end{bmatrix}\right\} + \begin{bmatrix} \boldsymbol{E}_i & \boldsymbol{0} \\ \boldsymbol{F}_i & -\mathbf{I} \end{bmatrix}\begin{bmatrix} \boldsymbol{w}(t) \\ \boldsymbol{r}(t) \end{bmatrix}\right\} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\begin{bmatrix} \boldsymbol{A}_i & -\boldsymbol{B}_i\boldsymbol{K}_{1j} & -\boldsymbol{B}_i\boldsymbol{K}_j \\ \boldsymbol{C}_i & -\boldsymbol{D}_i\boldsymbol{K}_{1j} & -\boldsymbol{D}_i\boldsymbol{K}_j \end{bmatrix}\begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{\eta}(t) \\ \hat{\boldsymbol{x}}(t) \end{bmatrix} + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\begin{bmatrix} \boldsymbol{E}_i & \boldsymbol{0} \\ \boldsymbol{F}_i & -\mathbf{I} \end{bmatrix}\begin{bmatrix} \boldsymbol{w}(t) \\ \boldsymbol{r}(t) \end{bmatrix} \end{aligned} \tag{18}$$

$$\begin{aligned} \dot{\hat{\boldsymbol{x}}}(t) &= \sum_{i=1}^{r} h_i(z(t))\left\{\boldsymbol{A}_i\hat{\boldsymbol{x}}(t) + \boldsymbol{B}_i\left(-\sum_{j=1}^{r} h_j(z(t))[\boldsymbol{K}_j, \boldsymbol{K}_{1j}]\begin{bmatrix} \hat{\boldsymbol{x}}(t) \\ \boldsymbol{\eta}(t) \end{bmatrix}\right)\right. \\ &\quad \left. + \boldsymbol{H}_i\left(\sum_{j=1}^{r} h_j(z(t))\{\boldsymbol{C}_j\boldsymbol{x}(t) + \boldsymbol{D}_j\boldsymbol{u}(t) + \boldsymbol{F}_j\boldsymbol{w}(t)\} - \sum_{j=1}^{r} h_j(z(t))\{\boldsymbol{C}_j\hat{\boldsymbol{x}}(t) + \boldsymbol{D}_j\boldsymbol{u}(t)\}\right)\right\} \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))[\boldsymbol{H}_i\boldsymbol{C}_j, -\boldsymbol{B}_i\boldsymbol{K}_{1j}, \boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_j - \boldsymbol{H}_i\boldsymbol{C}_j]\begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{\eta}(t) \\ \hat{\boldsymbol{x}}(t) \end{bmatrix} \\ &\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))[\boldsymbol{H}_i\boldsymbol{F}_j \quad \boldsymbol{0}]\begin{bmatrix} \boldsymbol{w}(t) \\ \boldsymbol{r}(t) \end{bmatrix} \end{aligned} \tag{19}$$

are obtained. Combine (18) and (19), we obtain the following system representations:

$$\begin{aligned} \begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{\eta}}(t) \\ \dot{\hat{\boldsymbol{x}}}(t) \end{bmatrix} &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\begin{bmatrix} \boldsymbol{A}_i & -\boldsymbol{B}_i\boldsymbol{K}_{1j} & -\boldsymbol{B}_i\boldsymbol{K}_j \\ \boldsymbol{C}_i & -\boldsymbol{D}_i\boldsymbol{K}_{1j} & -\boldsymbol{D}_i\boldsymbol{K}_j \\ \boldsymbol{H}_i\boldsymbol{C}_j & -\boldsymbol{B}_i\boldsymbol{K}_{1j} & \boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_j - \boldsymbol{H}_i\boldsymbol{C}_j \end{bmatrix}\begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{\eta}(t) \\ \hat{\boldsymbol{x}}(t) \end{bmatrix} \\ &\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\begin{bmatrix} \boldsymbol{E}_i & \boldsymbol{0} \\ \boldsymbol{F}_i & -\mathbf{I} \\ \boldsymbol{H}_i\boldsymbol{F}_j & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{w}(t) \\ \boldsymbol{r}(t) \end{bmatrix} \end{aligned} \tag{20}$$

