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Adaptive Control via Backstepping Technique and Neural Networks of a Quadrotor Helicopter

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Abstract: A nonlinear adaptive controller for the quadrotor helicopter is proposed using backstepping technique mixed with neural networks. The backstepping strategy is used to achieve good tracking of desired translation positions and yaw angle while maintaining the stability of pitch and roll angles simultaneously. The knowledge of all physical parameters and the exact model of the quadrotor are not required for the controller, only some properties of the model are needed. In fact, online adaptation of neural networks and some parameters is used to compensate some unmodeled dynamics including aerodynamic effects. Under certain relaxed assumptions, the proposed control scheme can guarantee that all the signals in the closed-loop system are Uniformly Ultimately Bounded (UUB). The design methodology is based on Lyapunov stability. One salient feature of the proposed approach is that the controller can be applied to any type of quadrotor helicopter of different masses and lengths within the same class. The feasibility of the control scheme is demonstrated through simulation results.

1. INTRODUCTION

The quadrotor helicopter is an ideal solution to fulfill the requirements of several civilian applications such as monitoring of traffic, recognition and surveillance of vehicles, search and rescue operations [1]. This kind of helicopter is simple and effective since the slope of the blades is not controlled. The quadrotor has some advantages over conventional helicopters. In fact, four rotors which generate a propeller forces are used to simplify the displacement. It has more lift thrust therefore it offers better payload. The controller system regulates the four speed of rotors, the slope of the helicopter on the right, on the left, ahead, behind, and rotation on itself.

Various advanced control techniques have recently been adopted to meet increasing demands on the quadrotor performance, like exact linearization [2], sliding mode control [3], and backstepping technique [4]. The latter seems to be more applicable for the quadrotor since it is an under-actuated system. The use of the classical backstepping controller approaches requires the exact expression of the dynamic model which may limit their practical utility. Numerous adaptive approaches are proposed to extend the applicability of the backstepping control technique [5]. The main problem with this approach is that the approximated functions must be linear in the unknown parameters, and some very tedious analysis is needed to determine regression matrices. The adaptive controller based on Neural Networks (NN) can be designed without significant prior knowledge of the system dynamics [6].

The aerodynamic effects are highly complicated and depend on many physical variables [1]. It is very difficult to identify them exactly. The difference between the mathematical model and the real system may cause performance degradation. To overcome this drawback for nonlinear quadrotor systems, a controller using adaptive backstepping and neural networks is proposed in this paper. We use adaptive multi-layer NN to estimate some unknown nonlinearity of the quadrotor helicopter model, including the aerodynamic effects in order to compensate them. The NN weights are tuned on-line with no learning phase required. The ignorance of all physical parameters of the quadrotor is also considered and they are estimated by other adaptation algorithms. Therefore, the controller does not require the exact knowledge of the dynamic model or the parameters of the system. This is a significant advantage since our controller can be applied to any type of quadrotor of different masses and lengths within the same class. The objective of our controller is to achieve good tracking of desired positions and yaw angle while maintaining the stability of pitch and roll angles.

The paper is outlined as follows. In section II, the dynamic model of the quadrotor helicopter is presented. Then, in section III, the closed-loop stability of the proposed controller is demonstrated. In section IV, some simulation results are carried out to show the efficiency of the controller. Finally, some conclusions are given in section V.

2. MODELING OF A QUADROTOR

The quadrotor, shown in figure 1, has four rotors to generate the propeller forces F_1, F_2, F_3 and F_4. Its configuration simplifies the displacement and increases the lift force. On varying the rotor speeds altogether with the same quantity, the lift forces will change, affecting in this case the altitude of the vehicle. The two pairs of rotors $(1, 3)$ and $(2, 4)$ turn in opposite directions in order to

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balance the moments and produce yaw motion as needed. Yaw angle is obtained by speeding up or slowing down the clockwise motors depending on the desired angle direction. The motion direction according to the horizontal plan depends on the sense of yaw angle and tilt angles (pitch and roll), whether they are positives or negatives.

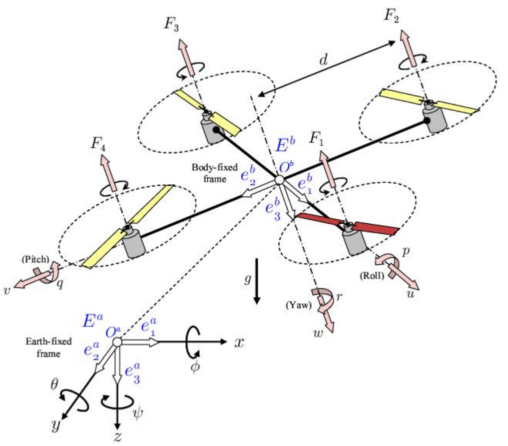

Fig. 1. Body-fixed frame and earth-fixed frame for the quadrotor.

