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An improved off-line approach for output feedback robust model predictive control

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Abstract: In this paper, we present a new off-line formulation of output feedback robust model predictive control for systems with polytopic uncertainty. The method is based on the use of several Lyapunov functions for each different vertex of the polytope. First we construct nested invariant ellipsoids and their corresponding state feedback gains off-line, therefore we are able to analyze closed-loop robust stability, and guarantee it by adjusting design parameters. On-line, we calculate control law by bisection search regarding to estimator state position between two adjacent ellipsoids in state space. Proposed algorithm reduces conservatism of available off-line methods due to using several Lyapunov functions and novel estimator design method. Moreover proposed technique is very efficient. The algorithm is illustrated with an example.

1. INTRODUCTION

Model predictive control (MPC) schemes have established themselves as the preferred control strategy for large number of industrial processes (Kouvaritakis & Canon, 2001). Their ability to handle constrained multivariable processes, and their technical feasibility are two main reasons for their popularity. MPC solves an optimization problem at each sampling time to calculate control law (Camacho & Bordons, 2004). Since exact model is seldom available, it is very important for MPC algorithms to be robust to model uncertainty.

RMPC methods mainly use a min-max optimization to guarantee that the system constraints are satisfied for all possible values of the unknown model parameters. Kothare *et al.* (1996), it was shown that infinite horizon state feedback RMPC problem can be formulated into LMIs. Via using LMIs, explicit plant uncertainty was explicitly incorporated in problem formulation. Later, Cuzzola *et al.* (2002) formulated RMPC for polytopic uncertain system in less conservative way, by using several Lyapunov functions each one corresponding to a different vertex of the polytope. One main drawback of Kothare *et al.* (1996) algorithm is that the on-line computational demand will grow significantly with problem size. Recently, this problem has been extensively investigated. Kouvaritakis *et al.* (2000), state feedback gain F is designed off-line to generate state predictions, and then on-line, only a summation of 2-norm of the perturbations on F is minimized. By setting control horizon M=1, Mayne *et al.* (2000) reduced on-line computation burden. Based on Kothare *et al.* (1996) formulation, Wan & Kothare, (2003) directly solved optimization problem off-line, so that N state feedback gains with corresponding nested ellipsoidal domains are constructed. Then On-line, the control law is calculated by using proper search method.

Most of existing approaches assume measurable states. Quite recently, some efforts were made to formulate output feedback RMPC problem (Mayne *et al.*, 2006; Wan & Kothare, 2003b). In most cases output feedback formulations use simple estimator design, and calculate control law based on estimator state.

To reduce conservatism of existing off-line methods, the technique described here is based on use of several Lyapunov functions for each different vertex of the polytope. This paper involves off-line controller and estimator design to reduce on-line computation burden. Firstly, controller is used to generate sequence of explicit control laws off-line, hence we are able to analyze closed-loop robust stability, and guarantee it by parameter adjustments. Finally, we calculate and implement control laws on-line based on estimator state using generated off-line control laws.

The paper is organized as follows. In section 2, we review a previous state feedback RMPC method and formulate it as an off-line approach. In section 3 we present off-line output feedback RPMC. In section 4 we illustrated the algorithm with an example.

Notations. For any vector x, and matrix ψ, $\| x \|_{\psi}^{T} = x^T \psi x$. The symbol * induces a symmetric structure in the matrix.