The non-singular transformation

$$\begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{\eta}(t) \\ \tilde{\boldsymbol{x}}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & -\mathbf{I} \end{bmatrix}\begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{\eta}(t) \\ \hat{\boldsymbol{x}}(t) \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{\eta}(t) \\ \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t) \end{bmatrix} \tag{21}$$

is employed, where $\tilde{\boldsymbol{x}}(t) \triangleq \boldsymbol{x}(t) - \hat{\boldsymbol{x}}(t)$ is observing error, then

$$\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{\eta}}(t)\\ \dot{\tilde{\boldsymbol{x}}}(t)\end{bmatrix}=\begin{bmatrix}\mathbf{I}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{I}&\mathbf{0}\\ \mathbf{I}&\mathbf{0}&-\mathbf{I}\end{bmatrix}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))\begin{bmatrix}\boldsymbol{A}_i&-\boldsymbol{B}_i\boldsymbol{K}_{\mathrm{I}j}&-\boldsymbol{B}_i\boldsymbol{K}_j\\ \boldsymbol{C}_i&-\boldsymbol{D}_i\boldsymbol{K}_{\mathrm{I}j}&-\boldsymbol{D}_i\boldsymbol{K}_j\\ \boldsymbol{H}_i\boldsymbol{C}_j&-\boldsymbol{B}_i\boldsymbol{K}_{\mathrm{I}j}&\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_j-\boldsymbol{H}_i\boldsymbol{C}_j\end{bmatrix}\right\}\begin{bmatrix}\mathbf{I}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{I}&\mathbf{0}\\ \mathbf{I}&\mathbf{0}&-\mathbf{I}\end{bmatrix}^{-1}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{\eta}(t)\\ \tilde{\boldsymbol{x}}(t)\end{bmatrix}$$
$$+\begin{bmatrix}\mathbf{I}&\mathbf{0}&\mathbf{0}\\ \mathbf{0}&\mathbf{I}&\mathbf{0}\\ \mathbf{I}&\mathbf{0}&-\mathbf{I}\end{bmatrix}\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))\begin{bmatrix}\boldsymbol{E}_i&\mathbf{0}\\ \boldsymbol{F}_i&-\mathbf{I}\\ \boldsymbol{H}_i\boldsymbol{F}_j&\mathbf{0}\end{bmatrix}\right\}\begin{bmatrix}\boldsymbol{w}(t)\\ \boldsymbol{r}(t)\end{bmatrix} \tag{22}$$

Further

$$\begin{bmatrix}\dot{\boldsymbol{x}}(t)\\ \dot{\boldsymbol{\eta}}(t)\\ \dot{\tilde{\boldsymbol{x}}}(t)\end{bmatrix}=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))\begin{bmatrix}\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_j&-\boldsymbol{B}_i\boldsymbol{K}_{\mathrm{I}j}&\boldsymbol{B}_i\boldsymbol{K}_j\\ \boldsymbol{C}_i-\boldsymbol{D}_i\boldsymbol{K}_j&-\boldsymbol{D}_i\boldsymbol{K}_{\mathrm{I}j}&\boldsymbol{D}_i\boldsymbol{K}_j\\ \mathbf{0}&\mathbf{0}&\boldsymbol{A}_i-\boldsymbol{H}_i\boldsymbol{C}_j\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{\eta}(t)\\ \tilde{\boldsymbol{x}}(t)\end{bmatrix}$$
$$+\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))\begin{bmatrix}\boldsymbol{E}_i&\mathbf{0}\\ \boldsymbol{F}_i&-\mathbf{I}\\ \boldsymbol{E}_i-\boldsymbol{H}_i\boldsymbol{F}_j&\mathbf{0}\end{bmatrix}\begin{bmatrix}\boldsymbol{w}(t)\\ \boldsymbol{r}(t)\end{bmatrix} \tag{23}$$