The equations describing the altitude and the attitude motions of a quadrotor helicopter are basically those of a rotating rigid body with six degrees of freedom [7]. Let there be two main reference frames (see figure 1): the earth-fixed inertial reference frame $E^a(O^a, \overrightarrow{e_1^a}, \overrightarrow{e_2^a}, \overrightarrow{e_3^a})$ such that $\overrightarrow{e_3^a}$ denotes the vertical direction downwards into the earth and the body-fixed reference frame $E^b(O^b, \overrightarrow{e_1^b}, \overrightarrow{e_2^b}, \overrightarrow{e_3^b})$ fixed at the center of mass of the quadrotor. The absolute position of the quadrotor is described by $X = [x, y, z]^T$ and its attitude by the *Euler* angles $\Theta = [\psi, \theta, \phi]^T$, used corresponding to aeronautical convention. The attitude angles are respectively called *Yaw* angle (ψ rotation around z-axis), *Pitch* angle (θ rotation around y-axis) and *Roll* angle (ϕ rotation around x-axis). Let $V = [u, v, w]^T \in E^b$ denote the linear velocity and $\Omega = [p, q, r]^T \in E^b$ denote the angular velocity of the airframe expressed in the body-fixed-frame. The relation between the velocities vectors (V, Ω) and $(\dot{X}, \dot{\Theta})$ is given by

$$\begin{cases} V = R^T(\Theta)\dot{X} \Longrightarrow \dot{X} = R(\Theta)V \\ \Omega = M(\Theta)\dot{\Theta} \end{cases} \quad (1)$$

where $R(\Theta)$ and $M(\Theta)$ are respectively the transformation velocity and the rotation velocity matrices between E^a and E^b, such as [1]:

$$R(\Theta) = \begin{bmatrix} C_\psi C_\theta & C_\psi S_\theta S_\phi - S_\psi C_\phi & C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi C_\theta & S_\psi S_\theta S_\phi + C_\psi C_\phi & S_\psi S_\theta C_\phi - C_\psi S_\phi \\ -S_\theta & C_\theta S_\phi & C_\theta C_\phi \end{bmatrix}$$

and

$$M(\Theta) = \begin{bmatrix} -S_\theta & 0 & 1 \\ C_\theta S_\phi & C_\phi & 0 \\ C_\theta C_\phi & -S_\phi & 0 \end{bmatrix}$$

The derivation of (1) with respect to time gives

$$\begin{cases} \ddot{X} = R(\dot{V} + \Omega \times V) \\ \dot{\Omega} = M\ddot{\Theta} + (\frac{\partial M}{\partial \phi}\dot{\phi} + \frac{\partial M}{\partial \theta}\dot{\theta})\dot{\Theta} \end{cases} \quad (2)$$

Using *Newton's* laws in the body-fixed reference frame E^b, about the quadrotor subject to forces ΣF_{ext} and moments ΣT_{ext} applied to the center of mass, one can obtain the dynamic equation motions [2]:

$$\begin{cases} \Sigma F_{ext} = m\dot{V} + \Omega \times (mV) \\ \Sigma T_{ext} = J\dot{\Omega} + \Omega \times (J\Omega) \end{cases} \quad (3)$$

where m and J are respectively the mass and the total inertia matrix of quadrotor (considering the symmetry of the quadrotor structure, we can suppose that $J = diag[I_x, I_y, I_z]$ where $I_x = I_y = I_1$ and $I_z = I_2$), ΣF_{ext} and ΣT_{ext} include the external forces/torques developed in the center of mass of the quadrotor according to the direction of the reference frame E^b, such as:

$$\begin{cases} \Sigma F_{ext} = F_{prop} + F_{aero} + F_{grav} \\ \Sigma T_{ext} = T_{prop} + T_{aero} \end{cases} \quad (4)$$

where the forces $\{F_{prop}, F_{aero}, F_{grav}\}$ and the torques $\{T_{prop}, T_{aero}\}$ are explained in the table 1 with $e_3 = [0, 0, 1]^T$; g is the gravity; U is the velocity of the helicopter with respect to the air; $\{A_F(U) = [A_u(U), A_v(U), A_w(U)]^T$, $A_T(U) = [A_p(U), A_q(U), A_r(U)]^T\}$ are two complex nonlinear function vectors which represent respectively aerodynamic forces and torques; d is the distance from the center of mass to the rotor axes and $c > 0$ is the drag factor.

Model	Source
$F_{prop} = -\Sigma_{i=1}^4 F_i e_3$ $T_{prop} = \begin{bmatrix} d\ (F_2 - F_4) \\ d\ (F_1 - F_3) \\ c\ \Sigma_{i=1}^4\{(-1)^{i+1}F_i\} \end{bmatrix}$	Propeller system
$F_{aero} = -A_F(U)$ $T_{aero} = -A_T(U)$	Aerodynamic effect
$F_{grav} = mgR^T e_3$	Gravity effect

Table 1. Physical effects acting on a quadrotor expressed in the body-fixed reference frame

Using (3) and (4) the equation of the dynamics of rotation of the quadrotor, expressed in the reference frame E^a, will be:

$$\begin{cases} \ddot{X} = \frac{1}{m}R[F_{prop} - A_F(U)] + ge_3 \\ \ddot{\Theta} = (JM)^{-1}[\,T_{prop} - J(\frac{\partial M}{\partial \phi}\dot{\phi} + \frac{\partial M}{\partial \theta}\dot{\theta})\dot{\Theta} \\ \qquad -A_T(U) - (M\dot{\Theta}) \times (JM\dot{\Theta})] \end{cases} \quad (5)$$

The aerodynamic functions $A_i(U)$ are highly nonlinear and dependent on numerous physical variables such as the angle between airspeed and the body-fixed frame and geometric form of the helicopter. Generally, they are approximated from the non-dimensional coefficients C_i as $A_i(U) = \frac{1}{2}\rho C_i U^2$ where ρ is the air density [1].

