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Closed-loop LPV identification of the time-varying dynamics of a variable-speed wind turbine ⋆

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Abstract: In this paper we present a closed-loop LPV identification algorithm that uses a periodic scheduling sequence to identify the rotational dynamics of a wind turbine. In the algorithm we assume that the system undergoes the same time variation several times, which make it possible to use time-invariant identification methods since the input and output data are chosen from the same point in the variation of the system. We use closed-loop time-invariant techniques to identify a number of extended observability matrices and state sequences that are, inherent to subspace identification, identified in a different state basis. We show that by formulating an intersection problem all the states can be reconstructed in a general state basis from which the system matrices can be estimated. The novel algorithm is applied on a wind turbine model operating in closed loop.

Keywords: LPV, subspace identification, nonlinear identification

1. INTRODUCTION

The trend with offshore wind turbines is to increase the rotor diameter as much as possible. The reason is that the foundation costs of offshore wind turbines amount to a large part of the total costs. Therefore, designers want to increase the energy yield per wind turbine, which increases with the square of the rotor diameter, as much as possible to reduce the costs. The increasing dimensions have led to the relative increase of the loads on the wind turbine structure.

Because of the increasing rotor size it is necessary to react on turbulence in a more detailed way: each blade separately and at several separate radial distances. This first item is dealt with in Individual Pitch Control (IPC), motivated by the helicopter industry; Bossanyi [2005], Engelen et al. [2007], which is the latest development in the wind turbine industry to minimize the loads. With this concept each blade is pitched individually to suppress the harmonic loads. The controllers are designed using linear controller synthesis and are gain scheduled afterwards to compensate for the non-linear behavior of the variable-speed wind turbines. However, this method does not guarantee any stability or performance; Shamma and Athans [1991]. In recent work of Bianchi et al. [2005], Lescher et al. [2006], the Linear Parameter Varying (LPV) framework is proposed for the design of feedback controllers in the wind energy. The main advantage of the LPV controller synthesis problem is that it results in robust gain-scheduled controllers which have the property to have a guaranteed performance and stability over the complete operation envelop.

For LPV control it is important to have an accurate mathematical model of the system under consideration. Common practice in the wind industry is to model the dynamics using first principles; Molenaar [2003]. This approach has a number of disadvantages: time consuming, over/under modeling, uncertainties, and complexity. However, efficient methods exist to obtain mathematical models from measurement data, these methods are referred to as system identification. Using the available measurements only the most important dynamics is modeled. This implies that system identification gives a compact sized model which is suitable for controller (re)design, load calculations and model validation.

An overview of past literature in LPV identification setting can be found in Verdult [2002]. It is possible to distinguish between identification techniques for different types of scheduling sequences. For the case where this sequence can be randomly varying, the identification problem has proven to be challenging. The subspace identification method proposed in Verdult and Verhaegen [2002], and later improved in Verdult and Verhaegen [2005] has the inherent drawback that it requires an approximation; neglecting certain terms and possibly leading to biased results. However, this method can be used as an initial estimate for a parametric identification method such as proposed in Verdult [2002]. Because of these difficulties, it is interesting to investigate whether the use of dedicated scheduling sequences facilitates the identification of LPV systems. Specific cases of scheduling sequences have been studied, such as the case of abrupt switching, which leads to piecewise affine (hybrid) systems; Wingerden et al.

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[2007], Wingerden and Verhaegen [2007], white noise; Santos et al. [2006], and periodic scheduling; Felici et al. [2007].

In this paper we tackle a part of the LPV system identification problem for wind turbines. We will consider the rotational dynamics of the wind turbine where the scheduling sequence undergoes the same time variation several times, which makes it possible to use a number well-established steps from LTI system identification. The main contribution of this paper is that we extend the work in Felici et al. [2007] to the closed-loop setting, and the application on a challenging wind turbine model. The first property is essential for wind turbines because wind turbines are unstable systems and consequently have to operate with a stabilizing controller.

The remainder of this paper is setup as follows. In Section 2, the theoretic framework is presented for the identification of closed-loop LPV systems using a dedicated scheduling sequence, while in Section 3, the algorithm is applied to an LPV model of a wind turbine. In the final section we present the conclusions of this paper.