Therefore the T-S fuzzy multi-variable integral control system with state observer can be represented as

$$\begin{aligned}\dot{\boldsymbol{X}}(t)&=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))\boldsymbol{G}_{ij}\boldsymbol{X}(t)+\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))\boldsymbol{T}_{ij}\boldsymbol{U}(t)\\ &=\sum_{i=1}^{r}h_i(z(t))h_i(z(t))\boldsymbol{G}_{ii}\boldsymbol{X}(t)+2\sum_{i=1}^{r}\sum_{i<j}^{r}h_i(z(t))h_j(z(t))\frac{\boldsymbol{G}_{ij}+\boldsymbol{G}_{ji}}{2}\boldsymbol{X}(t)\\ &\quad+\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))\boldsymbol{T}_{ij}\boldsymbol{U}(t)\end{aligned} \tag{24}$$

where

$$\boldsymbol{X}(t)=\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{\eta}(t)\\ \tilde{\boldsymbol{x}}(t)\end{bmatrix},\ \boldsymbol{U}(t)=\begin{bmatrix}\boldsymbol{w}(t)\\ \boldsymbol{r}(t)\end{bmatrix},\ \boldsymbol{T}_{ij}=\begin{bmatrix}\boldsymbol{E}_i&\mathbf{0}\\ \boldsymbol{F}_i&-\mathbf{I}\\ \boldsymbol{E}_i-\boldsymbol{H}_i\boldsymbol{F}_j&\mathbf{0}\end{bmatrix},$$
$$\boldsymbol{G}_{ij}=\begin{bmatrix}\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_j&-\boldsymbol{B}_i\boldsymbol{K}_{\mathrm{I}j}&\boldsymbol{B}_i\boldsymbol{K}_j\\ \boldsymbol{C}_i-\boldsymbol{D}_i\boldsymbol{K}_j&-\boldsymbol{D}_i\boldsymbol{K}_{\mathrm{I}j}&\boldsymbol{D}_i\boldsymbol{K}_j\\ \mathbf{0}&\mathbf{0}&\boldsymbol{A}_i-\boldsymbol{H}_i\boldsymbol{C}_j\end{bmatrix}.$$

2.5 Stability Analysis

According to Theorem 1 in Tanaka *et al*. (1998), the stability condition for the above T-S fuzzy control system is obtained as:

THEOREM 1: The equilibrium of the continuous fuzzy control system described by (24) is globally asymptotically stable if there exists a common positive definite matrix $\boldsymbol{P}$ such that

$$\boldsymbol{G}_{ii}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\boldsymbol{G}_{ii}<\mathbf{0} \tag{25}$$

$$\left(\frac{\boldsymbol{G}_{ij}+\boldsymbol{G}_{ji}}{2}\right)^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}\left(\frac{\boldsymbol{G}_{ij}+\boldsymbol{G}_{ji}}{2}\right)<\mathbf{0} \tag{26}$$

$$i<j\quad \text{s.t.}\quad h_i\cap h_j\neq\varnothing \tag{27}$$

2.6 Separate Peroperty

According to (23), the characteristic equation of the closed-loop control system is obtained as

$$\det\left\{s\mathbf{I}-\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))\begin{bmatrix}\boldsymbol{A}_i-\boldsymbol{B}_i\boldsymbol{K}_j&-\boldsymbol{B}_i\boldsymbol{K}_{\mathrm{I}j}&\boldsymbol{B}_i\boldsymbol{K}_j\\ \boldsymbol{C}_i-\boldsymbol{D}_i\boldsymbol{K}_j&-\boldsymbol{D}_i\boldsymbol{K}_{\mathrm{I}j}&\boldsymbol{D}_i\boldsymbol{K}_j\\ \mathbf{0}&\mathbf{0}&\boldsymbol{A}_i-\boldsymbol{H}_i\boldsymbol{C}_j\end{bmatrix}\right\}=0 \tag{28}$$