3. CONTROLLER DESIGN

In this section, the controller for the quadrotor helicopter is proposed by using the backstepping technique, the NN

[1] the abbreviations $S_{(.)}$ and $C_{(.)}$ denotes respectively sin(.) and cos(.).

[2] the symbol $\times$ denotes the usual vector product.

universal estimator and the adaptive control technique. Our objective is to ensure the convergence of the positions $\{x(t), y(t), z(t), \psi(t)\}$ to the desired trajectories $\{x_d(t), y_d(t), z_d(t), \psi_d(t)\}$ respectively, and stabilize the pitch and the roll angles $\{\phi(t), \theta(t)\}$.

The control law is built in four steps. Firstly, we rewrite the dynamic model of a quadrotor in a state-space form suited for backstepping control design. Secondly, we use the NN to estimate some nonlinear functions of dynamic model. Thirdly, we propose six virtual control inputs and one real control input for a quadrotor. Finally, we perform an overall closed-loop stability analysis of the proposed adaptive controller.

3.1 State-space representation

The quadrotor model, developed in the section II, can be rewritten in a state-space form by using the following state vectors:

$$x_1 = \begin{bmatrix} x \\ y \end{bmatrix},\ x_3 = \begin{bmatrix} \phi \\ \theta \end{bmatrix},\ x_5 = \begin{bmatrix} z \\ \psi \end{bmatrix},\ x_7 = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix}$$
$$x_2 = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix},\ x_4 = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \end{bmatrix},\ x_6 = \begin{bmatrix} \dot{z} \\ \dot{\psi} \end{bmatrix}$$

Let $u = [\dot{F}_1, \dot{F}_2, \dot{F}_3, \dot{F}_4]^T$ be the considered control inputs and let $\{S_1, S_2, S_3\}$ denotes respectively an under-actuated subsystem, a fully-actuated subsystem and a propeller subsystem. The state-space equations for the whole system are given as follows:

$$\begin{aligned} S_1 &: \begin{cases} \dot{x}_1 = x_2 \\ D_0\dot{x}_2 = f_0(\chi_0) + g_0(x_5, x_7)\varphi_0(x_3) \\ \dot{x}_3 = x_4 \\ D_1\dot{x}_4 = f_1(\chi_1) + g_1(x_3)\varphi_1(x_7) \end{cases} \\ S_2 &: \begin{cases} \dot{x}_5 = x_6 \\ D_2\dot{x}_6 = f_2(\chi_2) + g_2(x_3)\varphi_2(x_7) \end{cases} \\ S_3 &: \{\ \dot{x}_7 = u \end{aligned} \tag{6}$$

where :

- The arguments χ_i are given by:

$$\begin{cases} \chi_0 = [\dot{x}, \dot{y}, \dot{z}, \psi, \theta, \phi]^T \\ \chi_1 = [\psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}, F_1, F_2, F_3, F_4]^T \\ \chi_2 = [\dot{x}, \dot{y}, \dot{z}, \psi, \theta, \phi, \dot{\psi}, \dot{\theta}, \dot{\phi}, F_1, F_2, F_3, F_4]^T \end{cases} \tag{7}$$

- The matrices $D_i = diag(\alpha_i)$ such as:

$$\alpha_0 = \begin{bmatrix} m \\ m \end{bmatrix}, \alpha_1 = \begin{bmatrix} I_1/d \\ I_1/d \end{bmatrix}, \alpha_2 = \begin{bmatrix} m \\ I_2/c \end{bmatrix}$$

- The matrices g_i are given by [3]:

$$\begin{aligned} g_0 = -\Sigma_{i=1}^4 F_i \begin{bmatrix} S_\psi & C_\psi \\ -C_\psi & S_\psi \end{bmatrix}, g_1 = \begin{bmatrix} 1 & S_\phi T_\theta \\ 0 & C_\phi \end{bmatrix}, \\ g_2 = \begin{bmatrix} -C_\phi C_\theta & 0 \\ 0 & C_\phi / C_\theta \end{bmatrix} \end{aligned} \tag{8}$$

- The vectors φ_i are:

$$\begin{aligned} \varphi_0 = \begin{bmatrix} S_\phi \\ C_\phi S_\theta \end{bmatrix},\ \varphi_1 = \begin{bmatrix} F_2 - F_4 \\ F_1 - F_3 \end{bmatrix}, \\ \varphi_2 = \begin{bmatrix} \Sigma_{i=1}^4 F_i \\ \Sigma_{i=1}^4 \{(-1)^{i+1} F_i\} \end{bmatrix} \end{aligned} \tag{9}$$

- The functions $f_0 = [f_x, f_y]^T$, $f_1 = [f_\phi, f_\theta]^T$ and $f_2 = [f_z, f_\psi]^T$ such as:

$$\begin{bmatrix} f_x \\ f_y \\ f_z \end{bmatrix} = -\frac{1}{m} R A_F (R^T \dot{X}) + g e_3 \tag{10}$$

$$\begin{aligned} \begin{bmatrix} f_\psi \\ f_\theta \\ f_\phi \end{bmatrix} = & -(JM)^{-1}[\, J(\frac{\partial M}{\partial \phi}\dot{\phi} + \frac{\partial M}{\partial \phi}\dot{\theta})\dot{\Theta} \\ & + A_F(M\dot{\Theta}) \\ & + (M\dot{\Theta}) \times (JM\dot{\Theta})] \\ & + \begin{bmatrix} \frac{d}{I_1}(S_\phi / C_\theta)\ (F_1 - F_3) \\ -\frac{c}{I_2} S_\phi \Sigma_{i=1}^4 \{(-1)^{i+1} F_i\} \\ \frac{c}{I_2} C_\phi T_\theta \Sigma_{i=1}^4 \{(-1)^{i+1} F_i\} \end{bmatrix} \end{aligned}$$