2. LPV IDENTIFICATION FOR DEDICATED SCHEDULING SEQUENCES

For Periodic Linear Time Varying (PLTV) systems it is well known that LTI system identification can be used to obtain accurate models of the PLTV system; Verhaegen and Yu [1995]. However, when the time variation is changing these models are not valid anymore. In this section an algorithm is presented that uses PLTV identification to construct an LPV model; valid for arbitrarily scheduling after the identification experiment. First, we describe a general problem formulation and explain the assumptions made. Then a number of extended observability matrices are estimated assuming that the same time-varying behavior is present a number of times. These observability matrices have the inherent drawback that they are identified in a different state basis. This can be solved by solving an intersection problem. When the similarity transformations are known the states can be transformed to the same global state basis and the system matrices can be reconstructed by solving a set of linear equations.

2.1 Problem Formulation

Consider the following LPV system

$$x_{k+1} = A_k x_k + B_k u_k + K e_k, \tag{1}$$
$$y_k = C_k x_k + D_k u_k + e_k, \tag{2}$$

where $x_k \in \mathbb{R}^n$, $u_k \in \mathbb{R}^r$, $y_k \in \mathbb{R}^\ell$, are the state, input, and output vectors; the noise sequence $e_k \in \mathbb{R}^\ell$ is a zero-mean white noise sequence. The proposed structure can be seen as the innovation form for LPV systems; well known for LTI system identification.

The time varying system matrices are given by

$$A_k = \sum_{i=1}^{m} A^{(i)} \mu_k^{(i)},$$

where m represents the number of LPV system matrices. In an identical manner the matrices B_k, C_k, and D_k are defined. In these expressions $A^{(i)} \in \mathbb{R}^{n\times n}$, $B^{(i)} \in \mathbb{R}^{n\times r}$, $C^{(i)} \in \mathbb{R}^{\ell\times n}$, $D^{(i)} \in \mathbb{R}^{\ell\times r}$, and $K \in \mathbb{R}^{n\times \ell}$ are referred to as the system matrices. The model weights are $\mu_k^{(i)} \in \mathbb{R}$. Note that the system matrices depend in a linear manner on the time-varying scheduling vector

$$\mu_k = \left(\mu_k^{(1)} \ \mu_k^{(2)} \ \cdots \ \mu_k^{(m)} \right)^T.$$

To include the case of affine dependence, one can set the first entry of the scheduling vector to unity: $\mu_k^{(1)} = 1 \ \forall\, k$. It is required that the terms of the scheduling sequence are linearly independent, such that:

$$\text{rank}\left([\, \mu_0 \ \mu_1 \ \cdots \ \mu_{t-1} \,]\right) = \text{m},$$

and $t > m$. The scheduling sequence μ_k is assumed to be known and periodic with period t, defined over $\bar{N}$ periods:

$$\mu_k = \mu_{k+\tau t} \quad \forall\, \tau = \{1, 2, \ldots, \bar{N}-1\},\ k = \{0, 1, \ldots, t-1\}$$

The total number of samples is thus equal to $N = t\bar{N}$. Because of the periodicity of the scheduling, the time variation of the system is periodic.

The state-space realization given in (1)-(2) can be written in the predictor form as:

$$x_{k+1} = \tilde{A}_k x_k + \tilde{B}_k u_k + K y_k, \tag{3}$$
$$y_k = C_k x_k + D_k u_k + e_k, \tag{4}$$

where $\tilde{A}_k = A_k - KC_k$ and $\tilde{B}_k = B_k - KD_k$. It is well-known that an invertible linear transformation of the state does not change the input-output behavior of a state-space system. Therefore, we can only determine the system matrices up to a similarity transformation $T \in \mathbb{R}^{n\times n}$: $T^{-1}A^{(i)}T$, $T^{-1}B^{(i)}$, $T^{-1}K$, $C^{(i)}T$, and $D^{(i)}$.

The identification problem can now be formulated as: given the input sequence u_k, the output sequence y_k, and the scheduling sequence μ_k; find all the LPV system matrices $A^{(i)}$, $B^{(i)}$, $C^{(i)}$, $D^{(i)}$, and K up to a global similarity transformation.

