Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Nonlinear Adaptive H_∞ Control of Robotic Manipulators under Constraint

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Abstract: The problem of constructing nonlinear adaptive H_∞ control of robotic manipulators under costraints is considered in this paper. In the proposed control scheme, the trajectory converges to the desired constrained trajectory, and the constraint force also follows the desired constraint one. The resulting control strategy is derived as a solution of certain H_∞ control problem, where estimation errors of tuning parameters and errors of constraint forces are regarded as external disturbances to the process.

1. INTRODUCTION

Motion control problems of mechanical systems are divided into two categories, that is, free motion control and constrained motion control. Free motion control problems of mechanical systems are seen in the situations where there is no contact between controlled processes and environments, and have been studied extensively as basic control problems of mechanical systems. Nonlinear or adaptive free motion control schemes for mechanical systems with uncertain parameters, have been also investigated based on several approaches including passivity methodology and Lyapunov function analysis (Arimoto (1996), Shen and Tamura (1999)). On the contrary, motion control problems of constrained mechanical systems are seen in the situations where there exists a contact between controlled processes and environments, and contact forces between end-effectors of mechanical systems and environments are generated. Compared with free motion control, constrained motion control has been a difficult problem, where not only constrained trajectory control but also simultaneous constraint force control should be considered (McClamroch and Wang (1988), Su et al. (1992)), and the adaptive control version of that problem for mechanical systems with parametric uncertainties, is a difficult but important problem from the practical point of view.

Recently, an adaptive control scheme of mechanical systems under constraint, was proposed in Yao and Tomizuka (1995), Yuan (1997), where constrained trajectory control together with control of constraint force were considered. That approach was analyzed via Lyapunov functions, and asymptotic stability of tracking errors of constrained trajectories, and the variables concerned with errors of constraint forces, is shown in that work. However, the control performance of that strategy was not discussed in detail.

Considering that previous adaptive control scheme (Yao and Tomizuka (1995), Yuan (1997)), the present manuscript provides design methods of nonlinear adaptive H_∞ control of constrained robotic manipulators based on the notion of inverse optimality (Krstić and Deng (1998), Miyasato (1999)). In those approaches, estimation errors of tuning parameters in the adaptation mechanism and errors of constraint forces are regarded as external disturbances to the process, and the resulting control strategy is derived as a solution of corresponding H_∞ control problems (Miyasato (2000), Miyasato (2002), Miyasato (2007)). Asymptotic stability of tracking errors of constrained trajectories and the variables concerned with errors of constraint forces, are assured, and $\mathcal{L}^2$ gains from those disturbances (errors of tuning parameters and constraint forces) to generalized outputs are prescribed by several design parameters, explicitly. The proposed control strategy contains a kind of nonlinear damping metohdology, and thus, attains good convergence and transient property with less control efforts.

2. PROBLEM STATEMENT

Consider a robotic manipulator whose trajectory is constrained geometrically.

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \tau + f, \tag{1}$$

where $\theta \in \mathbf{R}^n$ is a vector of joint angles, $M(\theta) \in \mathbf{R}^{n\times n}$ is a matrix of inertia, $C(\theta,\dot{\theta}) \in \mathbf{R}^{n\times n}$ is a matrix of Coriolis and centrifugal forces, $G(\theta) \in \mathbf{R}^n$ is a vector of gravitational torques, and τ is a vector of input torques (control input). It is assumed that the system parameters in $M(\theta)$, $C(\theta,\dot{\theta})$, and $G(\theta)$ are unknown. The trajectory θ of the robotic manipulator is subject to a constraint represented by a set of m geometric equations (holonomic constraint and frictionless, $m < n$) such that

$$\Phi(\theta) = 0, \quad \frac{d}{dt}\Phi(\theta) = 0, \quad (\Phi \in \mathbf{R}^m), \tag{2}$$

and f is a constraint force which is expressed as

$$f = J(\theta)^T\lambda, \quad (\lambda \in \mathbf{R}^m), \tag{3}$$

$$J(\theta) = \frac{\partial \Phi}{\partial \theta}, \quad (J(\theta) \in \mathbf{R}^{m\times n}), \tag{4}$$

where λ is a Lagrangian multiplier. It is assumed that the constraint force is measured by a force sensor mounted at the end-effector of the system.

