

## Adaptive temperature control in a freezer with on-off actuation

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**Abstract:** This manuscript describes an adaptive scheme for temperature control in household freezers. Peculiar of the proposed control scheme is the use of standard sensing and actuation equipment, with particular reference to the on-off compressor. After illustrating the rationale of the proposed control, some tests on a detailed simulation model are reported, to validate the approach and show its effectiveness with respect to both food preservation and energy consumption.

Keywords: On-off control; Adaptive control.

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### 1. INTRODUCTION AND PROBLEM STATEMENT

The context of the presented research is temperature control in household refrigerators and freezers. The importance of that research domain is witnessed by pioneering works such as Jarrett (1972), industry-related research like that described in Knoop et al. (1988), modelling and simulation efforts like those of Ikegami et al. (2001), and many other scientific and industrial publications.

In recent years, environmental and energy-related issues have caused a new and increased interest on the matter, see e.g. Tanaka (2001). The control of household refrigerators and freezers, especially if both food preservation and energy consumption are accounted for, is quite difficult a problem, that is further complicated by the typical presence of low-end sensing and actuating systems—a choice that is easily motivated in terms of final product cost. As a result of both the difficulty and the industrial interest of the problem, many control approaches were historically attempted, ranging from traditional schemes based on relay control up to fuzzy logic application, like e.g. Becker et al. (1994), and/or neural networks, such as Choi et al. (1998).

Omitting a detailed review, that would not fit in this manuscript, one can however observe that the developed applications can be divided in two categories: those employing a variable-speed compressor (e.g. Tassou and Qureshi (1994); Rasmussen and Ritchie (1997) and many others), and those with a fixed-speed (i.e., on-off) one, such as Jarrett (1972) and others. It is worth noting that the on-off solution appears to be less frequently investigated in the scientific literature, especially in recent years. Comparisons of the two different types of actuation also exist, see e.g. Wicks (2000), but when cost is an issue, the on-off solution is very frequently taken, and therefore the problem complexity is enhanced.

The typical appliance of the type considered herein has two cavities (one for the freezer and one for the refrigerator) and a single refrigerating source, composed of a standard cooling circuit with an on-off compressor. A damper is present between

the refrigerator and the freezer cavities, and a fan is installed to facilitate air circulation in the appliance. Both the damper and the fan have on-off actuators. The controlled variables are the freezer and the refrigerator temperatures, while the control signals are the compressor and fan activation, and the damper opening. The control schemes typically employed are based on simple relays with hysteresis, which make the controlled temperatures eventually reach limit-cycle behaviours.

The period and amplitude of those limit cycles influence both the food preservation and the energy consumption, not to mention the life of the compressor and of the other active components. In this work the focus is restricted to the control of a single temperature, namely that of the freezer. The presented scheme is being extended to the control of both the freezer and refrigerator temperatures, and further extensions can be envisaged to multi-zone temperature control; that activity will be presented in future works.

Clearly, the limit cycle reached by the freezer temperature, if this happens, is determined by the cooling circuit, the compressor relay (the other actuators will come into play later on), and the freezer contents. Given the circuit characteristics, that come with the appliance, if the freezer contents were constant one could tune the relay once to achieve the desired cycle characteristics, and then leave the relay parameters untouched, but such a relay tuning might be critical, and in the most general case there is no guarantee that the desired cycle characteristics can be achieved.

In any case, the relay tuning problem is dominated by the fact that the freezer contents is not constant at all, and therefore - if tight requirements on preservation and consumption are present - an adaptive control scheme is practically unavoidable. This manuscript proposes such a scheme, having as objective a limit cycle of the freezer temperature with the minimum possible amplitude, for apparent food preservation reasons, and the longest possible period, so as to minimise the compressor upset. The presented scheme employs on-off actuators, and is sufficiently simple to be implemented on the microcontroller installed in the appliance.