then

$$\det\left[\begin{array}{c|c}\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))\begin{bmatrix}s\mathbf{I}-\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{K}_j&\boldsymbol{B}_i\boldsymbol{K}_{\mathrm{I}j}\\ -\boldsymbol{C}_i+\boldsymbol{D}_i\boldsymbol{K}_j&s\mathbf{I}+\boldsymbol{D}_i\boldsymbol{K}_{\mathrm{I}j}\end{bmatrix}&\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))\begin{bmatrix}-\boldsymbol{B}_i\boldsymbol{K}_j\\ -\boldsymbol{D}_i\boldsymbol{K}_j\end{bmatrix}\\ \hline \sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))[\mathbf{0}\quad\mathbf{0}]&\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))(s\mathbf{I}-\boldsymbol{A}_i+\boldsymbol{H}_i\boldsymbol{C}_j)\end{array}\right]=0 \tag{29}$$

Further,

$$\det\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))\begin{bmatrix}s\mathbf{I}-\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{K}_j&\boldsymbol{B}_i\boldsymbol{K}_{\mathrm{I}j}\\ -\boldsymbol{C}_i+\boldsymbol{D}_i\boldsymbol{K}_j&s\mathbf{I}+\boldsymbol{D}_i\boldsymbol{K}_{\mathrm{I}j}\end{bmatrix}\right\}$$
$$\bullet\det\left\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(z(t))h_j(z(t))(s\mathbf{I}-\boldsymbol{A}_i+\boldsymbol{H}_i\boldsymbol{C}_j)\right\}=0 \tag{30}$$

It is concluded from (30) that the dynamics of multi-variable integral control system based on T-S fuzzy model is independent to that of the fuzzy observer of original plant. This means that the designs of the control law and the observer can be carried out independently, and when they are used together in this way, the poles remain unchanged. Therefore the control system (24) remains the separation principle in the designs of linear systems.

3. THE COORDINATED CONTROL SYSTEM DESIGN FOR POWER UNIT

3.1 Boiler-turbine dynamics

The boiler-turbine model representing the power unit in this study is a 3rd order nonlinear dynamic developed by Bell and Åström in 1987 (Bell *et al*. 1987). The model is based on the basic conservation laws which govern the boiler operation while maintaining an emphasis on simpler structure. Parameter estimation was made from dynamic data measured from the Sydvenska Kraft AB Plant in Malmo, Sweden. The plant is oil-fired and the rated power is 160 MW. Although the model is of low order, it is capable of capturing the major dynamic behaviors of the real plant. The plant dynamics is given by (Chen *et al*., 2004)

$$\frac{\mathrm{d}p}{\mathrm{d}t}=-0.0018u_2p^{\frac{9}{8}}+0.9u_1-0.15u_3 \tag{31}$$

$$\frac{\mathrm{d}P_{\mathrm{O}}}{\mathrm{d}t}=(0.073u_2-0.016)p^{\frac{9}{8}}-0.1P_{\mathrm{O}} \tag{32}$$

$$\frac{\mathrm{d}\rho_f}{\mathrm{d}t}=\frac{(141u_3-(1.1u_2-0.19)p)}{85} \tag{33}$$

where p denotes drum pressure (kgf/cm^2), P_{O} denotes power output (MW), and ρ_{f} denotes fluid density (kg/cm^3). The inputs to the system are the valve positions for fuel flow u_1, steam control u_2, and feedwater flow u_3. These control inputs are subject to magnitude and rate saturations

$$\left|\frac{\mathrm{d}u_1}{\mathrm{d}t}\right| \le 0.007/\sec,\ 0 \le u_1 \le 1 \tag{34}$$

$$-2/\sec \le \frac{\mathrm{d}u_2}{\mathrm{d}t} \le 0.02/\sec,\ 0 \le u_2 \le 1 \tag{35}$$

$$\left|\frac{\mathrm{d}u_3}{\mathrm{d}t}\right| \le 0.05/\sec,\ 0 \le u_3 \le 1 \tag{36}$$