2.2 Definitions and assumptions

We define the stacked output vector $\bar{y}_k^i$ as

$$\bar{y}_k^i = \left(y_k^T \ y_{k+1}^T \ \cdots \ y_{k+i-1}^T \right)^T,$$

and similarly the stacked input $\bar{u}_k^i$, stacked noise $\bar{e}_k^i$, and stacked scheduling $\bar{\mu}_k^i$ are defined. In parallel to what is done in LTI system identification, also for time-varying systems an observability matrix can be derived

$$\tilde{\mathcal{O}}_k^f = \begin{pmatrix} C_k \\ C_{k+1}\tilde{A}_k \\ C_{k+2}\tilde{A}_{k+1}\tilde{A}_k \\ \vdots \\ C_{k+f-1}\tilde{A}_{k+f-2}\ldots\tilde{A}_k \end{pmatrix} \in \mathbb{R}^{\ell f\times n},$$

where $\tilde{\mathcal{O}}_k^f$ is the extended observability matrix and time instance k, and f is referred to as the future window size. In what follows, it is assumed that $\tilde{\mathcal{O}}_k^f$ has full column rank for all k, which is equivalent to requiring that the system is observable on all intervals of length f according to the condition for observability of LPV systems in Rugh [1996].

We also define the matrices $\tilde{\Phi}_k^f$, and $\tilde{\Psi}_k^f$ which are the time-varying equivalent of the Toeplitz matrices related to the future input and output respectively, as defined in Jansson

[2005]. With the previous definitions it holds that for all $k = \{0, \ldots, N-1\}$

$$\bar{y}_k^f = \tilde{\mathcal{O}}_k^f x_k + \tilde{\Phi}_k^f \bar{u}_k^f + \tilde{\Psi}_k^f \bar{y}_k^f + \bar{e}_k^f. \tag{5}$$

This data equation is the starting point for many subspace identification schemes. However, now the matrices $\tilde{\mathcal{O}}_k^f$, $\tilde{\Phi}_k^f$, and $\tilde{\Psi}_k^f$ are time varying.

2.3 Step 1: Closed-loop PLTV identification

As we have seen in equation (5), due to the time varying nature of the system, the matrices will be different for each time step. However, due to the periodicity of the scheduling sequence, we know that the same matrices will appear periodically:

$$\begin{aligned} \tilde{\mathcal{O}}_k^f &= \tilde{\mathcal{O}}_{k+t}^f \quad \forall k \in [0, \cdots, \overline{N}-1], \\ \overline{\mu}_k^{p+f} &= \overline{\mu}_{k+t}^{p+f} \quad \forall k \in [0, \cdots, \overline{N}-1], \end{aligned}$$

and in a similar way this can be done for $\tilde{\Phi}_k^f$, and $\tilde{\Psi}_k^f$.

The state at time instance $k+p$ is a function of the known past inputs, $\overline{u}_k^p$, and outputs, $\overline{y}_k^p$, and the the initial state. This state x_{k+p}, where p is referred to as the past window length, is given by

$$x_{k+p} = \left(\tilde{A}_{k+p}\tilde{A}_{k+p-1}\ldots\tilde{A}_k\right) x_k + \left(\tilde{\mathcal{C}}_k^p \;\; \tilde{\mathcal{K}}_k^p\right) \begin{pmatrix} \overline{u}_k^p \\ \overline{y}_k^p \end{pmatrix}, \tag{6}$$

where $\tilde{\mathcal{C}}_k^p$, and $\tilde{\mathcal{K}}_k^p$ are matrices depending on $\tilde{A}_k$, $\tilde{B}_k$, and K with their LTI variant given in Jansson [2005].

We assume that the LPV system given in (3)-(4) is stable [1]. By choosing p large enough the contribution of the initial state to the state x_{k+p} can be made arbitrarily small. In a number of LTI subspace methods it is well known to disregard the effect of the initial state resulting in a biased estimate, although this bias can be made arbitrarily small by choosing p large, see Jansson [2005], Chiuso [2007]. Now we define the future matrices:

$$Y_k^f = \left(\overline{y}_{p+k}^f,\, \overline{y}_{p+k+t}^f,\, \overline{y}_{p+k+2t}^f,\, \cdots,\, \overline{y}_{p+k+(\overline{N}-1)t}^f\right),$$

and on a similar way for U_k^f, and E_k^f. In a similar way we define matrices for the past:

$$Y_k^p = \left(\overline{y}_k^p,\, \overline{y}_{k+t}^p,\, \overline{y}_{k+2t}^p,\, \cdots,\, \overline{y}_{k+(\overline{N}-1)t}^p\right),$$

and on a similar way for U_k^p and we define:

$$X_k = \left(\overline{x}_{p+k},\, \overline{x}_{p+k+t},\, \overline{x}_{p+k+2t},\, \cdots,\, \overline{x}_{p+k+(\overline{N}-1)t}\right).$$

