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Fuzzy Guaranteed Cost Control for Fuzzy Time Delay Systems ⋆

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Abstract: Based on T-S fuzzy model, the guaranteed cost control problem of a class of uncertain nonlinear systems with input and state time delay is discussed. By constructing state-feedback controller, a sufficient condition that for the given performance index and control law, the closed-loop system is quadratic guaranteed cost stable is presented and expressed in terms of linear matrix inequality (LMI). A numerical example shows that the proposed design method is effective.

1. INTRODUCTION

Since proposed by Zadeh(1965), fuzzy logic control has been developed into a conspicuous and successful branch of automation and control theory. With the development of fuzzy systems, some fuzzy control systems design methods have appeared in fuzzy control field. Among various kinds of fuzzy control methods, Takagi and Sugeno(1985) proposed a design and analysis method for overall fuzzy systems, in which the qualitative knowledge of a system was first represented by a set of local Takagi and Sugeno(T-S) fuzzy model. In this approach, the T-S fuzzy model substitutes the consequent fuzzy sets in a fuzzy rule by a linear equation of the input variables. Local dynamics in different state-space regions are represented by linear models and the overall model of the system is represented as the fuzzy interpolation of these linear models. Therefore, it has a convenient dynamic structure so that some well-established linear systems theory can be easily applied out based on the fuzzy model via the so-called parallel distributed compensation (PDC) scheme(Takagi and Sugeno,1992;H.O.Wang et al.1996). The idea is that for each local linear model, a linear feedback control is designed and the resulting overall controller, which is nonlinear in general, is fuzzy blending of each individual linear controller. Just because of this, T-S fuzzy model has attracted considerable attention and is widely used in the control design of nonlinear systems(B.S. Chen et al. 2000; M.C. Teixeira and S.H. Zak,1999;X. G. Yang et al. 2005). In traditional T-S models, there is no delay in the state and input. On the other hand, time-delay often occur in many dynamical systems such as rolling mill systems, biological systems, metallurgical processing systems, network systems, and so on. It is shown that the existence of delay usually becomes the source of instability and deteriorating performance of systems. In recent years, some authors have paid their attention to control of nonlinear systems with time-delay systems. In(Y. Y. Cao and P. M. Frank,2000) , the stability analysis and synthesis of nonlinear systems with time-delay systems via linear T-S fuzzy models was addressed. The Krasovskii-Lyapunov function was employed to develop a sufficient condition of delay-independent stability. And then, the fuzzy controller design schemes for state feedback were proposed in terms of linear matrix inequality (LMI). By using a quadratic Lyapunov functional approach instead of the Krasovskii-Lyapunov function approach, the similar work was done in (Y. Y. Cao and P. M. Frank,2001). The results of both (Y. Y. Cao and P. M. Frank,2000) and (Y. Y. Cao and P. M. Frank,2001) are obtained under the assumption that the systems contain no uncertainty. In (K. R. Lee et al. 2000), the problem of robust output feedback H_∞ control has been discussed for fuzzy dynamical systems with time-delay. Sufficient conditions for the existence of H_∞ controller are given by means of matrix inequalities. Recently, the reliable fuzzy control design approach for the T-S fuzzy model systems with time-delay was proposed in (B.S. Chen and X.P. Liu,2004). This paper mainly focuses on a class of uncertainty fuzzy control design problem for fuzzy control systems with time-delay. The robust stabilization schemes via state feedback and output feedback have been proposed by means of LMI.

In addition to the simple stabilization, there have been various efforts to assign certain performance criteria when designing a controller, such as quadratic cost minimization, H_∞ norm minimization, pole placement, etc. Among

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them, the guaranteed cost control aims at stabilizing the systems while maintaining an adequate level of performance represented by quadratic cost function(S. H. Esfahain and S.O. R. Moheimani,1998;S. H. Esfahain and I.R.Petersen,2000; Y. S. Lee et al. 2001;L. Yu and J. Chu,1999).Although it is an important problem to design a guaranteed cost controller for nonlinear systems, it seems that this field is still open to fuzzy control theory.

Unlike the results of above cited papers on T-S fuzzy systems with time-delay, in this paper, we mainly focus on the problem of fuzzy guaranteed cost control for T-S fuzzy systems with input and state time-delay. A quadratic cost function is used as a guaranteed performance index. The Krasovskii -Lyapunov function approach is employed to analyse the stability and design a guaranteed cost controller. The problem of guaranteed cost control via state feedback, its guaranteed cost controller design scheme is developed in terms of LMI. The controller designed here minimized a bound on a quadratic performance index and ensure the resulting closed-loop system is asymptotically stable. It is shown that the problem addressed here can be solved in terms of the feasibility of some linear matrix inequalities. An LMI-based design procedure is proposed for the guaranteed cost control problems of nonlinear systems with time-delay systems via state feedback control.