## 2. THE PROPOSED ADAPTIVE SCHEME

The well known describing function approximation states that a linear system, subject to relay feedback, will enter a permanent oscillation (or limit cycle) if the Nyquist curve  $P(j\omega)$  of that linear system intersects the critical point locus of the relay (we omit for brevity to discuss the limit cycle stability, see e.g. Åström and Hägglund (1991) for a complete discussion). Hence, given the linear system and the relay characteristics, there can be only one possible limit cycle. For example, in the absence of relay hysteresis, the limit cycle is at the frequency  $\omega_u$  at which  $P(j\omega)$  intersects the real negative axis. That point is frequently termed the 'ultimate point', and it is widely used in the so called 'relay-based identification' (Åström and Hägglund, 1991; Yu, 1999; Wang et al., 2003), where relay feedback is a way to figure out some information on the frequency response of an unknown system.

If a linear block  $F(s)$  is inserted in the loop, the frequency response of the linear dynamics seen by the relay becomes  $F(j\omega)P(j\omega)$ . The describing function approximation still holds, apparently, and a limit cycle may arise in the same hypotheses, simply replacing  $P$  with  $FP$ . This principle is used in the relay-based identification domain, where by altering  $F$  one may 'explore' different points of  $P(j\omega)$  (Leva, 1993). For example, if  $F(s) = 1/s$  and the relay has no (or negligible) hysteresis, the point found on  $P(j\omega)$  is the one with phase  $-90^\circ$ , since the limit cycle yields the ultimate point of  $F(j\omega)P(j\omega)$ , and  $\arg^\circ(F(j\omega)) = -90^\circ$ . The above principle is illustrated in figure 1.

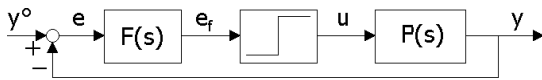


Fig. 1. Basic scheme for relay-based identification exploring  $P(j\omega)$ .

The same idea can be used in another way, however, orienting its purpose to control rather than identification. By suitably designing  $F(s)$  in the scheme of figure 1, it is in fact possible to select the characteristics of the induced oscillation so that they match some pre-specified criteria, if this is possible given  $P(j\omega)$ . To the best of the authors' knowledge, the idea above has not yet been explored in the literature—at least, not for the application and with the adaptive features treated herein.

Such a scheme is particularly interesting for applications like that considered herein, as both food preservations and energy consumption requirements can be expressed as desired characteristics of the obtained limit cycle. Clearly, for the design of a scheme like that of figure 1 to succeed, it is necessary that the dynamic behaviour of the device under control be represented by a known, constant-parameter  $P(j\omega)$  in all the operating conditions of interest. If this is not the case, one may devise an *adaptive* scheme, where the filter  $F(s)$  is modified on-line so that the limit cycle gets to match the specified criteria. Such an adaptive scheme is summarised in figure 2.

The problem addressed herein requires adaptation because the freezer dynamics depend on its contents, that are inherently time varying, although they may remain constant for long periods of time, and in any case are substantially unknown. Applying the ideas sketched out in figure 2 gives rise to an adaptive on-off regulator, as the control output comes directly from the relay, and provides the basis of the proposed solution.

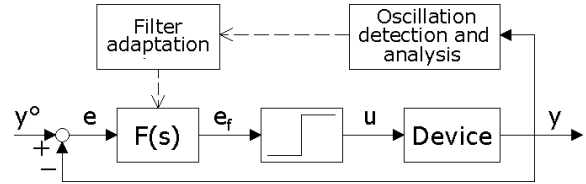


Fig. 2. Conceptual scheme of the proposed solution.

This chapter explains that solution and investigates its validity with respect to the addressed problem.

### 2.1 The main idea

The control goal is to achieve an oscillation of the freezer temperature  $T_{scfz}$  (or, almost equivalently, of its measurement  $T_{scfzf}$ , provided by a conveniently located probe), the amplitude of which is less or equal to a given  $A_{max}$ , and the frequency of which is less or equal to a given  $\omega_{max}$ . Since the amplitude  $D$  of the relay command is fixed in the considered application, the problem corresponds to inducing a sustained oscillation at a frequency  $\omega_{ox}$  such that

$$\begin{cases} \omega_{ox} \leq \omega_{max} \\ |P(j\omega_{ox})| \leq P_{max} \end{cases} \quad (1)$$

where

$$P_{max} = \frac{\pi A_{max}}{4D}. \quad (2)$$

To do this, the idea is to iteratively modify the filter  $F(s)$ . Under the assumption that both the magnitude and the phase of  $P(j\omega)$  decrease monotonically with  $\omega$  in all the frequency range of interest, that iterative modification can be carried out in a straightforward way. That assumption is very reasonable and certainly verified in the case at hand, as it corresponds to stating that the process dynamics has essentially a low-pass behaviour, and does not exhibit low-frequency zeroes, nor any resonance or antiresonance.