With these definitions, and disregarding the effect of the initial state; (6) can be substituted in (5) to obtain

$$Y_k^f \approx \tilde{\mathcal{O}}_k^f \left(\mathcal{C}_k^p \;\; \mathcal{K}_k^p\right) \begin{pmatrix} U_k^p \\ Y_k^p \end{pmatrix} + \tilde{\Phi}_k^f U_k^f + \tilde{\Psi}_k^f Y_k^f + E_k^f \tag{7}$$
$$\forall k \in [0, \cdots, \overline{N}-1].$$

The first step in the closed-loop system identification scheme is to use this approximation to get an estimate of $\tilde{\Phi}_k^f$, and $\tilde{\Psi}_k^f$. An estimate of the matrices $\tilde{\Phi}_j^f$ and $\tilde{\Psi}_j^f$ can be found by performing a linear regression (Jansson [2005], Chiuso and Picci [2005]) where we assume that the matrix $\tilde{\mathcal{O}}_k^f \left(\mathcal{C}_k^p \;\; \mathcal{K}_k^p\right)$ has full rank. Subtracting this estimate from (7) we end up with:

$$\begin{aligned} Z_k = Y_k^f - \hat{\Phi}_k^f U_k^f - \hat{\Psi}_k^f Y_k^f &\approx \mathcal{O}_k^f \left(\mathcal{C}_k^p \;\; \mathcal{K}_k^p\right) \begin{pmatrix} U_k^p \\ Y_k^p \end{pmatrix} + \overline{E}_k^f, \\ &\approx \tilde{\mathcal{O}}_k^f X_k + E_k^f, \end{aligned} \tag{8}$$

Equation (8) can be used to determine the observability matrix, $\tilde{\mathcal{O}}_k^f$, and the state sequence, X_k, up to a similarity transformation, using the SVD of the matrix Z_k

$$Z_k = \left(\mathcal{U}_n^k \;\; \mathcal{U}_{n\perp}^k\right) \begin{pmatrix} \Sigma_n^k & 0 \\ 0 & \Sigma_0^k \end{pmatrix} \begin{pmatrix} \mathcal{V}_n^k \\ \mathcal{V}_n^{k\perp} \end{pmatrix},$$

where Σ_n^k is the diagonal matrices containing the n dominant singular values and $\mathcal{U}_n^k$ is the corresponding column space. Note that we can find the dominant singular values by detecting a gap between the singular values. An estimate of the state and the extended observability can be obtained:

$$\hat{\tilde{\mathcal{O}}}_k^f = \mathcal{U}_n^k T_k, \tag{9}$$

$$\hat{X}_k = T_k^{-1} \Sigma_n^k \mathcal{V}_n^k,. \tag{10}$$

This can be done for all $k = \{0, \cdots, \overline{N}-1\}$, obtaining t different observability matrices. The similarity transformations T_k will also be different at each time, so the models are identified in a different basis. We have to stress that if the identified states are in the same state basis the LPV system identification problem is solved.

2.4 Step 2: Relating the t extended observability matrices

For this step we need to relate the different observability matrices to the same basis. This can be done by writing the observability matrices of the different repeating scheduling sequences as a product between a matrix containing only the scheduling terms and a constant matrix which depends only on the system matrices $\tilde{A}^{(i)}$, $C^{(i)}$. This factorization was introduced in Felici et al. [2007]. First define the m-tuple $\mathcal{A} = \{\tilde{A}^{(1)}, \ldots, \tilde{A}^{(m)}\}$ containing all matrices $A^{(i)}$ and similarly the m-tuple $\mathcal{C} = \{C^{(1)}, \ldots, C^{(m)}\}$ consisting of all matrices $C^{(i)}$. Then define the operator $\mathcal{P}_j$ on these two tuples which returns the block-matrix of all ordered products between one element from $\mathcal{C}$ and $j-1$ elements from $\mathcal{A}$ (m^j possible combinations). Formally, the ξ^{th} block row $\mathcal{P}_j^\xi(\mathcal{C}, \mathcal{A}) \in \mathbb{R}^{\ell \times n}$ of $\mathcal{P}_j(\mathcal{C}, \mathcal{A}) \in \mathbb{R}^{\ell m^j \times n}$ is given by:

$$\mathcal{P}_j^\xi(\mathcal{C}, \mathcal{A}) = C^{(i_1^\xi)} A^{(i_2^\xi)} A^{(i_3^\xi)} \cdots A^{(i_j^\xi)},$$

with $i_1^\xi, \ldots i_j^\xi \in \{1, \ldots, m\} \; \forall \; \xi \in \{1, \ldots, m^j\}$ and ordered by $\rho_{\xi+1} > \rho_\xi$ where

$$\rho_\xi = \left(i_1^\xi \;\; i_2^\xi \;\; \cdots \;\; i_j^\xi\right) \begin{pmatrix} m^{j-1} \\ m^{j-2} \\ \vdots \\ m^0 \end{pmatrix}.$$