This paper is organized as follows. Section 2 provides preliminaries and the formulation of the fuzzy guaranteed cost control problem. In Section 3, a state feedback guaranteed cost control law for the fuzzy systems with the state and input time-delay is proposed based on the parallel distributed compensation in terms of linear matrix inequalities approach. A numerical example is given in Section 4 to illustrate the design methods presented in this paper. These are followed by some concluding remarks in Section 5.

2. PROBLEM FORMULATION

The continuous fuzzy dynamic model was proposed fuzzy model is a piecewise interpolation of several linear models through membership functions. The fuzzy model is presented by fuzzy IF-THEN rules and will be employed here to deal with the control design problem for the nonlinear uncertain system with the state and input time-delay,which can be described by the following T-S fuzzy time-delay systems.

Plant Rule i :

IF $\xi_1(t)$ is M_{1i} and$\cdots$and $\xi_p(t)$ is M_{pi}, THEN

$$\begin{aligned}\dot{x}(t) &= (A_i + \Delta A_i(t))x(t) + (A_{1i} + \Delta A_{1i}(t))x(t-\tau_1) \\ &\quad + B_i u(t) + B_{1i}u(t-\tau_2) \\ y(t) &= C_i x(t) \\ x(t) &= \psi(t), t \in [-\tau_0, 0], i = 1, \cdots, r\end{aligned} \tag{1}$$

where M_{ij} is the fuzzy set, $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input, $y(t) \in R^q$ is the output vector, $A_i, A_{1i}, B_i, B_{1i}, C_i$ are some constant matrices of compatible dimension, r is the number of IF-THEN rules, and $\xi(t) = [\xi_1(t), \cdots, \xi_p(t)]$ are the premise variables. It is assumed that the premise variables do not depend on the input variable $u(t)$. $\psi(t) \in C_{n,\tau_0}$ is a vector-valued initial continuous function. $\Delta A_i(t), \Delta A_{1i}(t)$ are the time-varying uncertain matrices. It is also assumed that the following.

Assumption 1. The uncertainty of the systems can be described as:

$$[\Delta A_i(t), \Delta A_{1i}(t)] = HF(t)[E_{1i}, E_{2i}]$$

where H, E_{1i}, E_{2i} are given constant matrices,$F(t)$ is not known function matrix and satisfied $F^T(t)F(t) \le I$.

Given a pair of $(x(t), u(t))$, the final output of the fuzzy time-delay system is inferred as follows:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r} \lambda_i(\xi(t))[(A_i + \Delta A_i(t))x(t) \\ &\quad + (A_{1i} + \Delta A_{1i}(t))x(t-\tau_1) \\ &\quad + B_i u(t) + B_{1i}u(t-\tau_2)] \\ y(t) &= \sum_{i=1}^{r} \lambda_i(\xi(t))C_i x(t)\end{aligned} \tag{2}$$

where $\lambda_i(\xi(t)) = \frac{\beta_i(\xi(t))}{\sum_{j=1}^{r}\beta_j(\xi(t))}, \beta_j(\xi(t)) = \prod_{k=1}^{p} M_{kj}(\xi_j(t))$,

$M_{kj}(\xi_j(t))$ is the grade of membership of $\xi_j(t)$ in M_{kj}. It is assumed that $\sum_{i=1}^{r}\lambda_i(\xi(t)) = 1, \lambda_i(\xi(t)) \geqslant 0, i = 1, \cdots, r$ for all t.

Given positive-definite symmetric matrices Q, R,we consider the cost function

$$J = \int_0^\infty \{x^T(t)Qx(t) + u^T(t)Ru(t)\}dt \tag{3}$$

Associated with the cost function (3), the fuzzy guaranteed cost control is defined as follows.

Definition (B.S. Chen and X.P. Liu,2005): Consider the system (2). If there exists a fuzzy control law $u(t)$ and a scalar J_0 such that the closed-loop system is asymptotically stable and the closed-loop value of the cost function (3) satisfies $J \le J_0$, then J_0 is said to be a guaranteed cost and the control law $u(t)$ is said to be a guaranteed cost control law for (2).

The objective of this paper is to develop a procedure to design a state-feedback guaranteed cost control law.