### 2.2 Feasibility criterion and applicability limits

The use of the filter  $F(s)$  in the proposed solution is illustrated in figure 3. As can be seen, the magnitude and phase effect of  $F(s)$  can cause the oscillation to occur at various points of  $P(j\omega)$ , i.e., to have different frequencies and amplitudes.

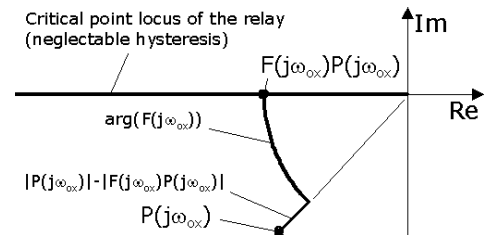


Fig. 3. Use of the filter to make the oscillation occur at various points of  $P(j\omega)$ .

Apparently, however, it is not possible to achieve *any* (amplitude, frequency) couple, as the amplitude and the frequency of the oscillation must correspond to a point of  $P(j\omega)$ . That is, if the available control signal is the output of a relay, the process dynamics reflects into an admissible set of oscillations. Note that changing the relay amplitude and/or hysteresis would simply modify its critical point locus, and therefore the admissible

oscillations' set would remain unchanged: to widen that set, it is necessary to introduce a *modulating* control action.

The considerations above lead to define the applicability limits of the proposed solution as in figures 4 and 5, where those limits are illustrated on the Nyquist and on the Bode magnitude diagrams. For a feasible solution to exist, there must be some points of  $P(j\omega)$  with frequency and magnitude fulfilling both constraints (1).

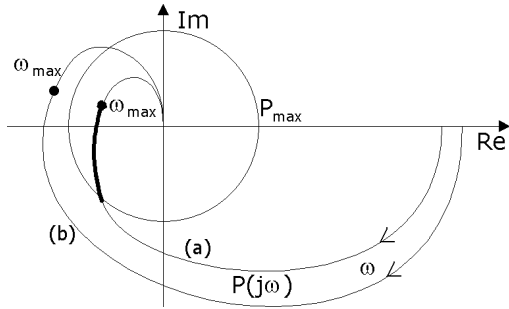


Fig. 4. The feasibility criterion of the proposed solution as seen on the Nyquist diagram; case (a) is feasible, case (b) is not.

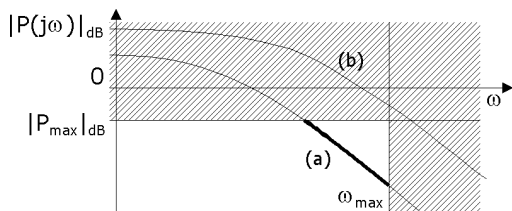


Fig. 5. The feasibility criterion of the proposed solution as seen on the magnitude Bode diagram; case (a) is feasible, case (b) is not.

Given the meaning of  $P(j\omega)$  in the present context, and the reasonable continuity assumptions introduced thereof, a simple frequency domain identification procedure, based on relay tests or also on open-loop sine experiments (which means trusting the filtering action hypothesis completely), can easily lead to decide whether or not the proposed solution is applicable to a given device or, equivalently, how much the constraints on the oscillation frequency and amplitude can be narrowed on that particular device.