Let define the *Jacobian* matrices $J_0 = \frac{\partial \varphi_0(x_3)}{\partial x_3}$, $J_1 = \frac{\partial \varphi_1(x_7)}{\partial x_7}$ and $J_2 = \frac{\partial \varphi_2(x_7)}{\partial x_7}$, therefore:

$$\begin{aligned} J_0 = \begin{bmatrix} C_\phi & 0 \\ -S_\phi S_\theta & C_\phi C_\theta \end{bmatrix}, \\ J_1 = \begin{bmatrix} 0 & +1 & 0 & -1 \\ +1 & 0 & -1 & 0 \end{bmatrix},\ J_2 = \begin{bmatrix} +1 & +1 & +1 & +1 \\ +1 & -1 & +1 & -1 \end{bmatrix}. \end{aligned} \tag{11}$$

One can synthesize the control law forcing the states x_1 and x_5 of a quadrotor to follow the desired trajectory $x_{1d}(t) = [x_d(t), y_d(t)]^T$ and $x_{5d}(t) = [z_d(t), \psi_d(t)]^T$ by using the backstepping technique. In this purpose, the following assumptions are needed:

A1: The signals X, Θ, $\dot{X}$ and $\dot{\Theta}$ can be measured or estimated by on-board sensors.
A2: The velocities $\dot{X}$, $\dot{\Theta}$ and the forces F_i $(i = 1, \ldots, 4)$ are bounded.
A3: The yaw, pitch and roll angels are limited to $(-\pi < \psi < \pi)$, $(-\frac{\pi}{2} < \theta < \frac{\pi}{2})$ and $(-\frac{\pi}{2} < \phi < \frac{\pi}{2})$.

According to the assumptions, it should be noted that the arguments (7) are bounded and all matrices given in (8) are invertible. Moreover, the matrix J_0 given in (11) is invertible.

3.2 Neural network approximations

The considered approximate function is a Multi-Layer Perceptron (MLP) Neural Network (NN) with one hidden layer having N artificial neurons and the output is linear. The approximation of the C^1 non-linear function $f : \Re^n \to \Re^m$ have the structure $w_1^T \sigma(w_2^T \chi) \in \Re^m$ where $\chi \in \Re^n$ is the input vector signal of the neural network, $w_2 \in \Re^{n \times N}$ is the input-hidden layer weights of the neural network, $w_1 \in \Re^{N \times m}$ is the hidden-output weights of the neural network and $\sigma : \Re^N \to \Re^N$ provides an activation function of the hidden neurons.

Assume that the nonlinear functions $f_i : \Re^{n_i} \to \Re^2$, $i = 0, 1, 2$ in (6) can be represented by 3-layer MLP neural networks having N_i artificial neurons in the hidden layers and some ideal constant weights $w_{i1} \in \Re^{N_i \times 2}$ and $w_{i2} \in \Re^{N_i \times n_i}$ for the input layer and the output layer respectively, i.e.,

$$f_i(\chi_i) = w_{i1}^T \sigma_i(w_{i2}^T \chi_i) + \epsilon_i(\chi_i) \tag{12}$$

[3] The abbreviation $T_{(.)}$ denotes tan(.) .

where $\|\epsilon_i(\chi_i)\| < \bar{\epsilon}_i(\chi_i)$, with a known and sufficiently small $\bar{\epsilon}_i(\chi_i) \in C^1$. The activation functions σ_i are of sigmoidal form.

The NN estimations of the functions f_i in (12) are given by:

$$\hat{f}_i(\chi_i) = \hat{w}_{i1}^T \sigma_i(\hat{w}_{i2}^T \chi_i) \tag{13}$$

where $\hat{w}_{i1}$ and $\hat{w}_{i2}$ are neural networks parameters which will be provided by an adaptation algorithm based on a stability analysis.

With regard to linear parameterized networks [8], the advantage of the MLP networks is the relatively reduced number of parameters. It is clear that this number depends on the dimension of the input, nevertheless this dependence is not exponential. The drawback of this type of networks is their non-linear parametrization. However, an alternative to treat these nonlinearities is to use development in *Taylor* series of functions $\sigma_i(w_{i2}^T \chi_i)$ around the estimated parameter $(\hat{w}_{i2}^T \chi_i)$. It can be written as follows [4]:

$$\sigma_i(w_{i2}^T \chi_i) = \sigma_i(\hat{w}_{i2}^T \chi_i) - \sigma_i'(\hat{w}_{i2}^T \chi_i)\tilde{w}_{i2}^T \chi_i - O_i(\hat{w}_{i2}^T \chi_i)$$

where $\sigma_i'(\hat{x}) = \left.\frac{\partial \sigma_i(x)}{\partial x}\right|_{x=\hat{x}}$, and $O_i(\hat{w}_{i2}^T \chi_i)$ represents terms of superior order, their values are:

$$\begin{aligned} O_i(\hat{w}_{i2}^T \chi_i) = & \left[\sigma_i(\hat{w}_{i2}^T \chi_i) - \sigma_i(w_{i2}^T \chi_i)\right] \\ & - \sigma_i'(\hat{w}_{i2}^T \chi_i)\tilde{w}_{i2}^T \chi_i \end{aligned} \tag{14}$$

It is well known that sigmoidally functions σ_i and their derivatives σ_i' are bounded, then for (14) we can determine the approximation error bounds with *Taylor* series, that are such as [5]:

$$\left\|O_i(\hat{w}_{i2}^T \chi_i)\right\| \le c_{i1} + c_{i2} \|\tilde{w}_{i2}\|_F \|\chi_i\| \tag{15}$$

where c_{i1} and c_{i2} are positive constants calculated from the expressions of σ_i and σ_i'.