To illustrate this definition, notice that for $m = 2$ one obtains:

$$\mathcal{P}_1 = \begin{pmatrix} C^{(1)} \\ C^{(2)} \end{pmatrix}, \quad \mathcal{P}_2 = \begin{pmatrix} C^{(1)}\tilde{A}^{(1)} \\ C^{(1)}\tilde{A}^{(2)} \\ C^{(2)}\tilde{A}^{(1)} \\ C^{(2)}\tilde{A}^{(2)} \end{pmatrix}.$$

[1] Observe that we assume that the matrix $\tilde{A}$ is stable and not A because the rotational dynamics of a wind turbine includes a pure integrator.

The amount of block-rows grows exponentially as m^j. The operator $\mathcal{P}_j$ is used to define

$$S = \begin{pmatrix} \mathcal{P}_1(\mathcal{C}, \mathcal{A}) \\ \mathcal{P}_2(\mathcal{C}, \mathcal{A}) \\ \vdots \\ \mathcal{P}_f(\mathcal{C}, \mathcal{A}) \end{pmatrix} \in \mathbb{R}^{q \times n}. \tag{11}$$

Now define

$$M_k^f = \begin{pmatrix} \mu_k^T & 0 & \cdots & 0 \\ 0 & \mu_k^T \otimes \mu_{k+1}^T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mu_k^T \otimes \cdots \otimes \mu_{k+f-1}^T \end{pmatrix} \otimes I_\ell, \tag{12}$$

with $M_k^f \in \mathbb{R}^{f\ell \times q}$. Then it can be shown by simple substitution and using (5) that

$$\tilde{\mathcal{O}}_k^f = M_{k+p}^f S$$

where $\tilde{\mathcal{O}}_k^f$ is known up to an unknown similarity transformation (9), M_{k+p}^f depends on the known scheduling sequence (12), and S is an unknown matrix defined in (11). Note that the number of rows of S (columns of M_{k+p}^f), denoted by q, increases exponentially with f according to the relation $q = \sum_{j=1}^{f} \ell m^j$. We now give a result that relates the different observability matrices. We present the following result for the noiseless case and for the case that an unbiased estimate has been obtained. Now we define

$$\begin{aligned} \tilde{\mathcal{U}} &= \operatorname{diag}\left(\mathcal{U}_{\mathrm{n}}^0, \cdots, \mathcal{U}_{\mathrm{n}}^{\mathrm{t}-1}\right) \in \mathbb{R}^{\mathrm{dt} \times \mathrm{nt}}, \\ \tilde{\Gamma} &= \operatorname{diag}\left(\mathcal{O}_1 0^{\mathrm{f}}, \cdots, \mathcal{O}_{\mathrm{t}-1}^{\mathrm{f}}\right) \in \mathbb{R}^{\mathrm{ft} \times \mathrm{nt}}, \\ \hat{T} &= \left((T_0)^T, \cdots, (T_{t-1})^T\right)^T \in \mathbb{R}^{nt \times n}, \\ \tilde{M} &= \left(\left(M_p^f\right)^T, \cdots, \left(M_{t+p-1}^f\right)^T\right)^T \in \mathbb{R}^{nt \times q}, \end{aligned}$$

where M_k^f is defined in (12). Also, define $\tilde{T} \in \mathbb{R}^{n \times n}$ equal to T up to an unknown square invertible matrix $\hat{T}$ now the following relations hold

$$\tilde{T} = \hat{T}T, \qquad \text{and} \qquad \tilde{\mathrm{S}} = \mathrm{ST}.$$

Now we can define

$$\operatorname{null}\left(\left[\,\tilde{\mathcal{U}}\ \tilde{M}\,\right]\right) = \begin{pmatrix} \phi \\ \psi \end{pmatrix}, \tag{13}$$

which can also be formulated as an intersection problem. With the condition $\phi \in \mathbb{R}^{n\mathrm{v} \times \sigma}$, $\psi \in \mathbb{R}^{q \times \sigma}$, and $\sigma = q + n\mathrm{v} - \operatorname{rank}\left(\left[\,\tilde{O}\ \tilde{M}\,\right]\right)$ and $\sigma = n$. Then

$$\psi = ST = \tilde{S} \qquad \phi = \hat{T}T = \tilde{T} \tag{14}$$

This implies that when the rank conditions hold the matrices S and $\hat{T}$ can be found up to an unknown similarity transformation.