As a final applicability remark, it must be noted that the entire describing function framework relies on the hypothesis that the dynamic block in the relay feedback loop is linear. This is apparently false in the case at hand, as the control inputs (particularly the damper, but also the compressor activation) act on the system (also) by modifying the thermal exchange coefficients between the masses where energy storages may be thought to be concentrated. Therefore, the effects of those inputs on the controlled temperatures cannot be described with a linear model. Nonetheless, based on an extensive experimental activity, two facts can be observed:

- (1) the response of the controlled variable to a step on any of the control inputs has a behaviour that can be represented by a linear, high-order model precisely enough (of course that linear model may vary depending on the control input used, and the state of the other inputs, as *in the large* the model is nonlinear);

- (2) if a relay feedback loop is set up having one of the possible control inputs as the manipulated variable, the device does enter an oscillatory condition, and the aspect of the obtained oscillations is 'reasonably sinusoidal', i.e., not less similar to a sine wave than the controlled variable's behaviour observed in several literature cases where the describing function approximation was used successfully (Yu, 1999).

Hence, one can apply the describing function approximation to estimate a 'point of the process frequency response', but bearing in mind that the so identified point belongs to a sort of local model, representing only the device's behaviour in that particular oscillatory condition. Any conclusion drawn (or action taken) with such a consciousness will make sense. In other words, one can proceed to the derivation of the proposed solution, that involves the concept of 'process frequency response' and refers to that response with the symbol  $P(j\omega)$ , by simply recalling that any point identified on  $P(j\omega)$  has to be thought of as belonging to the frequency response of a local model, representing the process dynamics in the vicinity of the corresponding frequency, and that a reasonable continuity exists among all those local models in the light of the 'reasonably linear' aspect of the step responses obtained from the model, that - conversely - covers all the frequencies.

To further explain the concept, given the invariance of the (non-linear) system under control, the feedback relay setting determines univocally the oscillatory behaviour, and its fundamental frequency component can be envisaged as the result of a given amplitude-phase characteristic of the system at that frequency. It is only for ease of representation, and for uniformity with respect to the notation used in the literature, that such characteristic is thought of as a single point of the frequency response of a (locally approximating in the frequency domain) linear system.

### 2.3 Structure of the adaptive on-off controller (AOOC)

The adaptive on-off controller (AOOC) proposed herein is composed of a relay, a linear filter, and the filter adaptation algorithm. The AOOC components are described in the following. The relay has ideally no hysteresis, and in practice it has only the amount of hysteresis needed to cope with measurement noise, yet allowing to safely assume that its critical point locus be the real negative semiaxis. Notice that it would be possible, and easy, to generalise the proposed algorithm to the use of a relay with hysteresis, as the only effect of hysteresis is that the critical point locus in not the real negative semiaxis, but a straight line parallel to it and located in the third quadrant. For simplicity and clarity, however, the algorithm is here derived and used with a hysteresis-free relay.

The linear filter used in the AOOC has the form

$$F(s) = \frac{1 + sT_z}{1 + sT_p} \quad (3)$$

so that, by means of convenient choice of the two time constants  $T_z$  and  $T_p$ , a phase contribution in the range  $(-90^\circ, +90^\circ)$  can be introduced at any frequency of interest. The gain of  $F(s)$  is equal to 1 for apparent convenience reasons. Parameters  $T_z$  and  $T_p$  are determined from the phase shift (or correction) that the filter has to introduce at the oscillation frequency.

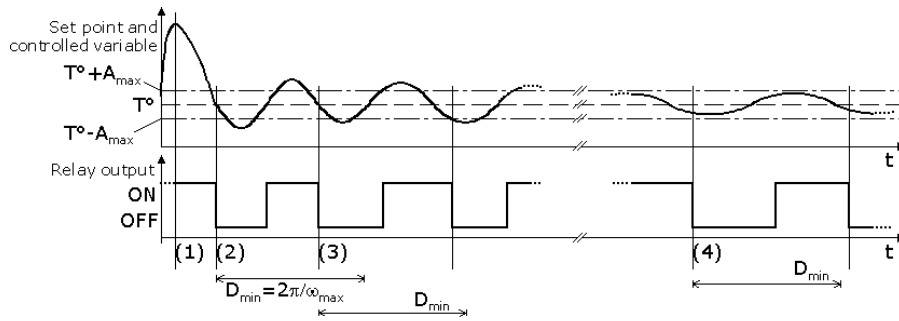


Fig. 6. Typical behaviour of the controlled variable during the operation of the proposed AOOC.