Robotic manipulators with rotational joints have the following properties (Spong and Vidyasagar (1989)).

Properties of Robotic Manipulators.

(1) $M(\theta)$ is a bounded, positive definite, and symmetric matrix.
(2) $\dot{M}(\theta) - 2C(\theta, \dot{\theta})$ is a skew symmetric matrix.
(3) The left-hand side of (1) can be written into the following form,
$$M(\theta)a + C(\theta, \dot{\theta})b + G(\theta) = \Omega(\theta, \dot{\theta}, a, b)^T \phi, \tag{5}$$
where $\Omega(\theta, \dot{\theta}, a, b)$ is a known function of θ, $\dot{\theta}$, a, b, and ϕ is an unknown system parameter.

The control objective is to synthesize a proper control law of τ such that the constrained trajectory θ and constraint force f follow the desired constrained trajectory $\theta_d(t)$ (differentiable on $t \in [0, \infty)$ and $\Phi(\theta_d) = 0$) and the desired constraint force f_d, respectively.
$$\tilde{\theta} \equiv \theta - \theta_d, \;\; \tilde{f} \equiv f - f_d, \quad (\tilde{\theta}, \, \tilde{f} \in \mathbf{R}^n), \tag{6}$$
$$\lim_{t\to\infty} \tilde{\theta}(t) = \lim_{t\to\infty} \tilde{f}(t) = 0. \tag{7}$$
Typical examples of that control problem are grinding, polishing, inserting, deburring, and scribing, etc, where the end-effector of the mechanical system exerts a desired force to the environment as the controlled process moves along a prescribed constrained trajectory.

3. SYSTEM DESCRIPTION INCLUDING CONSTRAINT

System descriptions of controlled processes which includes constraints implicitly, are to be obtained in the present section. The development of such descriptions is mainly owing to the previous study of McClamroch and Wang (1988).

According to the dimension m of the geometric constraint, the output θ is divided into θ^1 and θ^2, where
$$\theta = \begin{bmatrix} \theta^1 \\ \theta^2 \end{bmatrix}, \quad \theta^1 \in \mathbf{R}^{n-m}, \;\; \theta^2 \in \mathbf{R}^m. \tag{8}$$
Then, $J(\theta)$ is also described in the following decomposed form.
$$J(\theta) = \left[\frac{\partial \Phi}{\partial \theta^1}, \; \frac{\partial \Phi}{\partial \theta^2} \right] = \left[J_1(\theta), \; J_2(\theta) \right], \tag{9}$$
$$J_1(\theta) \in \mathbf{R}^{m\times(n-m)}, \;\; J_2(\theta) \in \mathbf{R}^{m\times m}. \tag{10}$$
There is a proper partition such that $\det J_2(\theta) \neq 0$. Since the next relation holds,
$$0 = \frac{d}{dt}\Phi(\theta) = J(\theta)\dot{\theta} = J_1(\theta)\dot{\theta}^1 + J_2(\theta)\dot{\theta}^2, \tag{11}$$
$\dot{\theta}^2$ is represented by $\dot{\theta}^1$ such as
$$\dot{\theta}^2 = -J_2(\theta)^{-1} J_1(\theta)\dot{\theta}^1, \tag{12}$$
and it follows that $\dot{\theta}$ is represented by utilizing $\dot{\theta}^1$.
$$\dot{\theta} = L(\theta)\dot{\theta}^1, \tag{13}$$
$$L(\theta) = \begin{bmatrix} I_{n-m} \\ -J_2(\theta)^{-1} J_1(\theta) \end{bmatrix}. \tag{14}$$
For $L(\theta)$, it is easily shown that the next relation holds.
$$L(\theta)^T J(\theta)^T = J_1(\theta)^T - J_1(\theta)^T = 0. \tag{15}$$
By utilizing the property of $L(\theta)$, the system description which includes constraint implicitly, is deduced. The substitution of (13) and the next relation
$$\ddot{\theta} = L(\theta)\ddot{\theta}^1 + \dot{L}(\theta, \dot{\theta})\dot{\theta}^1, \tag{16}$$
into (1) yields
$$\begin{aligned} &M(\theta)L(\theta)\ddot{\theta}^1 + M(\theta)\dot{L}(\theta, \dot{\theta}))\dot{\theta}^1 \\ &\quad + C(\theta, \dot{\theta})L(\theta)\dot{\theta}^1 + G(\theta) = \tau + f. \end{aligned} \tag{17}$$
By multiplying $L(\theta)^T$ to above equation, the following representation is derived.
$$M_1(\theta)\ddot{\theta}^1 + C_1(\theta, \dot{\theta}_1)\dot{\theta}_1 + G_1(\theta) = L(\theta)^T \tau, \tag{18}$$
$$M_1(\theta) = L(\theta)^T M(\theta) L(\theta), \tag{19}$$
$$C_1(\theta, \dot{\theta}) = L(\theta)^T (M(\theta)\dot{L}(\theta, \dot{\theta}) + C(\theta, \dot{\theta})L(\theta)), \tag{20}$$
$$G_1(\theta) = L(\theta)^T G(\theta), \tag{21}$$
where the following relation is also considered.
$$L(\theta)^T f = L(\theta)^T J(\theta)^T \lambda = 0. \tag{22}$$
The system description (18) does not contain constraint force nor geometric constraint, explicitly. Then, for given τ, constrained trajectories $\ddot{\theta}^1$, $\dot{\theta}^1$ and θ^1 are computed from (18), and $\ddot{\theta}^2$, $\dot{\theta}^2$ and θ^2 are also derived by considering (2), (13), (16). Finally, the constraint force f is computed from the relation $f = M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} + G(\theta) - \tau$.