The NN estimation error of the functions f_i, $(i = 0, 1, 2)$ are

$$\begin{aligned} \tilde{f}_i(\chi_i) &= \hat{f}_i(\chi_i) - f_i(\chi_i) \\ &= \hat{w}_{i1}^T \sigma_i(\hat{w}_{i2}^T \chi_i) - w_{i1}^T \sigma_i(w_{i2}^T \chi_i) - \epsilon_i(\chi_i) \end{aligned} \tag{16}$$

Adding and subtracting $w_{i1}^T \sigma_i(\hat{w}_{i2}^T \chi_i)$ in (16) comes

$$\begin{aligned} \tilde{f}_i(\chi_i) = & w_{i1}^T \left[\sigma_i(\hat{w}_{i2}^T \chi_i) - \sigma_i(w_{i2}^T \chi_i)\right] \\ & + \tilde{w}_{i1}^T \sigma_i(\hat{w}_{i2}^T \chi_i) - \epsilon_i(\chi_i) \end{aligned} \tag{17}$$

Now, adding and subtracting $\hat{w}_{i1}^T[\sigma_i(w_{i2}^T \chi_i) - \sigma_i(\hat{w}_{i2}^T \chi_i)]$ in (17)

$$\begin{aligned} \tilde{f}_i(\chi_i) = & \hat{w}_{i1}^T \left[\sigma_i(w_{i2}^T \chi_i) - \sigma_i(\hat{w}_{i2}^T \chi_i)\right] \\ & - \tilde{w}_{i1}^T \left[\sigma_i(w_{i2}^T \chi_i) - \sigma_i(\hat{w}_{i2}^T \chi_i)\right] \\ & + \tilde{w}_{i1}^T \sigma_i(\hat{w}_{i2}^T \chi_i) - \epsilon_i(\chi_i) \end{aligned} \tag{18}$$

Using the NN approximations (13) and development in the first order *Taylor* series of activation function, the equation (18) can be rewritten:

$$\begin{aligned} \tilde{f}_i(\chi_i) = & \tilde{w}_{i1}^T \left[\sigma_i(\hat{w}_{i2}^T \chi_i) + \sigma_i'(\hat{w}_{i2}^T \chi_i)\hat{w}_{i2}^T \chi_i\right] \\ & + \hat{w}_{i1}^T \sigma_i'(\hat{w}_{i2}^T \chi_i)\tilde{w}_{i2}^T \chi_i + e_i(\chi_i) \end{aligned} \tag{19}$$

where $e_i(\chi_i) = \epsilon_{\sigma_i}(\chi_i) - \epsilon_i(\chi_i)$ with

$$\epsilon_{\sigma_i}(\chi_i) = \tilde{w}_{i1}^T \sigma_i'(\hat{w}_{i2}^T \chi_i) w_{i2}^T \chi_i + w_{i1}^T O_i(\chi_i) \tag{20}$$

are disturbances due to the first order *Taylor* series approximations.

While using the *Frobenius* norm, we can write:

$$\begin{aligned} \|e_i(\chi_i)\| = & \|\epsilon_i(\chi_i)\| + c_{2i} \|\tilde{w}_{i1}\|_F \|w_{i2}\|_F \|\chi_i\| \\ & + \|w_{i1}\|_F \|O_i(\chi_i)\| \end{aligned} \tag{21}$$

3.3 Control design

The core concept of our design controller is like this: We treat x_2, $\varphi_0(x_3)$, x_4, $\varphi_1(x_7)$, x_6 and $\varphi_2(x_7)$ as six virtual control inputs, i.e., we use backstepping approach to design the virtual controllers $\{v_1, v_2, v_3, v_4, v_5, v_6\}$ for the signals $\{x_2, \varphi_0(x_3), x_4, \varphi_1(x_7), x_6, \varphi_2(x_7)\}$ respectively. Each v_i is designed with the aim to reduce the tracking error in the previous design step $(i - 1)$.

Using the same methodology shown in our work [4], the stabilization of the quadrotor can be obtained by using the following control law:

$$\begin{aligned} v_1 &= A_1 z_1 + \dot{x}_{1d} \\ v_2 &= g_0^{-1}[z_1 + A_2 z_2 - \hat{f}_0 + diag(\dot{v}_1)\hat{\alpha}_0] \\ v_3 &= J_0^{-1}[g_0^T z_2 + A_3 z_3 + \dot{v}_2] \\ v_4 &= g_1^{-1}[J_0^T z_3 + A_4 z_4 - \hat{f}_1 + diag(\dot{v}_3)\hat{\alpha}_1] \\ v_5 &= A_5 z_5 + \dot{x}_{5d} \\ v_6 &= g_2^{-1}[z_5 + A_6 z_6 - \hat{f}_2 + diag(\dot{v}_5)\hat{\alpha}_2] \\ u &= \begin{bmatrix} J_1 \\ J_2 \end{bmatrix}^{-1} \left(\begin{bmatrix} g_1 & 0_{2\times2} \\ 0_{2\times2} & g_2 \end{bmatrix}^T \begin{bmatrix} z_4 \\ z_6 \end{bmatrix} + \begin{bmatrix} \dot{v}_4 \\ \dot{v}_6 \end{bmatrix} + A_7 z_7 \right) \end{aligned} \tag{22}$$