3. FUZZY GUARANTEED COST CONTROLLER DESIGN VIA STATE-FEEDBACK

In this section, we consider the design of a fuzzy guaranteed cost controller via state-feedback. Suppose the following fuzzy controller is employed to deal with the design of a fuzzy control system (2).

Control Rule i:

IF $\xi_1(t)$ is M_{1i} and$\cdots$and $\xi_p(t)$ is M_{pi}, THEN

$$u(t) = K_i x(t), i = 1, \cdots, r \tag{4}$$

Hence the overall fuzzy control law is represented by

$$u(t) = \sum_{i=1}^{r} \lambda_i(\xi(t))K_i x(t) \tag{5}$$

where $K_i(i = 1, \cdots, r)$ are the local control gains. The design of a fuzzy guaranteed cost controller is to determine feedback gains $K_i(i = 1, \cdots, r)$ and a positive scalar J_0 such that the resulting closed-loop system is asymptotically stable and closed-loop value of cost function (3) satisfies $J \le J_0$. With control law (5), the overall closed-loop system can be written as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i(\xi(t))\lambda_j(\xi(t))[(A_i + \Delta A_i(t) + B_iK_j)x(t) \\ &\quad +(A_{1i} + \Delta A_{1i}(t))x(t-\tau_1) + B_{1i}K_jx(t-\tau_2)] \\ y(t) &= \sum_{i=1}^{r}\lambda_i(\xi(t))C_ix(t) \end{aligned} \tag{6}$$

Then, the main result on the guaranteed cost control via state-feedback for the continuous-time T-S fuzzy model with the state and input time delay is summarized in the following theorem.

Theorem 1. Consider the system (6) associated with cost function (3). Suppose that there exist matrices $K_i(i = 1, \cdots, r), P > 0, S_1 > 0, S_2 > 0$ satisfying the following LMI:

$$\begin{bmatrix} \Lambda & P(A_{1i} + \Delta A_{1i}(t)) & PB_{1i}K_j \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} < 0 \tag{7}$$

where $\Lambda = (A_i + \Delta A_i + B_iK_j)^TP + P(A_i + \Delta A_i + B_iK_j) + S_1 + S_2 + Q + \bar{K}^TR\bar{K}$ and "*" denotes the transposed elements in the symmetric positions. Then, the control law (5) is a fuzzy guaranteed cost control law and $K_i = G_iX^{-1}, i = 1, \cdots, r$.

Proof. Choose a Lyapunov function candidate for (6) as

$$V(x,t) = x^T(t)Px(t) + \int_{t-\tau_1}^{t} x^T(\alpha)S_1x(\alpha)d\alpha + \int_{t-\tau_2}^{t} x^T(\alpha)S_2x(\alpha)d\alpha$$

For the simplicity,denote $\lambda_i(\xi(t))$ by λ_i.By differentiating $V(x,t)$ along the trajectory of the system (6), we obtain

$$\begin{aligned} \dot{V}(x,t) &= \dot{x}^T(t)Px(t) + x^T(t)P\dot{x}(t) + x^T(t)(S_1 + S_2)x(t) \\ &\quad -x^T(t-\tau_1)S_1x(t-\tau_1) - x^T(t-\tau_2)S_2x(t-\tau_2) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i\lambda_j[x^T(t), x^T(t-\tau_1), x^T(t-\tau_2)]\times \end{aligned}$$

$$\begin{bmatrix} \Lambda' & P(A_{1i} + \Delta A_{1i}(t)) & PB_{1i}K_j \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix}$$

where $\Lambda' = (A_i + \Delta A_i + B_iK_j)^TP + P(A_i + \Delta A_i + B_iK_j) + S_1 + S_2$. And (5) is equivalent to the following matrix inequalities:

$$\begin{aligned} \dot{V}(x,t) &< \sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i\lambda_j\left[x^T(t)\,x^T(t-\tau_1)x^T(t-\tau_2)\right] \\ &\quad \begin{bmatrix} -(Q + \bar{K}^TR\bar{K}) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix} \\ &= -\sum_{i=1}^{r}\sum_{j=1}^{r}\lambda_i\lambda_jx^T(t)(Q + \bar{K}^TR\bar{K})x(t) \\ &< 0 \end{aligned} \tag{8}$$

