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Adaptive LQ Approach Used in Conductivity Control inside Continuous-Stirred Tank Reactor

Jiri Vojtesek*. Petr Dostal*

**Department of Process Control, Faculty of Applied Informatics,*
Tomas Bata University in Zlin, Czech Republic
(Tel: 00420576035199; e-mail: {vojtesek,dostalp}@fai.utb.cz).

Abstract: This paper deals adaptive control of the real model represented by the Continuous-stirred tank reactor (CSTR). This type of reactor belongs to the class of systems with lumped-parameters. The chemical process inside the reactor is dilution of the salt with the clean water. The resulted mixture has specific conductivity, which is about to be controlled, depends on the content of the salt inside the reactant. Used adaptive approach is based on recursive identification of the system's parameters during the control. A polynomial approach used for the controller synthesis has satisfied control requirements and moreover, it could be used for systems with negative properties such as nonlinearity, non-minimum phase etc.

1. INTRODUCTION

Technological processes not only in the chemical industry belongs have nonlinear properties. CSTR is typical member of nonlinear systems with lumped parameters. Some types of reactors are characterized with the non-minimum phase behaviour, time delay, changing sign of gain etc. which make them uneasy to control (Ingham *et al*., 2000).

One way how to deal with these negative properties is to create mathematical model of this system and simulate the static and dynamic behaviour with the use of mathematical software like Matlab, Mathematica etc. This, so called simulation methods, are very popular today because of benefits over an experiment on a real system, which is sometimes not feasible and can be dangerous, or time and money demanding.

The main disadvantage of the simulation is that the complexity of the system must be reduced for example with the use of simplifications, approximations etc (Lyuben, 1989). These improvements help solvability but reduce reliability of the mathematical model. It is requested to verify simulation results on a real model.

Real models are small and cheap representatives of the real, usually big and expensive, plants. Results are than in smaller scale but much closer to real ones. Disadvantage of the real model is that it is not as configurable as the simulation model in the computer, experiments are usually more time demanding because they include operations like preparation, loading, cleaning, unloading etc. and experiments are more expensive. The real model is in this case represented by the CSTR which is one part of the Armfield's multifunctional Process Control Teaching (PCT40) system.

This paper is focused on the adaptive control of the conductivity inside the CSTR where chemical dilution of the sodium chloride (NaCl) in water (H_2O). This chemical is non-toxic and the conductivity control could be carried out in safe conditions of temperature and pressure. The adaptive approach used in this work is based the choice of the External Linear Model (ELM) of the originally nonlinear system parameters of which are estimated recursively during the control. The polynomial approach together with the Linear-Quadratic (LQ) method satisfies basic control requirements such as the stability, the disturbance attenuation and the reference tracking (Kucera, 1993).

The resulted controller was verified for two basic control configurations with the one degree-of-freedom (1DOF) and two degrees-of-freedom (2DOF) each for more values of the weighting factor which affects the control response (Grimble, 1994).

The control routines are implemented via Matlab's RealTime toolbox and Humusoft MF624 technological card.

2. CONTINUOUS STIRRED TANK REACTOR

The CSTR is one of the most common used equipment used in the chemical industry because they could be easily controlled. The real model used in this case is CSTR which is part of the multifunctional process control teaching system – the Armfield PCT40. This device is designed especially for teaching of a wide range of technological and chemical processes, such as temperature control in heat exchangers, flow control, level control in water tanks, pressure control and finally conductivity and pH control in additional PCT 41 and 42 units, which is CSTR. The schematic representation of the model is displayed in Fig. 1.

PCT40 unit consists of two process vessels, several pumps, sensors and connection to the computer. Additional PCT 41 and 42 units represent a chemical reactor with a stirrer inside and a cooling/heating lid.

Water can be injected inside the reactor via a normally closed solenoid valve (SOL1) or by a Proportional Solenoid Valve (PSV). The third option how to feed water inside the system is with the use of one of peristaltic pumps, A or B, and the second pump could be used for reactant feeding. This option was used in the following studies.