where:

$$z_1 = (x_{1d} - x_1),\ z_2 = (v_1 - x_2),\ z_3 = (v_2 - \varphi_0(x_3)),$$
$$z_4 = (v_3 - x_4),\ z_5 = (x_{5d} - x_5),\ z_6 = (v_5 - x_6),$$
$$z_7 = \begin{bmatrix} v_4 - \varphi_1(x_7) \\ v_6 - \varphi_2(x_7) \end{bmatrix},$$

and the adaptation laws of $\{\hat{f}_0, \hat{f}_1, \hat{f}_2\}$ and $\{\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2\}$ are given in the following section.

3.4 Closed loop stability

We will perform a detailed treatment of stability of the proposed backstepping controller. Using *Lyapunov* stability theory we will carry out the stability analysis.

Let define [6]

$$\begin{gathered} Z = \left[z_1^T, \ldots, z_7^T\right]^T \in \Re^{16}, \\ A = diag(A_1, \ldots, A_7) \in \Re^{16\times16}, \\ D = diag(I_{2\times2}, D_0, I_{2\times2}, D_1, I_{2\times2}, D_2, I_{4\times4}) \in \Re^{16\times16}, \\ \tilde{F} = \begin{bmatrix} 0_{2\times1} \\ \tilde{f}_0 \\ 0_{2\times1} \\ \tilde{f}_1 \\ 0_{2\times1} \\ \tilde{f}_2 \\ 0_{4\times1} \end{bmatrix} \in \Re^{16},\ K = \begin{bmatrix} 0_{2\times1} \\ diag(\dot{v}_1)\tilde{\alpha}_0 \\ 0_{2\times1} \\ diag(\dot{v}_3)\tilde{\alpha}_1 \\ 0_{2\times1} \\ diag(\dot{v}_5)\tilde{\alpha}_2 \\ 0_{4\times1} \end{bmatrix} \in \Re^{16} \end{gathered} \tag{23}$$

[4] $\widetilde{(.)} = \widehat{(.)} - (.)$

[5] the symbol $\|.\|_F$ denotes the *Frobenius* norm, i.e., given a matrix A, the *Frobenius* norm is given by $\|A\|_F = \sum_{i,j} a_{ij}^2$

[6] The notations $I_{n\times n}$ and $0_{n\times m}$ denote respectively the $(n \times n)$ identity matrix and the $(n \times m)$ null matrix.

The error dynamics of the closed loop system using the controller (22) can be expressed in terms of the above quantities as

$$D\dot{Z} = -AZ + MZ + \tilde{F} - K \tag{24}$$

where $M \in \Re^{16\times16}$ is skew-symmetric matrix given in [4].

Let W, $\hat{W}$ and $\tilde{W}$ matrices that contain respectively, all parameters, $w_{\{i1,i2\}}$, all estimations $\hat{w}_{\{i1,i2\}}$, and all parameter errors $\tilde{w}_{\{i1,i2\}}$, such as

$$\begin{cases} W = diag(w_{01}, w_{02}, w_{11}, w_{12}, w_{21}, w_{22}) \\ \hat{W} = diag(\hat{w}_{01}, \hat{w}_{02}, \hat{w}_{11}, \hat{w}_{12}, \hat{w}_{21}, \hat{w}_{22}) \\ \tilde{W} = diag(\tilde{w}_{01}, \tilde{w}_{02}, \tilde{w}_{11}, \tilde{w}_{12}, \tilde{w}_{21}, \tilde{w}_{22}) \end{cases} \tag{25}$$

The following assumptions are needed for the analysis:

A4: The ideal weights are bounded by known positive values $W_{\max}$ so that $\|W\|_F \le W_{\max}$ which are quite common in the neural networks literature [6].

A5: The desired trajectories $x_{\{1d,5d\}}$, the virtual control inputs $v_{\{1,\dots,6\}}$ and its derivatives are bounded.

By using the assumptions **A2**, **A3**, **A4** and manipulating (15) and (21) we can write

$$\|E\| \le \delta_1 + \delta_2 ||\tilde{W}||_F \tag{26}$$

where $E = \left[0_{1\times2}, e_0^T, 0_{1\times2}, e_1^T, 0_{1\times2}, e_2^T, 0_{1\times4}\right]^T \in \Re^{16}$ is the estimation error vector and $\{\delta_1, \delta_2\}$ are the positive constants.