4. ADAPTIVE CONTROL UNDER CONSTRAINT

First, we introduce the conventional adaptive control for constrained systems (Yuan (1997)). Define the following signals.
$$\tilde{\theta}^1 = \theta^1 - \theta_d^1 \; (\in \mathbf{R}^{n-m}), \tag{23}$$
$$\tilde{\theta}^2 = \theta^2 - \theta_d^2 \; (\in \mathbf{R}^{m}), \tag{24}$$
$$\dot{\theta}_r^1 = \dot{\theta}_d^1 - \Lambda\tilde{\theta}^1 \; (\in \mathbf{R}^{n-m}), \tag{25}$$
$$\dot{\theta}_r = L(\theta)\dot{\theta}_r^1 \; (\in \mathbf{R}^{n}), \tag{26}$$
$$s = \dot{\theta}^1 - \dot{\theta}_r^1 = \dot{\tilde{\theta}}^1 + \Lambda\tilde{\theta}^1 \; (\in \mathbf{R}^{n-m}), \tag{27}$$
$$\tilde{f} = f - f_d \; (\in \mathbf{R}^{n}), \tag{28}$$
$$(\Lambda \in \mathbf{R}^{(n-m)\times(n-m)}; \; \Lambda = \Lambda^T > 0),$$
where θ_d^1 is a subset of elements in θ_d which corresponds to θ^1. μ is a variable to handle the force control part, and is synthesized from $\tilde{f}$ such as
$$\dot{\mu} = -\kappa\mu - \kappa\tilde{f}, \quad (\mu \in \mathbf{R}^n), \quad (\kappa > 0). \tag{29}$$
Also σ and ν are introduced as follows:
$$\sigma \equiv Ls + \mu \; (= \dot{\theta} - \nu) \; (\in \mathbf{R}^n), \tag{30}$$
$$\nu \equiv \dot{\theta}_r - \mu \; (\in \mathbf{R}^n). \tag{31}$$
For σ and ν, we obtain the following relations.
$$\dot{\sigma} = L\dot{s} + \dot{L}s - \kappa(\mu + \tilde{f}), \tag{32}$$
$$\dot{\nu} = L\ddot{\theta}_r^1 + \dot{L}\dot{\theta}_r + \kappa(\mu + \tilde{f}). \tag{33}$$
The substitution of above relations into (1) yields
$$\begin{aligned} &M(\theta)\dot{\sigma} + C(\theta, \dot{\theta})\sigma \\ &\quad + M(\theta)\dot{\nu} + C(\theta, \dot{\theta})\nu + G(\theta) = \tau + f, \end{aligned} \tag{34}$$
or
$$M(\theta)\dot{\sigma} + C(\theta, \dot{\theta})\sigma + \Omega(\theta, \dot{\theta}, \dot{\nu}, \nu)^T \phi = \tau + f. \tag{35}$$
This corresponds to the error equation of the traditional adaptive control (Narendra and Annaswamy (1989), Ioannou and Sun (1996)), where ϕ is an unknown system parameter vector. For that error system, the control input is synthesized such as