Which implies that (6) is asymptotically stable. Integrating (8) from 0 to T produces

$$\begin{aligned} &x^T(T)Px(T) - x^T(0)Px(0) \\ &+ \int_{T-\tau_1}^{T} x^T(\alpha)S_1x(\alpha)d\alpha \\ &- \int_{-\tau_1}^{0} x^T(\alpha)S_1x(\alpha)d\alpha + \int_{T-\tau_2}^{T} x^T(\alpha)S_2x(\alpha)d\alpha \\ &- \int_{-\tau_2}^{0} x^T(\alpha)S_2x(\alpha)d\alpha \\ &< -\int_{0}^{T} x^T(t)(Q + K_j{}^TRK_j)x(t)dt \end{aligned} \tag{9}$$

Because of closed-loop system (6) is asymptotically stable, thus, $\lim_{T\to\infty} x^T(T)Px(T) = 0$, $\lim_{T\to\infty}\int_{T-\tau_1}^{T} x^T(\alpha)S_1x(\alpha)d\alpha = 0$, $\lim_{T\to\infty}\int_{T-\tau_2}^{T} x^T(\alpha)S_2x(\alpha)d\alpha = 0$ and using zero initial condition. Therefore, the following matrix inequalities can be obtained:

$$\begin{aligned} &\int_{0}^{\infty} x^T(t)\{Q + K_j^TRK_j\}x(t)dt \\ &< \varphi^T(0)P\varphi(0) + \int_{-\tau_1}^{0} \varphi^T(\alpha)S_1\varphi(\alpha)d\alpha \\ &\quad + \int_{-\tau_2}^{0} \varphi^T(\alpha)S_2\varphi(\alpha)d\alpha \end{aligned} \tag{10}$$

$$\begin{aligned} J_0 &= \varphi^T(0)P\varphi(0) + \int_{-\tau_1}^{0} \varphi^T(\alpha)S_1\varphi(\alpha)d\alpha \\ &\quad + \int_{-\tau_2}^{0} \varphi^T(\alpha)S_2\varphi(\alpha)d\alpha \end{aligned} \tag{11}$$

This completes the proof.

Comment 1: There exists uncertainties in (7), we solve controller parameter after processing the uncertainties. For the guaranteed cost control for the uncertainties system (6), we have the following lemma.

Lemma 2. Suppose that there exist matrices of compatible dimension Y, H, E, R, where $R > 0$ is symmetric, then

$$Y + HF(t)E + E^T F^T(t) H^T < 0$$

where $F^T(t)F(t) \le R$, for all $F(t)$,if only and if there exist $\varepsilon > 0$ satisfying the following inequality:

$$Y + \varepsilon^2 HH^T + \varepsilon^{-2} E^T RE < 0$$

Theorem 3. Consider the system (6),inequality (7) holds if only and if there is common positive-definite symmetric matrices$X, W_1, W_2, M_j, L_j, j = 1, \cdots, r$ as well as scalar $\varepsilon > 0$ satisfying the following inequality:

$$\begin{bmatrix} \Xi & Z_{1i} & Z_{2ij} & H & Z_{3i} & M_j^T & X & X & X \\ * & -W_1 & 0 & 0 & Z_{3i} & 0 & 0 & 0 & 0 \\ * & * & -W_2 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\varepsilon^2 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon^2 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -R^{-1} & 0 & 0 & 0 \\ * & * & * & * & * & * & -Q^{-1} & 0 & 0 \\ * & * & * & * & * & * & * & -W_1 & 0 \\ * & * & * & * & * & * & * & * & -W_2 \end{bmatrix}$$

$$< 0, i, j = 1, \cdots, r \tag{12}$$

where $\Xi = (A_i X + B_i M_j)^T + (A_i X + B_i M_j), Z_{1i} = A_{1i} W_1, Z_{2ij} = B_{1i} L_j, Z_{3i} = X E_{1i}^T$ and "*" denotes the transposed elements in the symmetric positions, then the control law (5) is a fuzzy guaranteed cost control law and $K_j = G_j X^{-1}, j = 1, \cdots, r$.