#### 2.4 Description of the AOOC algorithm

To obtain the AOOC, the scheme of figure 2 is therefore implemented and operated as follows. To illustrate its operation, a possible behaviour of the controlled variable is schematically shown in figure 6, where the main facts and time instants concerning the AOOC operation are evidenced.

- (1) The AOOC is activated at the end of a defrosting operation. Referring to figure 6, this is time (1).
- (2) When the AOOC is activated, the relay without hysteresis is connected to the control input. The relay input is the error filtered by  $F(s)$ . Initially,  $F(s)$  is set to 1, or initialised so as to provide a pre-specified phase shift at a pre-specified frequency. This may be useful for tailoring (off-line) the AOOC behaviour to a particular device.
- (3) The AOOC waits for the controlled variable to cross the set point. In figure 6, this is time (2). The AOOC then starts monitoring the oscillation until the next relay toggle in the same direction as that of time (2). In figure 6, the oscillation ends at time (3).
- (4) At the end of any oscillation, its characteristics are checked against the specification constraints, i.e., it is checked whether or not the oscillation amplitude is smaller than  $A_{max}$ , and its duration larger than  $D_{min} = 2\pi/\omega_{max}$ . If this is the case,  $F(s)$  remains unchanged and the AOOC continues operating as a fixed relay-plus-filter controller. Any subsequent oscillation is checked, however, so that  $F(s)$  may be modified when required.  
 It is also possible to decide whether or not an oscillation has to be considered depending on the state of the system, and particularly of its control inputs. In fact, some of those inputs (typically, for example, the damper) produce an apparent and sensitive change of the process dynamics' time scale, and in some of the possible situations it may be beneficial not to modify the filter. Taking such decisions has to be based essentially on experiments, and therefore the AOOC algorithm provides the necessary functionalities.
- (5) If the oscillation does not comply with the specifications, the filter is modified. In figure 6, this happens for example at time (3). To modify the filter, first it is determined whether the oscillation frequency must be increased or decreased. Recalling figures 4 and 5, this also means reducing or increasing the oscillation amplitude, respectively. If the oscillation frequency has to be increased, the filter time constants are computed so that, at the current oscillation frequency, the phase of the filter decreases of a pre-specified quantity; in the opposite case, the filter time constants are computed so that, at the current os-

cillation frequency, the phase of the filter increases of a pre-specified quantity. If the overall filter phase is negative (i.e., if a lag is required),  $T_z$  is set to zero and only a pole is used; in the opposite case, a structure with zero and high-frequency pole is parametrised.

- (6) The AOOC iterates the same behaviour for every subsequent oscillation.

Note that, strictly speaking, any conclusion drawn based on the describing function approximation should wait for the oscillation considered to become *permanent*. For efficiency reasons, however, in the AOOC the filter is modified at every complete oscillation. This is justified by the fact that, in virtually any device the AOOC may come across, it is possible to relate the characteristics of the permanent oscillations to their startup phase, as proposed e.g. in Panda and Yu (2003); Thyagarajan and Yu (2003) for a number of different dynamic structures.

As discussed in the quoted works, such relationships may be, in general, quite complex—see especially the detailed analysis reported in Panda and Yu (2003), in conjunction with the 'shape factor' approach documented in Thyagarajan and Yu (2003). In the particular case considered herein, however, the only conclusion drawn from the analysis of the first oscillation is *de facto* the *sign* of the phase shift variation that is required on the part of the filter  $F(s)$ : such a simple decision can be taken safely based on the first oscillation.

In the light of the considerations above, a final - yet important and more general - remark is in order. The filter design, as carried out herein, is largely independent of the appliance model, the only important fact being the magnitude and phase monotonicity, in the sense stated above. In fact, at each algorithm iteration, the required system information is 'condensed' at the oscillation frequency (see e.g. Leva (2005) for a discussion on that specific matter), and re-gathered based on the actual device's behaviour.

This remark allows to be confident on the method's capability to deal with machine-to-machine variability, while the simulation tests performed prove that the method is suitable for the typical, 'average' dynamics of the considered appliances.