2. PROBLEM STATEMENT

2.1 Model of Uncertain System

Consider uncertain time-varying system described by the following state space equations

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$$x(k+1) = A(k)x(k) + B(k)u(k),$$
$$y(k) = Cx(k), \quad (1)$$
$$[A(k) \quad B(k)] \in \Omega,$$

where $u(k) \in \Re^{n_u}$ is the control input, $x(k) \in \Re^{n_x}$ is the state of the plant and $y(k) \in \Re^{n_y}$ is the plant output, and Ω is the polytope $Co\{[A_1\, B_1],...,[A_L\, B_L]\}$, where Co denotes the convex hull of $[A_i\, B_i]$ vertices. Any $[A\, B]$ within the convex set Ω , is a linear combination of the vertices

$$\sum_{j=1}^{L} w_j = 1, 0 \le w_j \le 1,$$
$$A = \sum_{j=1}^{L} w_j A_j, B = \sum_{j=1}^{L} w_j B_j \quad (2)$$

2.2 On-line RMPC

The purpose is to design a predictive controller, which robustly stabilizes uncertain system (1). Consider following problem, which minimizes the worst case infinite horizon quadratic cost index

$$\min_{u(k+i|k)} \max_{[A(k+i|k)|B(k+i|k)]\in\Omega, i\ge 0} J_\infty(k) = \sum_{i=0}^{\infty}[\| x(k+i\,|\,k)\|_{\mathcal{Q}}^2 + \| u(k+i\,|\,k)\|_{\mathcal{R}}^2], \quad (3)$$

s.t.
$$\| u(k+i\,|\,k)\|_2 \le u_{\max}, \quad i,k \ge 0, \quad (4)$$
$$\| y(k+i\,|\,k)\|_2 \le y_{\max}, \quad i,k \ge 0, \quad (5)$$
$$| u_j(k+i\,|\,k)| \le u_{j,\max}, \quad i,k \ge 0 \quad (6)$$

Where $\mathcal{Q} \ge 0$ and $\mathcal{R} > 0$ are the weighting matrices.

At each step k a state feedback control law

$$u(k+i\,|\,k) = F(k)x(k+i\,|\,k), i \ge 0\,, \quad (7)$$

is used to minimize cost index. To obtain LMI formulation of the problem, define the following quadratic function

$$V(i,k) = x(k+i\,|\,k)^T P(k) x(k+i\,|\,k), P(k) > 0, \forall k \ge 0. \quad (8)$$

As in Kothare *et al.*, (1996), impose the following robust stability constraint

$$V(i+1,k) - V(i,k) \le -\| x(k+i\,|\,k)\|_{\mathcal{Q}}^2 - \| u(k+i\,|\,k)\|_{\mathcal{R}}^2, \quad \forall [A(k+i) \quad B(k+i)] \in \Omega, i \ge 0. \quad (9)$$

For stable closed-loop system, $x(\infty\,|\,k) = 0$ and $V(\infty,k) = 0$. Summing (8) from 0 to ∞ obtains

$$\max_{[A(k+i|k)|B(k+i|k)]\in\Omega, i\ge 0} J_\infty(k) \le V(0,k) \le \gamma\,, \quad (10)$$

where $\gamma > 0$ is a scalar, and it represents upper bound of cost function (3).

Theorem 1. (On-line state feedback RMPC)
Consider the uncertain system (1);

(a) At step k, the state feedback matrix $F(k)$ in (6) is given by $F(k) = YG^{-1}$, where G and Y are obtained from solution of the following optimization problem

$$\min_{Y,G,Q_j} \gamma \quad (11)$$

s.t.

$$\forall j = 1,2,...,L, \quad \begin{bmatrix} G+G^T-Q_j & * & * & * \\ A_j G + B_j Y & Q_j & * & * \\ \mathcal{Q}^{1/2} G & 0 & \gamma I & * \\ \mathcal{R}^{1/2} G & 0 & 0 & \gamma I \end{bmatrix} > 0, \quad (12)$$

$$\forall j = 1,2,...,L, \quad \begin{bmatrix} 1 & x(k\,|\,k)^T \\ x(k\,|\,k) & Q_j \end{bmatrix} > 0. \quad (13)$$

(b) Optimization constraints are satisfied if following additional LMIs are met

- Bound on the Euclidean norm of the control input: Constraint (4) is satisfied if

$$\forall j = 1,2,...,L, \quad \begin{bmatrix} u_{\max}^2 I & Y \\ Y^T & G+G^T-Q_j \end{bmatrix} \ge 0. \quad (14)$$

- Bound on absolute value of the jth component of the control input: Constraint (5) is satisfied if

$$\forall j = 1,2,...,L, \quad \begin{bmatrix} X & Y \\ Y^T & G+G^T-Q_j \end{bmatrix} \ge 0, \quad e_j^T X e_j \le u_{j,\max}^2. \quad (15)$$

  where e_j is jth column of the identity matrix of dimension n_u.