Proof. Set $\tilde{\Xi} = (A_i + B_i K_j)^T P + P(A_i + B_i K_j) + S_1 + S_2 + Q + K_j^T R K_j$, then inequality (7) is equivalent to the following matrix inequality:

$$\begin{bmatrix} \tilde{\Xi} & PA_{1i} & PB_{1i}K_j \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} + \begin{bmatrix} PH \\ 0 \\ 0 \end{bmatrix} F(t) \begin{bmatrix} E_{1i} & E_{2i} & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} E_{1i}^T \\ E_{2i}^T \\ 0 \end{bmatrix} F^T(t) \begin{bmatrix} H^T P & 0 & 0 \end{bmatrix} < 0$$

Applying lemma 2,we obtain

$$\begin{bmatrix} \tilde{\Xi} & PA_{1i} & PB_{1i}K_j \\ * & -S_1 & 0 \\ * & * & -S_2 \end{bmatrix} + \varepsilon^2 \begin{bmatrix} PH \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} H^T P & 0 & 0 \end{bmatrix}$$

$$+\varepsilon^{-2} \begin{bmatrix} E_{1i}^T \\ E_{2i}^T \\ 0 \end{bmatrix} \begin{bmatrix} E_{1i} & E_{2i} & 0 \end{bmatrix} < 0$$

using Schur complement, pre- and post-multiplying above inequality by $diag(P^{-1}, I, I, I, I, I, I, I, I)$, respectively, and let $X = P^{-1}, M_j = K_j P^{-1}, L_j = K_j W_2, W_1 = S_1^{-1}, W_2 = S_2^{-1}$, we obtain (12). This completes the proof.

Theorem 1 and 3 provide a sufficient condition for the existence of guaranteed cost controller via state-feedback for (2). When LMIs (7) or (12) are feasible, each guaranteed cost controller ensures the resulting closed-loop system is asymptotically stable and minimizes an upper bound of the closed-loop cost function. In view of this, it is desirable to find a guaranteed cost control law which minimizes the upper bound. For the fuzzy guaranteed cost control problem, it is given by the following theorem.

Theorem 4. Consider the system (1) associated with cost function (3). Suppose that the optimization problem

$$\min_{\alpha, \varepsilon, W_1, W_2, S_1, Q, M_j} \alpha + tr(V_1) + tr(V_2) \tag{13}$$

subject to

1) $LMI(12)$

$$2) \begin{bmatrix} -\alpha & \varphi^T(0) \\ \varphi(0) & -X \end{bmatrix} < 0 \tag{14}$$

$$3) \begin{bmatrix} -V_1 & N_1^T \\ N_1 & -W_1 \end{bmatrix} < 0 \tag{15}$$

$$4) \begin{bmatrix} -V_2 & N_2^T \\ N_2 & -W_2 \end{bmatrix} < 0 \tag{16}$$

Has solutions $\alpha, \varepsilon, W_1, W_2, S_1, Q, M_j$,where

$\int_{-\tau_1}^{0} \varphi(\alpha) \varphi^T(\alpha) d\alpha = N_1 N_1^T$;

$\int_{-\tau_2}^{0} \varphi(\alpha) \varphi^T(\alpha) d\alpha = N_2 N_2^T$;

$tr(\cdot)$ denotes the trace of the matrix $(\cdot)$. Then, the corresponding guaranteed cost controller (5) is an optimal guaranteed cost controller in the sense that with this controller the upper bound on the closed-loop cost function (3) in minimal.

Proof. In fact, according to

$$\int_{-\tau_1}^{0} \varphi(\alpha) S_1 \varphi^T(\alpha) d\alpha = \int_{-\tau_1}^{0} tr(\varphi(\alpha)) S_1 \varphi^T(\alpha) d\alpha$$

$$= tr(N_1 N_1^T S_1) = tr(N_1^T S_1 N_1)$$

$$\int_{-\tau_2}^{0} \varphi(\alpha) S_2 \varphi^T(\alpha) d\alpha = \int_{-\tau_2}^{0} tr(\varphi(\alpha)) S_2 \varphi^T(\alpha) d\alpha$$

$$= tr(N_2 N_2^T S_2) = tr(N_2^T S_2 N_2)$$

And subject to 2) in the problem (13) is equivalent to $N_1^T S_1 N_1 < V_1; N_2^T S_2 N_2 < V_2$, respectively. Thus, $\min\{\alpha + tr(V_1) + tr(V_2)\}$ guaranteed cost controller with respect to (13) is an optimal guaranteed cost controller.This completes the proof.