### 3. ACTIVATION OF THE AOOC

The AOOC is designed to be always active, as the way it was derived and it operates indicate that, in the absence of system perturbations, after a defrosting operation (or a door opening), the parameters of  $F(s)$ , and consequently the oscillation characteristics, reach some asymptotic value with no or minimum variations.

The only possible exception to the continuous activity of the AOOC could be when an abnormally high oscillation is detected, as results from a defrosting operation or a door opening. In this case, it may be advisable that the filter stays unchanged for one complete oscillation cycle, otherwise the 'abnormal' oscillation could lead to erratic results in the filter settings computed afterwards. As a consequence, data recording is restarted at the end of this oscillation and filter adaptation at the end of the subsequent one.

Notice that if no direct information is available, the 'abnormal' oscillation can be easily detected by monitoring the temperature behaviour for rapid upward drifts, and checking for crossings of a given threshold. Such a threshold can be determined experimentally, based on the temperature reference setting and the 'typical' use of the particular appliance considered—an activity that would stray from the scope of this work.

#### 4. SIMULATION EXAMPLES

This section reports a small sample of the numerous simulation tests performed to validate the proposed adaptation method, using a sufficiently detailed model of the freezer (not described here for space limitations), implemented in the Matlab programming language together with the control and adaptation algorithm. The reported activity refers to the Whirlpool Combi-Sobieski 2000 model.

For all the reported tests a set-point of  $-23.5\text{ }^{\circ}\text{C}$  is assumed for the freezer temperature probe, while the control specifications prescribe a  $\pm 2\text{ }^{\circ}\text{C}$  temperature swing, and a minimum oscillation period is assumed of 8 min. The performed tests include (1) standard control of the compressor and damper relays, (2) AOOC of the compressor relay, standard control of the damper relay, (3) AOOC of the compressor relay with the damper always open, and (4) AOOC of the compressor relay with the damper always closed. For brevity, we report here only one test of type 2 (test A) and one of type 4 (test B). For both tests, the corresponding figure (where the time axis is in hours) shows

- the behaviour of temperatures  $T_{scfzf}$  (freezer probe),  $T_{ev}$  (evaporator),  $T_{shfr}$  (refrigerator probe);
- the detail of  $T_{scfzf}$  (freezer probe), with the required range limits;
- the on-off commands of compressor activation (top), damper opening (middle), and fan activation (bottom);
- the measured oscillation period for filter adaptation (where appropriate).

As a general remark, it is worth observing that the presented simulations investigate the controlled appliance operation over a long period of time (typically, several hours). Long tests as those presented are reasonably consistent with the way freezers are normally operated, and also (which is more important) they reflect the procedures that are followed in order to assess the food preservation and energy consumption characteristics of an appliance. As a result of that panorama, quite intuitively, control applications of the type addressed herein may be difficult, but are not time-critical.

##### 4.1 Adaptive control of the compressor relay, standard control of the damper relay

In this test the adaptive filter relay control scheme is used to command the compressor activation, while the standard

hysteresis relay setting is adopted for the control of the damper opening, activating the adaptation after the defrosting phase. The results are in figure 7. After the defrosting phase 47 cycles are detected, some of which induce a filter modification. Notice that the oscillation characteristics may vary significantly (as an indirect effect of the 2nd relay) even when the filter transfer function is kept constant. As a result, specifications are apparently attained. When the damper is closed, just a temporary reduction of oscillation amplitude and period is observed, with a corresponding increased switching frequency of the compressor.

##### 4.2 Adaptive control of the compressor relay, damper always closed

In this test the adaptive filter relay control scheme is used for compressor control, while the damper is kept always closed. The fan is on when the compressor is on, off otherwise. The results are in figure 8. Also in this case, specifications are attained, though the oscillation period is shorter, and nearer to the 8 min limit. After an initial phase requiring adjustments, the filter is kept constant, resulting in a nearly stationary behaviour. It is worth noting that, since the presence of two independent relay controllers may generate double-oscillator type chaotic dynamics, in test A a steady state oscillation is not reached, and the filter is continuously re-tuned. Conversely, when the damper command is kept fixed during the whole operating time (either open or closed), either a permanent oscillation (a case not shown here) or a quasi-stationary behaviour (as in test B) are eventually obtained, so that the adaptive algorithm could even be switched off.