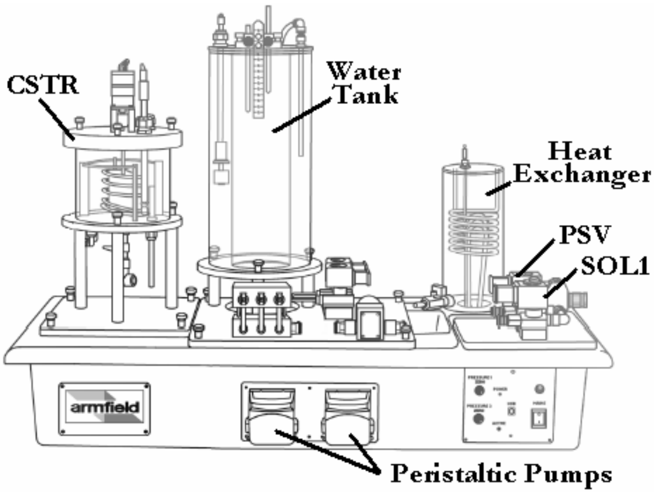

Fig. 1 Schematic representation of multifunctional process control teaching system PCT40

The technological parameters of the reactor are shown in the following table.

Table 1 Technological parameters of CSTR

Parameter	Range
Vessel diameter	0.153 *m*
Maximum vessel depth	0.108 *m*
Maximum operation volume	2 *l*
Minimum vessel depth	0.054 *m*
Minimum operation volume	1 *l*

Two types of connection are at the disposal. The first connection with Universal Serial Bus (USB) is included in the standard packing. A disadvantage of this system is that there is no possibility to implement other control strategies programmed in Matlab, C++ etc.

This disadvantage can be overcome with the second type of connection via a 60-way I/O connector or 50-way I/O connector to a technological PCI cart in the computer. The technological card used in this case is MF624 multifunction I/O card from Humusoft. This card has 8 inputs and 8 outputs, which is sufficient if we operate only one control exercise at a time. The whole system provides 9 inputs and 17 outputs if all exercises work at in the same time. That is why we use two MF624 cards.

The connection to PC via MF624 cards makes all control exercises fully programmable with the use of Matlab's Real-time toolbox and Simulink or from the Matlab's command window.

2.1 Description of the Chemical Process

The producer of PCT40 recommends dilution of potassium bicarbonate ($KHCO_3$) in water. This chemical is non-toxic and the conductivity control could be carried out in safe conditions of temperature and pressure. However, in our case potassium bicarbonate was replaced by ordinary sodium chloride (NaCl). The main reason of this substitution was the cost of experiment – twenty liters of 20% solution of potassium bicarbonate is made from 4.5 *kg* of dry $KHCO_3$ is 75 times more expensive than the same amount of NaCl. This substitution was made with the agreement of Armfield, the producer of the PCT40.

Thus, the chemical used in the experiments was 5% solution of NaCl, which means that 20 litres of the chemical consists of 1 *kg* dry NaCl solved in 19.5 litres of water. The conductivity of this solution is about 60 *mS*, which is relatively high and suitable for basic experiments. The conductivity, which will be controlled, changes with the degree of salinity. The chemical and water are fed in the reactor by peristaltic pumps. Volumetric flow rates of these pumps could be theoretically set in the range 0÷100 %; however, setting lower than 20 % results in very small revolutions of the rotor and the produced force is not high enough to transport the fluid from the barrel. The range set into the input recomputed to the volumetric flow rate is shown in Table 2.

Table 2 Speed of pumps A and B in % recomputed to the flow rate

Range set at the input to pumps A and B	Flow rate of pump A($l.min^{-1}$)	Flow rate of pump B ($l.min^{-1}$)
100%	1.12	1.11
75%	0.80	0.80
50%	0.48	0.51
30%	0.22	0.26

Although the system could be understood as multi-input (input flow rate of water and 5% salt solution) single-output (conductivity), only flow rate of water was used as a manipulated (input) variable. The flow rate of the chemical for all measurements is set to 30% (0.26 $l.min^{-1}$). It is expected that the system belongs to the class of lumped parameters systems which means that the state variable (in our case the conductivity) depends only on the time variable. This condition is fulfilled with the stirrer switched on during measurements. Even though the reactor has a heating/cooling coil, this equipment is not used in the experiments. The used clean water is ordinary cold water from the standard water distribution.