Theorem 1. Suppose assumptions **A1**, **A2**, **A3**, **A4** and **A5** are satisfied. Take the control law (22). Using the NN estimations given by (13) of the functions f_0, f_1 and f_2 with NN tuning be provided by

$$\begin{cases} \dot{\hat{w}}_{i1} = -\Gamma_{i1}\{\, k\,\|Z\|\,\hat{w}_{i1} \\ \qquad\qquad + \left[\sigma_i(\hat{w}_{i2}^T\chi_i) + \sigma_i'(\hat{w}_{i2}^T\chi_i)\hat{w}_{i2}^T\chi_i\right] z_i^T\,\} \\ \dot{\hat{w}}_{i2} = -\Gamma_{i2}\{k\,\|Z\|\,\hat{w}_{i2} + \chi_i z_i^T \hat{w}_{i1}^T \sigma_i'(\hat{w}_{i2}^T\chi_i)\} \end{cases} \tag{27}$$

for $i = 0, 1, 2$ with constant symmetric positive matrices $\{\Gamma_{i1}, \Gamma_{i2}\}$ and scalar positive constant k, and using the following adaptive estimation laws of α_0, α_1 and α_2:

$$\begin{cases} \dot{\hat{\alpha}}_0 = \Lambda_0 diag(\dot{v}_1) z_2 \\ \dot{\hat{\alpha}}_1 = \Lambda_1 diag(\dot{v}_3) z_4 \\ \dot{\hat{\alpha}}_2 = \Lambda_2 diag(\dot{v}_5) z_6 \end{cases} \tag{28}$$

where Λ_i $(i = 0, 1, 2)$ are positive definite matrices. Then the tracking error Z, the NN weight estimates $\hat{W}$ and the estimation errors $\tilde{\alpha} = [\tilde{\alpha}_0^T, \tilde{\alpha}_1^T, \tilde{\alpha}_2^T]^T$ are Uniformly Ultimately Bounded (UUB). The error Z can be kept as small as possible by increasing gains A.

Proof 1. Let the *Lyapunov* function candidate of the whole system [7]

$$V = \frac{1}{2}\left\{Z^T D Z + tr(\tilde{W}^T\Gamma^{-1}\tilde{W}) + \tilde{\alpha}^T\Lambda^{-1}\tilde{\alpha}\right\} \tag{29}$$

where $\Gamma = diag(\Gamma_{01}, \Gamma_{02}, \Gamma_{11}, \Gamma_{12}, \Gamma_{21}, \Gamma_{22})$ and $\Lambda = diag(\Lambda_0, \Lambda_1, \Lambda_2)$. Differentiating (29) and using (24), (27) and (28) gives

$$\begin{aligned} \dot{V} &= Z^T D\dot{Z} + tr(\tilde{W}^T\Gamma^{-1}\dot{\tilde{W}}) + \tilde{\alpha}^T\Lambda^{-1}\dot{\tilde{\alpha}} \\ &= -Z^T AZ - k\,\|Z\|\, tr[\tilde{W}^T(\tilde{W} + W)] + Z^T E \end{aligned} \tag{30}$$

[7] the function $tr(.)$ denotes the trace matrix, i.e., given a matrix A, the trace function is given by $tr(A) = \sum_i a_{ii}$

Applying the Schwartz inequality [9] to (30) we obtain [8]

$$\begin{aligned} tr[\tilde{W}^T(\tilde{W} + W)] &= ||\tilde{W}||_F^2 - \left\langle \tilde{W}, W \right\rangle_F \\ &\ge ||\tilde{W}||_F^2 - ||\tilde{W}||_F ||W||_F \end{aligned} \tag{31}$$

Then, we have

$$\begin{aligned} \dot{V} \le &-\lambda_{\min}\|Z\|^2 - k\,\|Z\|\,(||\tilde{W}||_F^2 - ||\tilde{W}||_F||W||_F) \\ &+ \|Z\|\,(\delta_1 + \delta_2||\tilde{W}||_F) \end{aligned} \tag{32}$$

where $\lambda_{\min}$ is the minimum eigenvalue of the matrix A. We can rewrite (32) as

$$\begin{aligned} \dot{V} \le -\|Z\|\,[\,&\lambda_{\min}\|Z\| + k(||\tilde{W}||_F^2 - ||\tilde{W}||_F W_{\max}) \\ &-(\delta_1 + \delta_2||\tilde{W}||_F)] \end{aligned} \tag{33}$$

which is negative as long as the term in square bracket in (33) is positive, i.e.:

$$\begin{aligned} &\lambda_{\min}\|Z\| + k(||\tilde{W}||_F^2 - ||\tilde{W}||_F W_{\max}) \\ &-(\delta_1 + \delta_2||\tilde{W}||_F) \ge 0 \end{aligned} \tag{34}$$

that is equivalent to

$$\begin{aligned} \lambda_{\min}\|Z\| + k[||\tilde{W}||_F - &\frac{(W_{\max} + \frac{\delta_2}{k})}{2}]^2 \\ -[\delta_1 + &\frac{(W_{\max} + \frac{\delta_2}{k})^2}{4}] \ge 0 \end{aligned} \tag{35}$$

this is true as long as

$$\|Z\| \ge \frac{1}{\lambda_{\min}}[\delta_1 + \frac{(W_{\max} + \frac{\delta_2}{k})^2}{4}] \tag{36}$$

or

$$||\tilde{W}||_F \ge \frac{(W_{\max} + \frac{\delta_2}{k})}{2} + \sqrt{\frac{1}{k}[\delta_1 + \frac{(W_{\max} + \frac{\delta_2}{k})^2}{4}]} \tag{37}$$

Thus, $\dot{V}$ is negative outside a compact set defined by (36) and (37). According to a standard *Lyapunov* theorem extension [10], this demonstrates the UUB of both Z, $\hat{W}$ and $\tilde{\alpha}$.