- Bound on the Euclidean norm of the system output: Constraint (5) is satisfied if

$$\forall j = 1,2,...,L, \begin{bmatrix} G+G^T-Q_j & * \\ C(A_j G + B_j Y) & y_{\max}^2 I \end{bmatrix} \ge 0. \quad (16)$$

(c) If (10) has a solution, then at each step k

$$\max_{[A(k+i|k)|B(k+i|k)]\in\Omega, i\ge 0} \| x(k+i\,|\,k)\|_{Q_m}^2 < 1 \quad (17)$$

where the Q_m matrix can be obtained as the solution of following optimization problem

$$\max_{\beta, Q_m} \beta \quad (18)$$

s.t. $$\beta I < Q_m \le Q_j, \quad \forall j = 1,2,...,L. \quad (19)$$

Proof. (See Cuzzola *et al.*, (2002))

Lemma 1. (Asymptotically Invariant ellipsoid (Wan and Kothare (2003a)). A subset $\varepsilon = \{x \in \Re^{nx} \mid x^T Q^{-1} x \le 1\}$ is said to be an asymptotically stable invariant ellipsoid, if it has the property that whenever $x(k) \in \varepsilon$, then $x(k+i) \in \varepsilon, i > 0$, hence $x(k) \to 0$ as $k \to \infty$.

From Theorem 1, part (c), we know that Q_m defines a state-invariant ellipsoid.

2.3 Off-line RMPC

In this section based on on-line RMPC formulation Theorem 1, and inspired by (Wan & Kothare, 2003a), a new off-line state feedback RMPC is presented.

Algorithm 1. Consider system (1), subject to constraints (4), (5), and (6). Given an initial feasible state, let $i = 1$, and continue as follows

Off-line

1. Solve (10) with an additional constraint $Q_{j,i} < Q_{j,i-1}, \forall j = 1,2,...,L$ (ignored at $i = 1$) to obtain corresponding $Q_{j,i}, \forall j = 1,2,...,L$, G_i , Y_i , and feedback gains $F_i = Y_i G_i^{-1}$ .Solve (16), to obtain corresponding invariant ellipsoid $\varepsilon_i = \{x \in \Re^{nx} \mid x^T {Q_{m,i}}^{-1} x \le 1\}$, and store the values obtained in a look-up table;
2. If $i < N$, choose a state x_{i+1} satisfying $\| x_{i+1} \|^2_{Q^{-1}_{m,i}} \le 1$, let $i = i+1$, and go to step 1;
3. Obtained values at each step are valid if following inequality is satisfied.

$$Q^{-1}_{m,i} - (A_j + B_j F_{i+1})^T Q^{-1}_{m,i} (A_j + B_j F_{i+1}) > 0, \quad \forall j = 1,...,L. \tag{20}$$

On-line

1. Given an initial state $x(0)$ satisfying $\| x(0) \|^2_{Q^{-1}_{m,1}} \le 1$, set $x(k) = x(0)$;
2. Perform a bisection search over $Q^{-1}_{m,i}$ in the look-up table to find the largest index i (smallest invariant ellipsoid), such that $\| x(k) \|^2_{Q^{-1}_{m,i}} \le 1$ .Adopt the following feedback law

$$u(k) = \begin{cases} F(\alpha_i(k))x(k), & x(k) \in \varepsilon_i, x(k) \notin \varepsilon_{i+1}, i \ne N \\ F_N x(k), & x(k) \in \varepsilon_N \end{cases}, \tag{21}$$

where $F(\alpha_i(k)) = \alpha_i(k) F_i + (1 - \alpha_i(k)) F_{i+1}$ and $\alpha_i(k)$ is calculated from the equality

$$x(k)^T [\alpha_i(k) Q_i^{-1} + (1 - \alpha_i(k)) Q_{i+1}^{-1}] x(k) = 1 . \tag{22}$$