2.2 Static and Dynamic Analyses

The static study displays steady-state values of the conductivity. It can be said that the system has mostly linear behaviour for the input variable u = 30÷70 %, which is represented by the volumetric flow rate of clean water through peristaltic pump A. As can be seen in Fig. 2, the static behaviour for interval u = 70÷100 % has nonlinear behaviour. The flow rate lower than 30% was not taken into the examination because the revolutions of the pump is too low – see the remarks above.

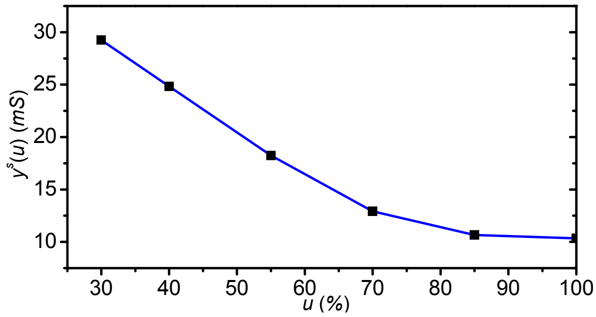

Fig. 2 Static analysis of the conductivity inside the reactor

Dynamic analysis was done for six step changes of volumetric flow rate – Δu = 30%, 40%, 55%, 70%, 85% and 100%. The time of the measurement was 720 s (12 min) and the sampling period for the values retrieval was $T_v = 1\ s$. The results are in Fig. 3. All step responses has similar behaviour as a system with first order transfer function.

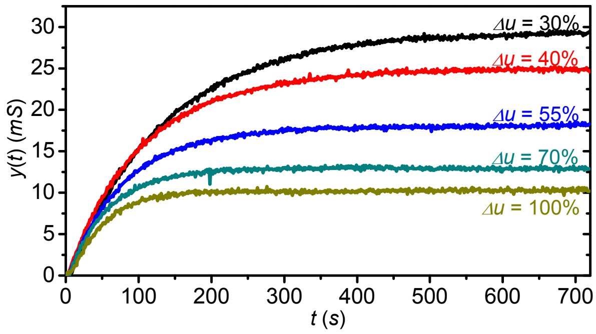

Fig. 3 Dynamic analysis of the conductivity inside the reactor

3. ADAPTIVE CONTROL

The basic idea of adaptive control is that parameters or the structure of the controller are adapted to parameters of the controlled plant according to the selected criterion (Bobal *et al.*, 2005).

The adaptive approach in this work is based on choosing an *external linear model* (ELM) of the original nonlinear system whose parameters are recursively identified during the control. Parameters of the resulted continuous controller are recomputed in every step from the estimated parameters of the ELM.

3.1 External Linear Model

The main types of ELM are *continuous-time* (CT) models and *discrete-time* models. The ELM used in this work is based on delta models (Mukhopadhyay *et al.*, 1992) which is a special type of discrete-time models where each discrete difference is related to the sampling period T_v.

Even though dynamic responses shown in Fig. 3 can be described by first order transfer functions, the external linear model of this process in the continuous time, which is used in the control analysis, is of the second order with relative order one, i.e.

$$G(s)=\frac{b(s)}{a(s)}=\frac{Y(s)}{U(s)}=\frac{b_1 s+b_0}{s^2+a_1 s+a_0} \tag{1}$$

where $U(s)$ and $Y(s)$ are Laplace transforms of the input and output variables.

3.2 Parameter Estimation

Equation (1) could be rewritten to:

$$a'(\delta)y(t')=b'(\delta)u(t') \tag{2}$$

where polynomials $a'(\delta)$, $b'(\delta)$ are discrete polynomials and their coefficients are different from those of the CT model $a(s)$ and $b(s)$ in (1). Time t' is discrete time.