Another quantity of interest is the water level deviation X_{w}, which is given by

$$X_{\mathrm{w}} = 0.05\left(0.13073\rho_f + 100\alpha_s + \frac{q_e}{9} - 67.975\right) \tag{37}$$

where steam quantity α_{cs} and evaporation rate q_e (kg/s) are given by

$$\alpha_{cs} = \frac{(1-0.001538\rho_f)(0.8p-25.6)}{\rho_f(1.0394-0.0012304p)} \tag{38}$$

$$q_e = (0.854u_2 - 0.147)p + 45.59u_1 - 2.514u_3 - 2.096 \tag{39}$$

Table 1 shows a collection of operating points for the plant.

Table 1 Operating points for plant

	#1	*#2*	*#3*	*#4*	*#5*	*#6*	*#7*
x_1^0	75.60	86.40	97.20	108.0	118.8	129.6	140.4
x_2^0	25.01	36.65	50.52	66.65	85.06	105.8	128.9
x_3^0	299.6	342.4	385.2	428.0	470.8	513.6	556.4
u_1^0	0.156	0.209	0.271	0.340	0.418	0.505	0.600
u_2^0	0.483	0.552	0.621	0.690	0.759	0.828	0.897
u_3^0	0.183	0.256	0.340	0.435	0.543	0.663	0.793

3.2 Control System Design

It is justified to claim that the change rate of the drum pressure p is slow compared to that of the power output P_{O} and fluid density ρ_{f} (Chen *et al.*, 2004). Therefore the nonlinearities are determined by the drum pressure p, which is used as the premise variable, i.e. $z(t)=[p]$, while the linearized models at each operating point are used as the consequents, thus T-S fuzzy models are constructed. The fuzzy sets in the relevant rules are M_i, whose membership functions are shown in Fig.1 Based on the T-S fuzzy model, the multi-variable integral control system with fuzzy state observer is designed.

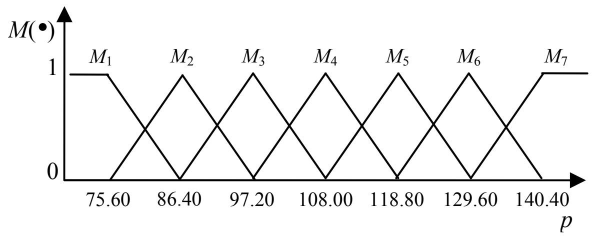

Fig. 1 Membership functions

Because the premise variable $z(t)=[p]$ is independent of the estimated variable $\hat{x}(t)$, the separation principle yet comes into existence, so it is able to obtain the gain of controller and the gain of observer separately (Tanaka *et al.*, 2001).

Each of $(\boldsymbol{A}_i, \boldsymbol{B}_i)$ is controllable, and $\mathrm{rank}\begin{bmatrix} \boldsymbol{A}_i & \boldsymbol{B}_i \\ \boldsymbol{C}_i & \boldsymbol{D}_i \end{bmatrix} = n + q$, i.e. the Augmented Plant (4) is local controllable, and each of $(\boldsymbol{A}_i, \boldsymbol{C}_i)$ is observable, i.e. the Original Plant (1) is local observable. Therefore the controllers and observers can be obtained by means of pole placement.