3. Set $k = k+1$, and go to step 2.

Theorem 2. Given the uncertain system (1), and a feasible initial state $x(0)$, Algorithm 1 robustly asymptotically stabilizes the closed loop system. Moreover, control law (20) in Algorithm 1 is continues function of the system state x .

Proof. For $x(k) \in \varepsilon_i$ and $Q_{j,i}, \forall j = 1,2,...,L$, G_i , Y_i satisfying optimization (10) conditions, then F_i is feasible and stabilizing. The additional condition $Q_{j,i} < Q_{j,i-1}$, $\forall j = 1,2,...,L$, results in $Q_{m,i} < Q_{m,i-1}$ (equivalently $Q^{-1}_{m,i-1} < Q^{-1}_{m,i}$), due to optimization (17). This implies that constructed asymptotically invariant ellipsoid ε_i have the property that $\varepsilon_i \subset \varepsilon_{i-1}$. Given an initial state $x(0)$ satisfying $x(0)^T Q^{-1}_{m,1} x(0) \le 1$ is given, the control law $u(k) = F_i x(k)$ corresponding to ellipsoid ε_i is guaranteed to keep the state within ε_i and converge it into the ellipsoid ε_{i+1} . Finally the smallest ellipsoid is guaranteed to keep the state within ε_N and converge it to the origin (Lemma 1). Satisfaction of condition (19) ensures system state x to be monotonically decreasing. □

3. OUTPUT FEEDBACK RMPC

3.1 Estimator Design

In many practical applications, only output variables are measured, or, not all state variables can be measured directly.

Following the leading of Wan & Kothare (2003b), the controller and the estimator are designed separately, and the interaction between them will be considered after design by testing the robust stability of the closed-loop system. We design a state estimator of the form

$$\hat{x}(k+1) = A_0 \hat{x}(k) + B_0 u(k) + L_{est}(y(k) - C_0 \hat{x}(k)), \forall k \ge 0 \tag{23}$$

where L_{est} is the estimator gain, and $\{A_0, B_0, C_0\}$ denote the nominal model. The error dynamics of the estimator are

$$e(k+1) = x(k+1) - \hat{x}(k+1) = (A_0 - L_0 C_0) e(k) + f(x(k), u(k)), \tag{24}$$

$\forall k \ge 0,$ with

$$f(x(k), u(k)) = [(A(k) - A_0) + (B(k) - B_0) u(k) - L_0 (C(k) - C_0)] x(k) . \tag{25}$$

The augmented closed-loop system for system (1) with state feedback $F\hat{x}(k)$ and estimator L_0 is

$$\Gamma(k+1) = A_{aug} \Gamma(k), \quad \Gamma = [\hat{x} \quad e]^T , \tag{26}$$

where

$$A_{aug}=\begin{bmatrix} A_0+B_0F+L_0(C(k)-C_0) & L_0C(k) \\ (A(k)-A_0)+(B(k)-B_0)F-L_0(C(k)-C_0) & A(k)-L_0C(k) \end{bmatrix}. \quad (27)$$

Despite Wan & Kothare (2003b), we consider all polytope vertices in designing the estimator to increase the robustness of output feedback implementation.
The following condition guarantees the stability of system

$$e(k)^T P^{-1} e(k) \le \rho^{2k} \quad , P>0, 0<\rho<1, \forall k \ge 0 \quad (28)$$

We take the term $f(.)$ in (22) as external signal.