Equation (2) has form of the differential equation for the identification:

$$\begin{aligned} y_\delta(k)=&-a_1 y_\delta(k-1)-a_0 y_\delta(k-2)+\\ &+b_1 u_\delta(k-1)+b_0 u_\delta(k-2)\end{aligned} \tag{3}$$

where y_δ is recomputed output to the δ-model computed via:

$$\begin{aligned} &y_\delta(k)=\frac{y(k)-2y(k-1)+y(k-2)}{T_v^2} \\ &y_\delta(k-1)=\frac{y(k-1)-y(k-2)}{T_v} \qquad u_\delta(k-1)=\frac{u(k-1)-u(k-2)}{T_v} \\ &y_\delta(k-2)=y(k-2) \qquad u_\delta(k-2)=u(k-2)\end{aligned} \tag{4}$$

where T_v is the sampling period, the data vector is

$$\boldsymbol{\phi}^T(k-1)=\left[-y_\delta(k-1),-y_\delta(k-2),u_\delta(k-1),u_\delta(k-2)\right] \tag{5}$$

and the vector of estimated parameters

$$\hat{\boldsymbol{\theta}}^T(k)=\left[a_1',a_0',b_1',b_0'\right] \tag{6}$$

could be computed from the ARX (Auto-Regressive eXogenous) model

$$y_\delta(k)=\hat{\boldsymbol{\theta}}^T(k)\boldsymbol{\phi}(k-1) \tag{7}$$

This model is very often used because the data vector consists only of variables which can be directly measured and there is no need to reconstruct them. The deterministic part can be optional, the estimated output variable is linear function of the measured data and a linear regression can be used for parameter estimation.

The Recursive Least-Squares (RLS) method is used for the parameter estimation in this work. The RLS method is well-known and widely used for the parameter estimation (Fikar and Mikles, 1999). It is usually modified with some kind of forgetting, exponential or directional (Kulhavy and Karny, 1984), because parameters of the identified system can vary during the control which is typical for nonlinear systems and the use of some forgetting factor could result in better output response.

The RLS method with exponential forgetting is describe by the set of equations:

$$\begin{aligned} &\varepsilon(k) = y(k) - \boldsymbol{\varphi}^T(k)\cdot\hat{\boldsymbol{\theta}}(k-1) \\ &\gamma(k) = \left[1+\boldsymbol{\varphi}^T(k)\cdot\mathbf{P}(k-1)\cdot\boldsymbol{\varphi}(k)\right]^{-1} \\ &\boldsymbol{L}(k) = \gamma(k)\cdot\mathbf{P}(k-1)\cdot\boldsymbol{\varphi}(k) \\ &\mathbf{P}(k) = \frac{1}{\lambda_1(k-1)}\left[\mathbf{P}(k-1) - \frac{\mathbf{P}(k-1)\cdot\boldsymbol{\varphi}(k)\cdot\boldsymbol{\varphi}^T(k)\cdot\mathbf{P}(k-1)}{\lambda_1(k-1)+\boldsymbol{\varphi}^T(k)\cdot\mathbf{P}(k-1)\cdot\boldsymbol{\varphi}(k)}\right] \\ &\hat{\boldsymbol{\theta}}(k) = \hat{\boldsymbol{\theta}}(k-1) + \boldsymbol{L}(k)\varepsilon(k) \end{aligned} \quad (8)$$

Several types of exponential forgetting can be used, e.g. like RLS with constant exponential forgetting, RLS with increasing exp. forgetting etc. RLS with the changing exp. forgetting is used for parameter estimation, where the changing forgetting factor λ_1 is computed from the equation

$$\lambda_1(k) = 1 - K\cdot\gamma(k)\cdot\varepsilon^2(k) \quad (9)$$

Where K is small number, in our case $K = 0.001$.

3.3 Control System Configuration

Two control system configurations were used in this work. The first configuration with one degree-of-freedom (1DOF) has controller in the feedback part - see Fig. 4.

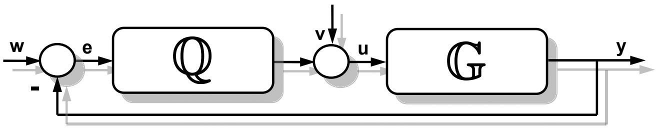

Fig. 4 1DOF control configuration

The configuration with two degrees-of-freedom (2DOF) displayed in Fig. 5 has controller divided into feedback part (Q) and feedforward part (R).