4. SIMULATION RESULTS

In this section, we will verify the effectiveness of the proposed controller by simulation of a quadrotor with the following parameters: $m = 2kg$, $d = 0.2m$, $c = 0.01m$, $I_x = I_y = I_z/2 = 1.2416 Nm.s^2/rad$, $C_u = C_v = C_w = 10^{-2}$ and $C_p = C_q = C_r = 10^{-3}$. Initially, the helicopter is in hover flight and charged by $m_0 = 250g$ payload mass. The initial conditions are: $x_i(0) = [0, 0]^T$ for $i = 1, \dots, 6$ and $x_7(0) \approx [0, 0, 0, 0]^T$.

The used controller parameters are: $A_1 = 10I$, $A_2 = 20I$, $A_3 = 30I$, $A_4 = 40I$, $A_5 = diag[100, 10]$, $A_6 = diag[120, 20]$, $A_7 = 10I$, $k = 0.01$, $\Gamma_{01} = \Gamma_{11} = \Gamma_{21} = 15I$, $\Gamma_{02} = \Gamma_{12} = \Gamma_{22} = 15I$, $\Lambda_0 = \Lambda_1 = 10^5 I$ and $\Lambda_2 = diag[10, 10^5]$. The initial conditions of all adaptive parameters are null. The number of neurons used in each of the hidden layer NNs is 15. We used sigmoidal activation function $\sigma(x) = 1/(1 + e^{-x})$.

For reason of derivations of the virtual controls, the desired trajectories are chosen in a manner to avoid initial conditions problem. So the chosen reference trajectories for $x_d(t)$, $y_d(t)$, $z_d(t)$ and $\psi_d(t)$ are that of the step response

[8] the notation $\langle .,. \rangle_F$ denotes the *Frobenius* norm product, i.e., given a appropriate matrices A and B, the *Frobenius* norm product is given by $\langle A, B\rangle_F = tr(A^T B)$

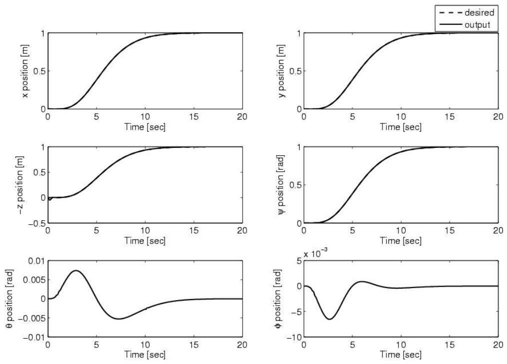

Fig. 2. Position outputs.

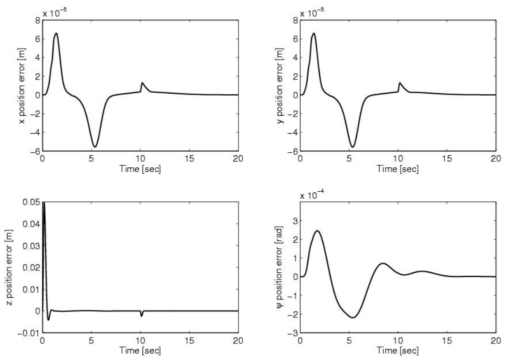

Fig. 3. Tracking errors.

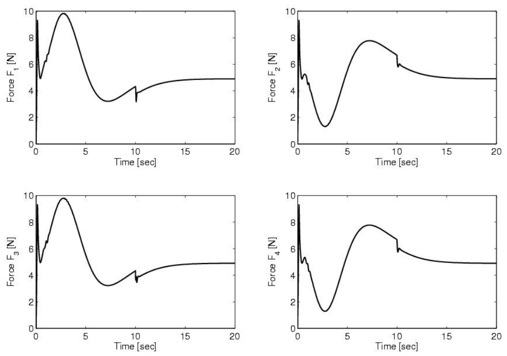

Fig. 4. Propeller forces.

of the transfer function defined by $1/(s+1)^6$ where s is the *Laplace* variable to make it smooth in curve and zero initial conditions before exciting the system.

The robustness test via a sudden fall of m_0 payload mass at $t = 10sec$ is simulated. Figure 2 shows the desired trajectories and the output positions of the quadrotor. Figure 3 shows the position tracking errors. It can be seen from these figures the good positions tracking of $\{x, y, z, \psi\}$ and the stabilization of the tilt angles $\{\phi, \theta\}$. Moreover, the tracking errors converge towards a vicinity of zero after a short time of adaptation. We can also notice that the robustness of the controller under payload changes is guaranteed. The helicopter is stable in spite of the sudden mass change perturbation. The figure 4 shows the obtained propeller forces. It should be noted from this figure that the control signals are acceptable and physically realizable.

5. CONCLUSION

A nonlinear adaptive controller for a six-degree-of-freedom quadrotor helicopter is proposed, and its stability is analyzed by using the *Lyapunov* theory. Although the behavior of the quadrotor, affected by aerodynamic forces and moments, is nonlinear and highly coupled, the adaptive backstepping technique mixed with neural networks, applied to the quadrotor, turns out to be a good starting point to avoid complex nonlinear control solutions. The dynamic model was divided into three interconnected subsystems to simplify the analysis. The proposed controller is able to stabilize the whole system and to drive the helicopter to desired trajectories. An adaptive neural networks algorithms and other adaptation laws are used respectively to compensate the aerodynamic effects and to estimate some physical parameters. The significant advantage of the proposed approach is that the dynamic model and all parameters are not required for the controller. Consequently, the controller can be applied to any type of quadrotor helicopter of different masses and lengths within the same class. Numerical simulations of the theoretical results show the good performance and the robustness of the proposed adaptive controller.

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