In the case of noise (13) will lead to a smaller (or even an empty) null space. This can be overcome by using an SVD to compute ϕ and ψ. The dimensions of the intersection problem formulated in (13) grows exponential with the window size f. However, this can be solved on a similar way as in Felici et al. [2007].

2.5 Step 3: Recovering of the LPV system matrices

In the previous step all the states sequences are transformed to the same global state basis, using (14) and (10). It is well known that when the state, input, output, and scheduling sequence are known the system matrices can be estimated. First we use (2) which is now a linear relation in $C^{(i)}$, and $D^{(i)}$, and where e_k represents a white noise. From this equation an estimate can be found of the $C^{(i)}$, and $D^{(i)}$ matrices, while also the noise sequence can be estimated. The estimated noise sequence is used to transform (1) into a linear expression depending on $A^{(i)}$, $B^{(i)}$, and K.

3. SIMULATION STUDY

In this paper we presented an identification approach to identify LPV systems assuming that the scheduling sequence is periodic. In this section we use a nonlinear model of the rotational dynamics to demonstrate how the algorithm works.

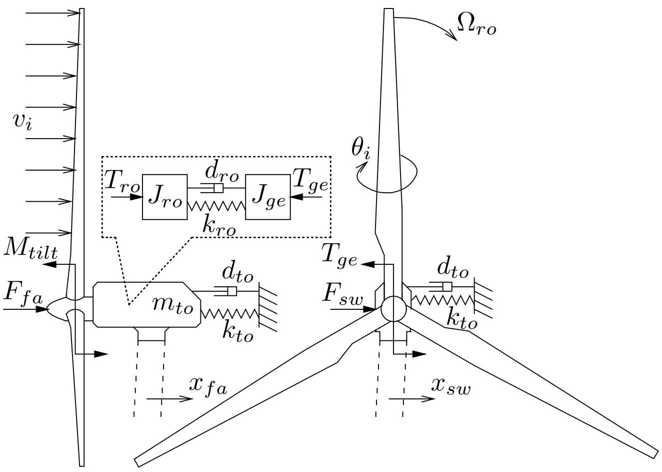

Fig. 1. Schematic representation of the wind turbine model

3.1 First principle model of a Horizontal Axis Wind Turbine (HAWT)

In this paper we consider a seven degrees of freedom model, as described in Engelen et al. [2007]. The model describes the rotational dynamics of a wind turbine around a particular operating point. The model contains degrees of freedom for the main rotation, first torsion mode of the drive train, the first fore-aft, and sideward bending mode of the tower. In this model, the blades are considered to be rigid. In Figure 1, a schematic representation of the model is given.

The system's, input, disturbance and output vectors are given by:

$$\begin{aligned} u &= \left(\,\delta\theta_1\ \delta\theta_2\ \delta\theta_3\ \delta T_{ge}\,\right)^T, \\ v &= \left(\,\delta v_1\ \delta v_2\ \delta v_3\,\right)^T, \\ y &= \left(\,\delta\Omega_{ge}\ \dot{x}_{fa}\ \dot{x}_{sw}\ \delta M_1\ \delta M_2\ \delta M_3\,\right)^T, \end{aligned}$$

respectively. This model contains thus the control inputs for the variation in generator torque δT_{ge} and the pitch angle $\delta\theta_i$ of each rotor blade. Furthermore, the model contains the inputs for the wind speed disturbance δv_i on each of the three rotor blades. The outputs are the variations in generator speed $\delta\Omega_{ge}$, the fore-aft velocity

$\dot{x}_{fa}$ and sideward velocity $\dot{x}_{sw}$ of the tower and the blade root bending moment δM_i of each rotor blade.

The model under consideration has a constant A matrix while the input and output matrices strongly depend on the azimuth angle, φ. In Engelen et al. [2007], the Coleman transformation is used to transform this model to an LTI model. However, this transformation can not cope with a failing sensor/actuator, gravity, and/or yaw misalignment. If the Coleman transformation is applied to these models still periodic components will be present in the dynamics. However, all the mentioned phenomena will again lead to an LPV model where the system undergoes the same time-variation a number of times. Still, in this paper we selected the model given in Engelen et al. [2007] based on its simplicity, available documentation, and that the mentioned phenomena will not change the proposed LPV system identification algorithm.