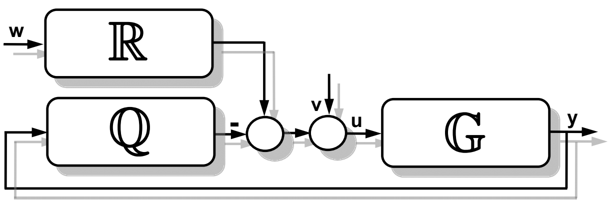

Fig. 5 2DOF control configuration

G in both configurations denotes transfer function (1) of controlled plant, w is the reference signal (wanted value), v is disturbance, e is used for control error, u is control variable and y is a controlled output.

The feedback and feedforward parts of the controller are designed with the use of polynomial synthesis:

$$Q(s) = \frac{q(s)}{s\cdot p(s)};\ R(s) = \frac{r(s)}{s\cdot p(s)} \quad (10)$$

where parameters of the polynomials $p(s)$, $q(s)$ and $r(s)$ are computed by the Method of uncertain coefficients which compares coefficients of individual s-powers from Diophantine equations (Kucera, 1993):

$$\begin{aligned} &a(s)\cdot s\cdot p(s) + b(s)\cdot q(s) = d(s) \\ &t(s)\cdot s + b(s)\cdot r(s) = d(s) \end{aligned} \quad (11)$$

The resulted, so called "hybrid", controller works in the continuous time but parameters of the polynomials $a(s)$ and $b(s)$ are identified recursively in the sampling period T_v. The polynomial $d(s)$ is a stable optional polynomial. It was proofed for example in (Stericker and Sinha, 1993) that the parameters of the delta model (2) for the small sampling period approach to the continuous ones in (1).

Polynomial $t(s)$ in the second Diophantine equation is an additive stable polynomial with random coefficients, because these coefficients are not used for computing of coefficients of the polynomial $r(s)$ in 2DOF configuration. All these equations are valid for step changes of the reference and disturbance signals.

The feedback controller $Q(s)$ ensures stability, load disturbance attenuation for both configurations and asymptotic tracking for 1DOF configuration. On the other hand, feedforward part $R(s)$ ensures asymptotic tracking in 2DOF configuration.

The polynomial $d(s)$ on the right side of Diophantine equations (11) is optional and could be designed for example with the use of Pole-placement method (Vojtesek *et al.*, 2004). The method used here uses Linear Quadratic (LQ) approach which is base on the minimizing of the cost function

$$J_{LQ} = \int_0^\infty \left\{\mu_{LQ}\cdot e^2(t) + \varphi_{LQ}\cdot\dot{u}^2(t)\right\}dt \quad (12)$$

where $\varphi_{LQ} > 0$ and $\mu_{LQ} \geq 0$ are weighting coefficients, $e(t)$ is control error and $\dot{u}(t)$ denotes derivative of the input variable. Polynomial $d(s)$ in this case is

$$d(s) = g(s)\cdot n(s) \quad (13)$$

Polynomials $n(s)$ and $g(s)$ are computed from spectral factorization

$$\begin{aligned} &(a\cdot f)^*\cdot\varphi_{LQ}\cdot a\cdot f + b^*\cdot\mu_{LQ}\cdot b = g^*\cdot g \\ &n^*\cdot n = a^*\cdot a \end{aligned} \quad (14)$$

and for control variable $u(t)$ and disturbance $v(t)$ from the ring of step functions $f(s) = s$. The resulted controller is strictly proper and the degree of $d(s)$ is computed via

$$\deg d = \deg(g\cdot n) = 2\deg a + 1 \quad (15)$$

3.4 Results of the Experiments

The output variable $y(t)$ is the conductivity of the chemical in (mS) and the input variable $u(t)$ is the flow rate of clean water through pump A in %. Transfer functions of the controller (10) are for the selected second order ELM (1)

$$\tilde{Q}(s) = \frac{q_2 s^2 + q_1 s + q_0}{s\cdot(s^2 + p_1 s + p_0)},\ \tilde{R}(s) = \frac{r_0}{s\cdot(s^2 + p_1 s + p_0)} \quad (16)$$

The sampling period was $T_v = 1$ *s*, time of the experiment is 720 *s* (15 *min*) and three step changes were done during this time. Experiments were done for different weighting factor ϕ_{LQ} = 0.001, 0.005 and 0.01. The second weighting factor was set to μ_{LQ} = 1 and results for 1DOF and 2DOF configurations are shown in the following figures.