$$\begin{aligned} e(k+1)^T P^{-1} e(k+1) &= \left\| [A(k)-L_0C(k)]^T e(k) \right\|_{P^{-1}} \\ &\le \rho^2 e(k)^T P^{-1} e(k), \end{aligned} \quad (29)$$

which is equivalent to

$$[A(k)-L_0C(k)]^T P^{-1}[A(k)-L_0C(k)]^T \le \rho^2 P^{-1}. \quad (30)$$

Let $Y=PL_0$, applying Schur complement, (29) can be written as following LMI constraints

$$P>0, \quad \begin{bmatrix} \rho^2 P & * \\ PA_j - YC & P \end{bmatrix} \ge 0 \quad , j=1,...,L. \quad (31)$$

The estimator gain is given by $L_0 = P^{-1}Y$.

3.2 Robust stability criteria

For output feedback implementation, the state feedback gain $F(k)$ in Algorithm 1 should be determined based on current estimator state $\hat{x}(k)$. $F(k)$ belongs to uncertain set $\Phi = Co\{F_1,F_2\} \cup ... \cup Co\{F_{N-1},F_N\}$.

Lemma 2. The augmented system (25), is stable if there exist symmetric positive definite matrices P_i and a matrix G such that for all vertices of Ω and all F_i in the set Φ

$$\begin{bmatrix} P_j & * \\ GA_{aug,i,j} & G+G^T-P_i \end{bmatrix} > 0 \quad , j=1,...,L, j=1,...,N. \quad (32)$$

Proof. If (30) is satisfied for all vertices of Ω and all F_i in the set Φ, then for an arbitrary plant $[A(k) \quad B(k)] \in \Omega$ and $F(k) \in \Phi$

$$\begin{bmatrix} P & * \\ GA_{aug} & G+G^T-P \end{bmatrix} > 0. \quad (33)$$

From De Oliveira *et al.* (1999) we know that (32) is robust stability condition of a discrete polytopic uncertain system, therefore any arbitrary plant within Ω and Φ is stable. □

Algorithm 2. (Off-line output feedback RMPC)
Obtain the look-up table $(Q_i, F_i, \varepsilon_i)$, using off-line part of Algorithm 1.

1. Specify decay rate ρ, and obtain a estimator gain satisfying (30).
2. Test robust stability condition in Lemma 2. If not satisfied, go back to step 2; if satisfied, continue implementing online part of Algorithm 1, based on estimator state.

4. NUMERICAL EXAMPLE

In this section we present an example to illustrate the implementation proposed algorithms. The simulations were done on PC with Pentium 4 processor (dual core 3GHz, 2MB of cache and 512MB total memory) in MATLAB environment, using YALMIP routine (Löfberg (2004)) as parser, and LMI control toolbox (Gahinet *et al.* (1995)) as solver of optimization problem.

In order to demonstrate effectiveness of proposed method, we revisit example reported in Kothare *et al.*, (1996). The system consist of a two-mass spring model whose discrete-time equivalent using Euler first order approximation with sampling time 0.1 is

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0.1 & 0 \\ 0 & 1 & 0 & 0.1 \\ -0.1\frac{K}{m_1} & 0.1\frac{K}{m_1} & 1 & 0 \\ 0.1\frac{K}{m_2} & -0.1\frac{K}{m_2} & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{0.1}{m_1} \\ 0 \end{bmatrix} u(k), \quad (34)$$
$$y = x_2,$$

where m_1 and m_2 are the two masses and K is the spring constant. The state variables x_1 and x_2 represent the position of the two masses while x_3 and x_4 are their velocities respectively.

Consider $m_1 = m_2 = 1$, control variable constraint $\|u(k)\| < 1$, initial condition $x_0 = [1,1,0,0]^T$, and weighting matrices $\mathcal{Q} = I$ and $\mathcal{R} = 1$. Take K as uncertainty parameter that belongs to the set $K \in [1, K_M]$. We consider following three cases for K_M value.