3.2 Simulation of the closed-loop wind turbine model

The LPV model of the HAWT is used to obtain the input, output, and the scheduling sequence for the identification algorithm. For this purpose, the equations are converted to discrete time using a naive zero-order hold discretization method with a sample time of 0.1 s. The naive approach omits the switching behaviors of the sampled scheduling signals. For our case, where the scheduling sequence is a function of the azimuth angles the scheduling sequences are given by the smooth signals

$$\varphi_k = \left(\sin\left(\frac{2\pi k}{\mathrm{v}}\right) \; \sin\left(\frac{2\pi k}{\mathrm{v}} + \frac{2\pi}{3}\right) \; \sin\left(\frac{2\pi k}{\mathrm{v}} + \frac{4\pi}{3}\right) \right)^T,$$

When an appropriate sample time is chosen this method gives a good approximation of the continuous time LPV system.

The wind turbine system is not asymptotically stable, it has an integrator, a collective pitch controller in a feedback loop is added to the system to stabilize the system. The controller used, can be found in Engelen et al. [2007] where the collective pitch controller is parameterized. For the pitch-angle inputs we take an additional zero-mean white noise with $\mathrm{var}\,(\theta_{k,i}) = 1$ deg, which is added to the control signal of the collective pitch controller. As input for the generator torque we take also a zero-mean white noise signal with $\mathrm{var}\,(T_{ge,k}) = 1 \cdot 10^6$ Nm. The wind disturbance signal is also zero-mean white noise with $\mathrm{var}\,(v_{k,i}) = 1$ m/s, but this signal is assumed to be unknown.

3.3 Closed-loop LPV subspace identification results

The collected data of u_k, y_k, and μ_k from the simulations are used in the identification experiments. The scheduling sequence can be rewritten as $\mu_k = \left(1 \; \varphi_{k,1} \; \varphi_{k,2} \right)^T$ to fulfill the assumption that this scheduling matrix must be of full rank. The third azimuth angle can be written as a linear combination of the other two angles. For the identification experiments we used $N = 1000$, $\mathrm{v} = 35$, $f = 16$ and $p = 10$.

The performance of the identified system is evaluated by looking at the eigenvalues of the A matrix and the value of the Variance-Accounted-For (VAF) on a data set different

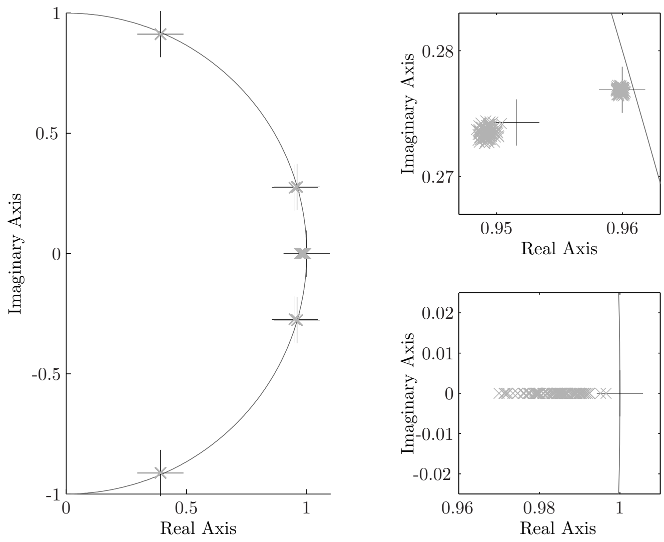

Fig. 2. Eigenvalues of the estimated A matrix in the complex plane, for 100 experiments. The big crosses correspond to the real values of the eigenvalues of the matrix. The boxes to the right show a magnification of three pole locations.