The starting vector of parameters is $\boldsymbol{\theta}_\delta(0) = [1.4425, -0.0141, -0.0090, -0.0033]^T$, covariance matrix $\mathbf{P}(0)$ has dimension 4x4 with $1{\cdot}10^6$ on the main diagonal, constant $K = 0.001$ and parameters $\gamma(0) = 0$, $\varepsilon(0) = 0$.

The experiments have shown that the control results are much better if we impose starting values of the vector of parameters $\boldsymbol{\theta}_\delta(0)$ than for arbitrary values. The values of this vector are taken from previous experiments. They could vary for each experiment but recursive identification would recomputed these parameters to correct ones after some time. The second finding which follows from practical experiments is that it was good to force this vector for some time at the beginning, in our case for 50 *s*. It means that identification runs from the beginning, but the estimated parameters are taken into account from the time of 20 *s* to the end of control. The parameters from time 0-50 *s* are same as in $\boldsymbol{\theta}_\delta(0)$. The results of control are then much better and smoother on the contrary the controller without this condition ends with unacceptable results for some cases.

Results presented in Fig. 6 show that increasing value of ϕ_{LQ} results mainly in the speed of the control response after the first step change. The speed of the control for the next step changes is nearly the same for whole parameters ϕ_{LQ}.

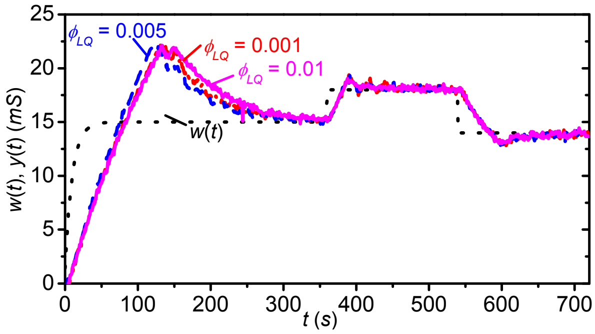

Fig. 6 Course of the output variable $y(t)$ for different values of the weighting factor ϕ_{LQ} with 1DOF controller

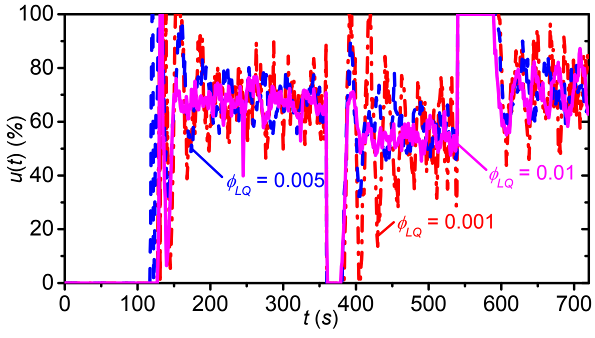

Fig. 7 Course of the input variable $u(t)$ for different values of the weighting factor ϕ_{LQ} with 1DOF controller

On the other hand, difference could be clearly seen in Fig. 7, where the highest value ϕ_{LQ} = 0.01 results in the smoothest course of the input variable which is in our case represented as a flow rate of the clean water.

Fig. 8 presents significantly the effect of the bigger value of the weighting factor to the output response. The line which represents output response for ϕ_{LQ} = 0.01 has the slowest rising time but with no overshoots especially at the beginning of the control.