3.3 Stability Analysis

For this system, we can find a positive definite matrix

$$\boldsymbol{P} = \begin{bmatrix} 4.1913 & -1.3193 & 0.1175 & 0.2395 & -0.0907 & -0.4280 & 1.7160 & 0.3433 & 0.0644 \\ -1.3193 & 2.3120 & 0.0167 & -0.1175 & 0.0569 & -0.2581 & -0.3076 & -0.4120 & 0.0086 \\ 0.1175 & 0.0167 & 0.1587 & -0.0107 & -0.0005 & 0.2719 & -0.0626 & -0.0080 & 0.0509 \\ 0.2395 & -0.1175 & -0.0107 & 0.0329 & -0.0109 & 0.0271 & 0.1176 & 0.0279 & -0.0034 \\ -0.0907 & 0.0569 & -0.0005 & -0.0109 & 0.0061 & -0.0246 & -0.0561 & -0.0085 & 0.0001 \\ -0.4280 & -0.2581 & 0.2719 & 0.0271 & -0.0246 & 5.8661 & 0.6305 & 0.0270 & 0.0433 \\ 1.7160 & -0.3076 & -0.0626 & 0.1176 & -0.0561 & 0.6305 & 3.3880 & 0.0747 & -0.0421 \\ 0.3433 & -0.4120 & -0.0080 & 0.0279 & -0.0085 & 0.0270 & 0.0747 & 1.5962 & -0.0065 \\ 0.0644 & 0.0086 & 0.0509 & -0.0034 & 0.0001 & 0.0433 & -0.0421 & -0.0065 & 1.2110 \end{bmatrix} > \boldsymbol{0} \tag{40}$$

such that Theorem 1 is satisfied. Therefore the equilibrium of the fuzzy control system designed above is globally asymptotically stable.

3.4 Simulation

The performance of the designed above control systems is tested with the following inputs to the system (31-33) (Dimeo *et al.*, 1995):

$$r_1(t) = 108 + (120 - 108) \times 1(t - 200) \tag{41}$$

$$r_2(t) = 66.65 + (120 - 66.65) \times 1(t - 600) \tag{42}$$

$$r_3(t) = 0 \tag{43}$$

This reference change in pressure and power represents a relatively large demand change in both variables.

The curves, depicted in Fig.2, show that the response curves of the three output variables, i.e., drum pressure p, power output P_{O} and drum water level X_{w}, which indicate that T-S fuzzy coordinated control system with fuzzy state observer can make the output regulation error to asymptotically decay to zero.

4. CONCLUSION

In this paper, a new control system design methodology for a boiler-turbine plant is presented. The T-S fuzzy coordinated control system with the original plant observer is presented and its application to the coordinated control system design for the power unit is discussed. The stability analysis of control system is described by means of linear matrix inequalities. The simulation result shows that the T-S fuzzy coordinated control system can achieve good steady-state tracking.

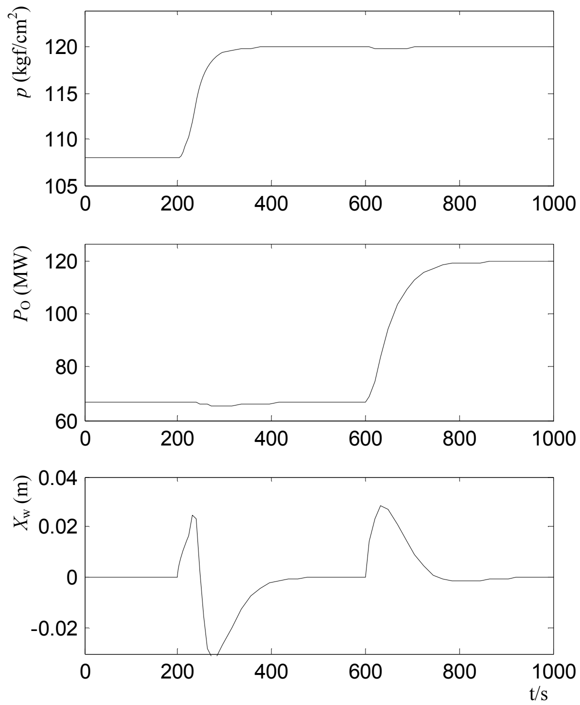

Fig.2 Curves of system output response

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