4. COMPUTER SIMULATION

In this section, to illustrate the proposed results, we apply the above design technique to design a fuzzy guaranteed cost controller for the following nonlinear systems.
Consider the following uncertain time delay system:

$$
\begin{aligned}
\dot{x}_1(t) &= 0.2\sin(t) + (0.25+\sin(t))x_1(t-4) \\
&\quad -1.5x_2(t) + 0.1x_1(t)x_2(t) \\
&\quad -u(t) + 0.1u(t-6) \\
\dot{x}_2(t) &= x_1(t) - (3+0.2\cos(t))x_2(t) \\
&\quad +(0.1+0.3\cos(t))x_2(t-4) \\
&\quad +0.1u(t) - 0.1u(t-6) \\
y(t) = \varphi(t) &= \begin{bmatrix} -e^{\frac{t}{2}} \\ -1.2e^{\frac{t}{2}} \end{bmatrix}, t \in [-6, 0]
\end{aligned}
$$

Setting up following fuzzy model:

Rule 1: IF $x_1(t)$ is $Max(M_1)$, THEN

$$
\begin{aligned}
\dot{x}(t) &= (A_1 + \Delta A_1)x(t) + A_{11}x(t-\tau_1) \\
&\quad +B_1u(t) + B_{12}u(t-\tau_2)
\end{aligned}
$$

Rule 2: IF $x_1(t)$ is $Min(M_2)$, THEN

$$
\begin{aligned}
\dot{x}(t) &= (A_2 + \Delta A_2)x(t) + A_{12}x(t-\tau_1) \\
&\quad +B_2u(t) + B_{22}u(t-\tau_2)
\end{aligned}
$$

The membership functions of fuzzy sets are chosen as

$$\mu_1(x_1(t)) = \tfrac{x_1(t)+1.5}{3},\ \mu_2(x_1(t)) = 1 - \mu_1(x_1(t))$$

We have

$$A_1 = \begin{bmatrix} 0 & -1 \\ 1 & -3 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}, B_1 = B_2 = \begin{bmatrix} -1 \\ 0.1 \end{bmatrix}$$

$$B_{11} = B_{12} = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}, F(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix}$$

$$A_{11} = A_{12} = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.1 \end{bmatrix}, H = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$E_{11} = E_{12} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, E_{21} = E_{22} = \begin{bmatrix} 1 & 0 \\ 0 & -0.3 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, R = 1, \varphi(0) = \begin{bmatrix} -1 \\ -1.2 \end{bmatrix}$$

$$N_1 = \begin{bmatrix} 0.6369 & 0.7590 \\ 0.7590 & 0.9152 \end{bmatrix}, N_2 = \begin{bmatrix} 0.7872 & 0.6147 \\ 0.6147 & 1.16 \end{bmatrix},$$

For the state-feedback guaranteed cost control, we choose $\varepsilon = 1$, applying Theorem 4, the feasible solutions to (13) are given as following:

$$P = \begin{bmatrix} 0.2923 & 0.0963 \\ 0.0963 & 0.4009 \end{bmatrix}, S_1 = \begin{bmatrix} 2.7416 & -0.2971 \\ -0.2971 & 0.4203 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 3.2150 & -0.434 \\ -0.434 & 0.3596 \end{bmatrix}, \alpha = 2.44, J_0 = 25.626$$

$$K_1 = [\,8.5153\ \ -0.8178\,], K_2 = [\,8.5328\ \ -0.8899\,]$$

$$V_1 = \begin{bmatrix} 3.9269 & 4.7122 \\ 4.7122 & 5.6547 \end{bmatrix}, V_2 = \begin{bmatrix} 4.9877 & 5.9847 \\ 5.9847 & 8.6173 \end{bmatrix}$$

With control law $u(t) = \mu_1 K_1 x(t) + \mu_2 K_2 x(t)$, the closed-loop system is asymptotically stable and a guaranteed cost of the closed-loop system is $J_0 = 25.626$ .The simulation result on state feedback guaranteed cost control is shown in Fig.1.

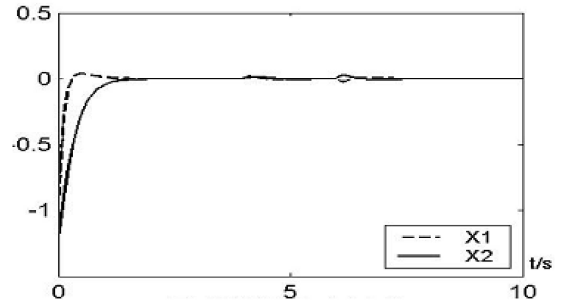

Fig. 1. Response of state via state feedback.

5. CONCLUSION

In this paper, we have considered the fuzzy guaranteed cost control design problem for nonlinear uncertain systems with state and input time-delay. The fuzzy guaranteed cost control design methodology for state feedback is developed in terms of the feasibility of linear matrix inequality. The controller designed achieves a closed-loop asymptotically stability and results in a minimal upper bound of the closed-loop value of cost function. A numerical example is also given to illustrate the design procedures and the effectiveness of the approach developed in this paper.

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