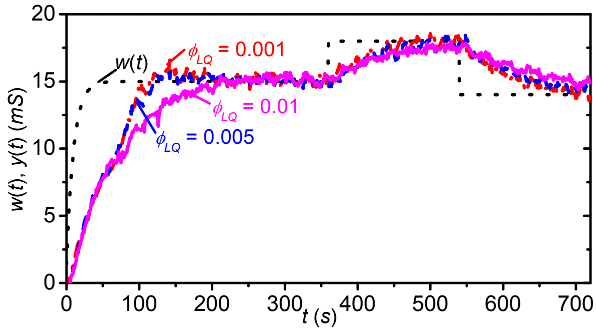

Fig. 8 Course of the output variable $y(t)$ for different values of the weighting factor ϕ_{LQ} with 2DOF controller

The course of the input variable $u(t)$ for 2DOF configuration displayed in Fig. 9 looks much worse than those for 1DOF. In fact, the changes are not so dramatic but they could cause real problems from the longer time horizon.

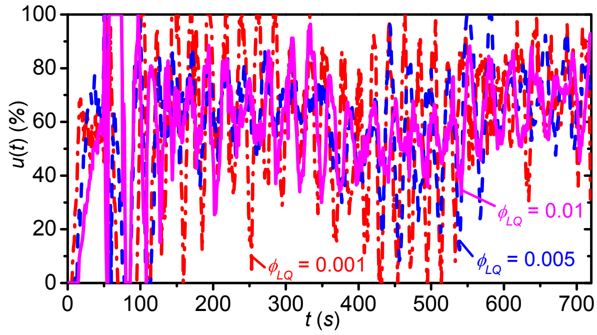

Fig. 9 Course of the input variable $u(t)$ for different values of the weighting factor ϕ_{LQ} with 2DOF controller

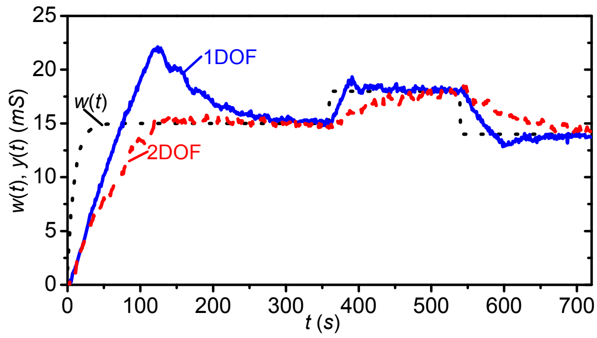

Fig. 10 Comparison of the course of the output variable $y(t)$ for the 1DOF and 2DOF controllers and ϕ_{LQ} = 0.005

The main advantage of the 2DOF control configuration is that this controller could eliminate overshoots mainly at the very beginning of the control – see Fig. 10. On the contrary, speed of the control is higher and the changes of the input variable $u(t)$ differ in smaller range (Fig. 11) for the 1DOF control configuration.

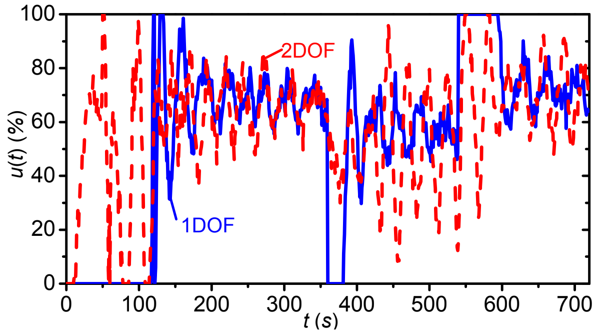

Fig. 11 Comparison of the course of the input variable $u(t)$ for the 1DOF and 2DOF controllers and $\phi_{LQ} = 0.005$

2. CONCLUSIONS

This paper presents results of the adaptive control of the conductivity inside the nonlinear system with lumped parameters, the continuous stirred tank reactor. Used adaptive control is based on the choosing of the external linear model in the range of delta models parameters of which are estimated recursively during the control. The controller could be tuned via weighting factor ϕ_{LQ}. The course of the output conductivity is quicker for decreasing value of this factor but very small values could produce overshoots. The work compares results for two control configurations – 1DOF and 2DOF and it can be clearly seen at the beginning of the control when the 1DOF controller produces quite big overshoot while 2DOF configuration approaches to the reference signal in much better way. On the other hand, the course of the output variable for the 2DOF configuration has slower course than those for 1DOF with the same weighting factor ϕ_{LQ}. All control results have shown applicability of this control method to the real system.

ACKNOWLEDGMENT

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