#### 5. CONCLUSIONS

An adaptive control scheme for temperature control in freezers was presented. The objective of the scheme is to achieve, by means of on-off control, an oscillatory behaviour of the temperature measured by the freezer probe, having an amplitude smaller than a given limit, and a duration longer than a given limit. The proposed scheme, the complexity of which is adequate for implementation in production controllers, was extensively analysed in simulation, so as to determine the validity limits of the underlying approach. Results are definitely encouraging.

The activity was carried out with reference to the Whirlpool Combi-Sobieski 2000 model, considering the (standardised test) condition in which the doors are closed, a cold and a hot pack are present both in the freezer and in the refrigerator, and the device has just undergone a defrosting operation. However, the generalisation of the proposed scheme to other products and operating conditions seems to be straightforward. Further work is underway to implement the presented scheme in the freezer controller, and test it experimentally. Also, research is being spent on the extension of the proposed scheme to the control of more than one temperature, so as to extend the scheme to the of appliances with multi-zone temperature control, and possibly multiple refrigerating sources.

#### ACKNOWLEDGEMENTS

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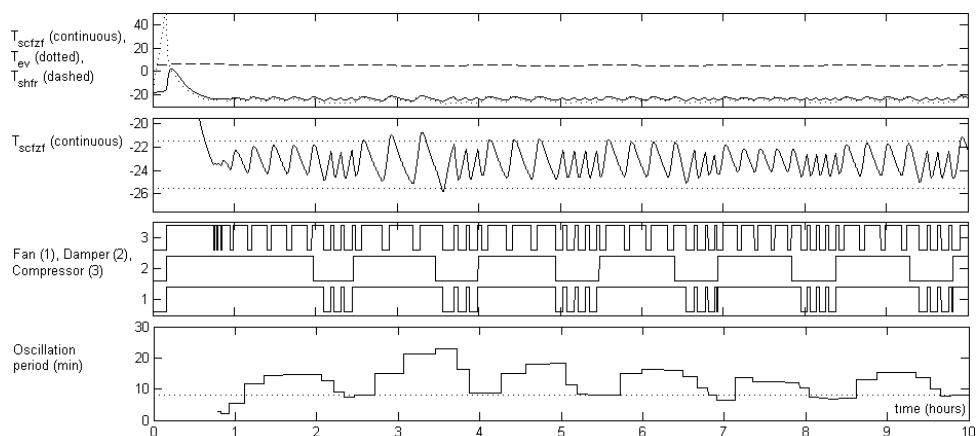


Fig. 7. Test A: adaptive control of the compressor relay, standard control of the damper relay.

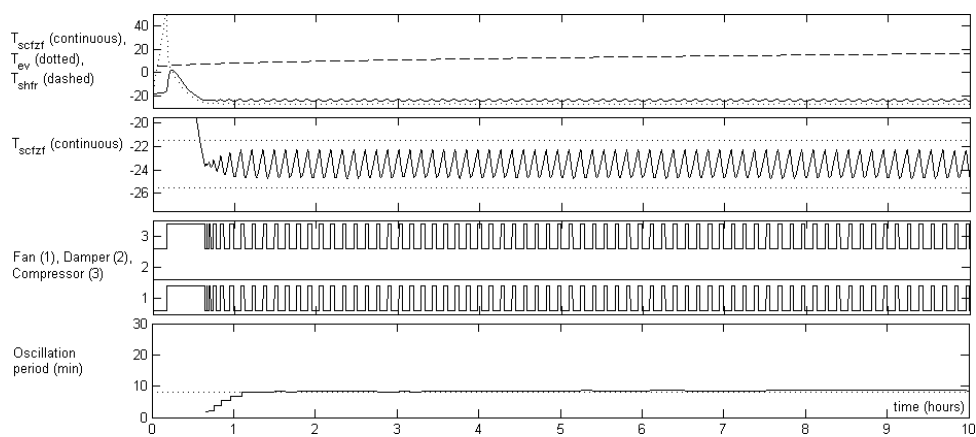


Fig. 8. Test B: adaptive control of the compressor relay, damper always closed.

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