Case 1. We vary K_M values until Theorem 1 LMI constraints become infeasible. $K_M = 98.3$ is the maximum feasible value for on-line Theorem 1 method. Using Algorithm 1, we implement off-line state feedback RMPC for $K_M = 98.3$. The time history of state variable x_2 is reported in Fig. 1. Implemented control law of Algorithm 1 regulates the system in less than 0.08 second while on-line procedure of Theorem 1 take 14.16 minutes to stabilize system completely with the same K_M, therefore, Algorithm 1 method is more than 10000 times faster than Theorem 1 on-line technique. The time history of other states is omitted here for brevity. Technique proposed in Wan & Kothare, (2003), become infeasible for $K_M \geq 60.4$.

Case 2. To investigate performance of Algorithm 1, we set $K_M = 30$, and then run both on-line and off-line methods. Fig. 2 shows that the off-line Algorithm 1 gives nearly the same performance as on-line RMPC algorithm Theorem 1 for $K = 20$. The average regulation time for the off-line RMPC is 14×10^{-3}, which is more than 25000 time faster than the 4.5 minutes it takes for on-line RMPC.

Case 3. Consider $K_M = 27$ and $K = 24$ for nominal plant. We set the design parameter $\rho = 0.99$ .and then implement Algorithm 2 to test the robustness and efficiency proposed off-line output feedback method. Fig. 3 shows time history of the plant output. Algorithm 2 regulates the plant output in less than 0.04 second, while Wan & Kothare (2003b) method fails to control the plant for $K_M > 2.4$. The time histories of estimation error of x_1 and x_2 and the control signal of proposed technique are displayed in Fig. 4. To compare RMPC out feedback proposed in this paper with Wan & Kothare (2003b), we setup up both methods. Fig. 5 demontartes that Wan & Kothare (2003b) method is unstable for $K = 2.5$, while algorithm 2 stabilizes the system smoothly.

4. CONCULTIONS

For polytopic uncertain systems, less conservative, off-line state feedback and output feedback RMPC algorithms based on the use of several Lyapunov functions were developed. State feedback closed-loop stability is guaranteed based on the concept of asymptotically stable invariant ellipsoids. Output feedback closed-loop stability is ensured by off-line robust stability analysis using proposed stability criteria.

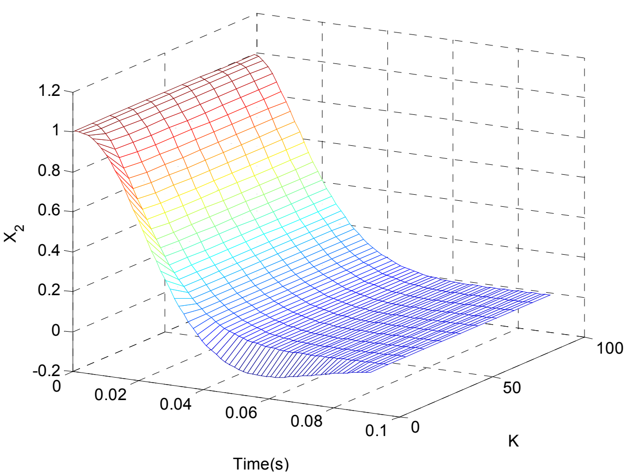

Fig. 1: Time history of x_2 for $1 \leq K \leq 98.3$. Off-line state feedback Algorithm 1 was applied.