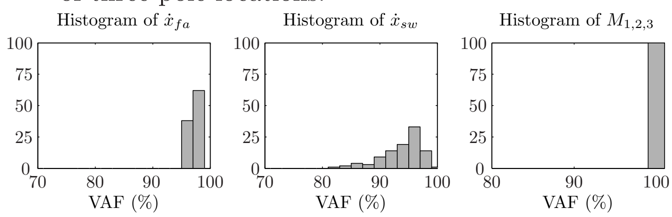

Fig. 3. Histogram of VAF values (%) of the outputs $\dot{x}_{fa}$, $\dot{x}_{sw}$ and $M_{1,2,3}$. The range of VAF values from 0 to 100% is divided into bins of 2%. For each bin, it is shown how many data sets out of the total 100 resulted in VAF values that fall into that bin.

from the one used for identification. The VAF is defined as $\mathrm{VAF} = \max\left\{1 - \frac{\mathrm{var}(y_k - \hat{y}_k)}{\mathrm{var}(y_k)}, 0\right\} \times 100$, where $\hat{y}_k$ denotes the output signal obtained by simulating the identified LPV system, y_k is the output signal of the true LPV system, and var is an operator that computes the variance. A small mismatch of the estimation of a pure integrator can cause drift in the time domain and consequently give large variations in the VAF values. For meaningful VAF values the system under consideration must be asymptotically stable, otherwise a small mismatch will give low VAF values due to the increasing or decreasing characteristic of the outputs.

This problem occurs for the output of the generator speed, therefore bode diagrams at a fixed scheduling vector are used to evaluate the performance at those specific channels

To investigate the sensitivity of the identification algorithm with respect the wind disturbances, a Monte-Carlo simulation with 100 runs was carried out. For each of the 100 simulations a different realization of the input u_k and wind disturbance v_k is used. In Figure 2 the eigenvalues of the estimated models are compared with the true values. It shows that the identified eigenvalues are very close to the true eigenvalues and that the variance

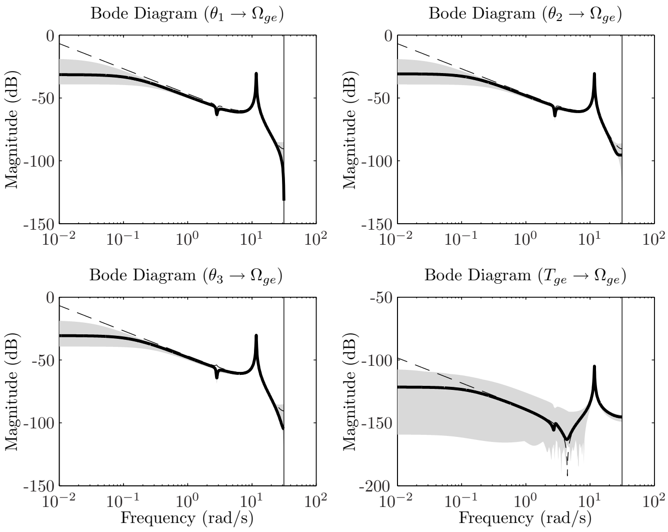

Fig. 4. Bode diagrams of the original transfer functions (dashed) and the identified transfer functions of the experiment with the highest mean VAF value (bold). The transfer functions of the other 99 experiments are within the gray region. The azimuth angles are fixed at the values $\varphi = \big(0, \sqrt{3}/2, -\sqrt{3}/2\big)$.

and bias is very small. Figure 3 shows the corresponding histograms of the VAF values on a fresh validation set with the same scheduling vector, however, without the wind disturbances. The outputs of the blade root moments M_1, M_2, and M_3 score very high VAF values, all within 98% and 100%. The outputs $\dot{x}_{fa}$ and $\dot{x}_{sw}$ are more affected by the wind disturbance and this results in a lower VAF value. The bode diagrams with the generator speed Ω_{ge} as output are given in Figure 4. For the transfer function between the generator torque and the generator speed, the low frequent behavior shows a large variance due to the high disturbance which has a significant effect on the estimation of pole belonging to the integrator. However, this is a well-known phenomena in LTI system identification.

4. CONCLUSION

Wind turbines are non-linear systems, although their non-linearity is linearly dependent on measurable scheduling signals and therefore they can be modeled in the LPV framework. With LPV controller synthesis, which is strongly related to robust controller design, gain-scheduled controllers can be calculated with guaranteed stability and performance margins. In this paper we discussed LPV system identification and we proposed a subspace algorithm to identify the rotational dynamics of a HAWT. We exploited the fact that the system experienced the same time-variation a number of times. We used LTI system identification techniques to identify a number of observability matrices and state sequences which are, inherent to subspace identification, identified in a different state basis. We showed that by formulating an intersection problem all the states can be reconstructed in a general state basis from which the system matrices could be estimated. We showed the working of the proposed algorithm on a nonlinear model of a wind turbine which was operating in closed loop.

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