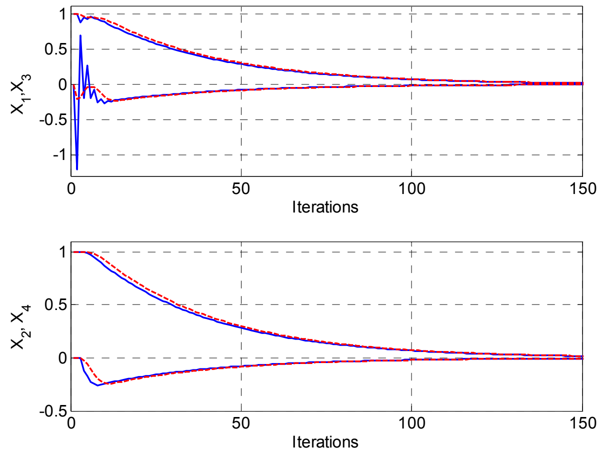

Fig. 2: Closed-loop responses for $K = 20$: dashed red lines, on-line RMPC in Theorem 1; solid blue line, off-line RMPC Algorithm 1.

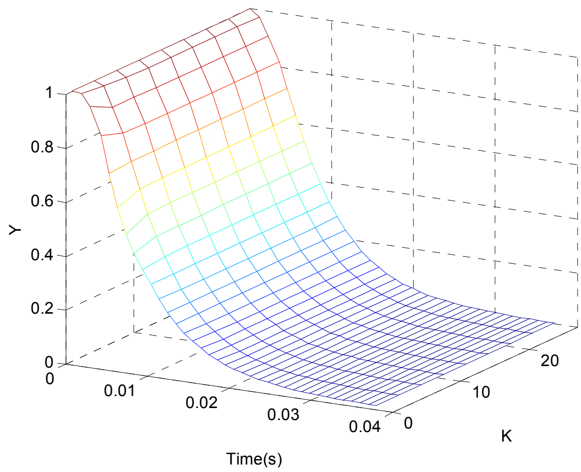

Fig. 3: Time history of y for $1 \leq K \leq 27$. Off-line output feedback Algorithm 2 was applied.

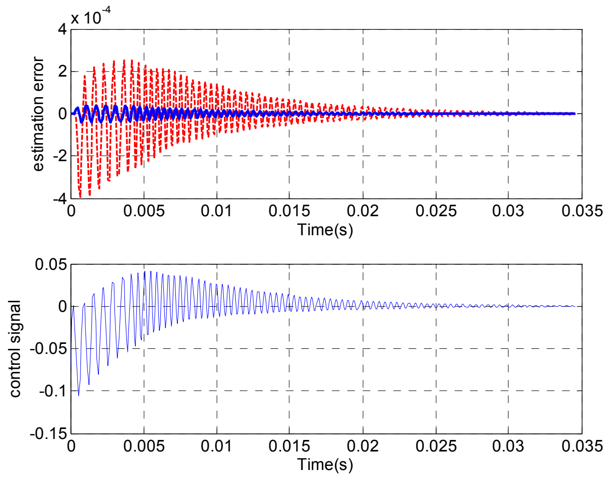

Fig. 4: Estimation error of x_1 (red dashed line) and x_2 (solid blue line), and the control signal for $K = 27$. Algorithm 2 was applied.

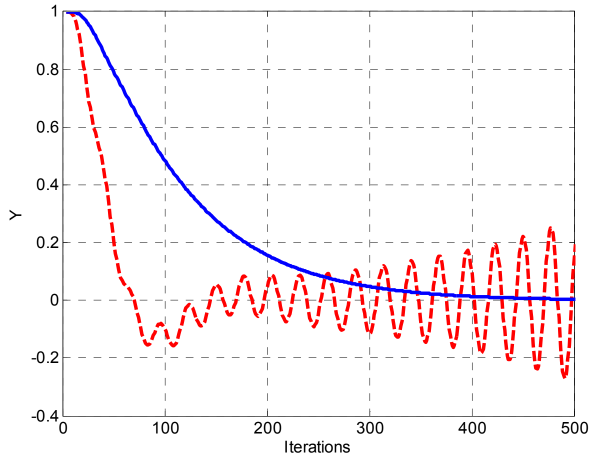

Fig. 5: Output performance profiles for $K = 2.5$: dashed red lines, output feedback RMPC algorithm of Wan & Kothare (2003b); solid blue line, output RMPC feedback Algorithm 2.

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