$$\tau = -K\sigma - f_d + \alpha\tilde{f} + \Omega^T\hat{\phi}, \tag{36}$$

$$(K \in \mathbf{R}^{n\times n} : \ K = K^T > 0, \quad \alpha > 0), \tag{37}$$

where $\hat{\phi}$ is a current estimate of ϕ, and is tuned by the following adaptive law.

$$\dot{\hat{\phi}} = -\Gamma\Omega\sigma, \quad (\Gamma = \Gamma^T > 0). \tag{38}$$

Then, the error equation becomes

$$M(\theta)\dot{\sigma} + C(\theta,\dot{\theta})\sigma = -K\sigma + (1+\alpha)\tilde{f} + \Omega^T\tilde{\phi}, \tag{39}$$

$$\tilde{\phi} = \hat{\phi} - \phi. \tag{40}$$

Here we define a positive function W

$$W = \frac{1}{2}\sigma^T M(\theta)\sigma + \left(\frac{1+\alpha}{2\kappa}\right)\mu^T\mu + \frac{1}{2}\tilde{\phi}^T\Gamma^{-1}\tilde{\phi}, \tag{41}$$

and take the time derivative of it along the trajectory of the original system.

$$\begin{aligned}\dot{W} &= -\sigma^T K\sigma + (1+\alpha)\sigma^T\tilde{f} \\ &\quad -(1+\alpha)\mu^T\mu - (1+\alpha)\mu^T\tilde{f} \\ &= -\sigma^T K\sigma - (1+\alpha)\mu^T\mu \le 0.\end{aligned} \tag{42}$$

Then it follows that $\sigma,\,\mu \in \mathcal{L}^2\cap\mathcal{L}^\infty$ and that $\hat{\phi}\in\mathcal{L}^\infty$. By considering the following relation

$$s = (L^T L)^{-1}L^T(\sigma - \mu), \tag{43}$$

it is shown that $s \in \mathcal{L}^2\cap\mathcal{L}^\infty$, if $(L^T L)^{-1}L^T \in \mathcal{L}^\infty$. Furthermore, by considering the next relation

$$s = \dot{\tilde{\theta}}^1 + \Lambda\tilde{\theta}^1 \in \mathcal{L}^2\cap\mathcal{L}^\infty, \tag{44}$$

we obtain $\tilde{\theta}^1, \dot{\tilde{\theta}}^1 \in \mathcal{L}^\infty$ and $\tilde{\theta}^1 \to 0$. Also, by seeing $\dot{\theta} = L\dot{\theta}^1$ and $\Phi(\theta) = 0$, it follows that $\tilde{\theta},\,\dot{\tilde{\theta}} \in \mathcal{L}^\infty$ and $\tilde{\theta}\to 0$, if $\theta^1\in\mathcal{L}^\infty$ implies $L\in\mathcal{L}^\infty$. Furthermore, $\dot{\theta}_r^1 = \dot{\theta}_d^1 - \Lambda\tilde{\theta}^1$ suggests that $\dot{\theta}_r^1\in\mathcal{L}^\infty$. Hence, it is shown that $\dot{\theta}_r = L\dot{\theta}_r^1 \in\mathcal{L}^\infty$, if $\theta^1\in\mathcal{L}^\infty$ implies $L\in\mathcal{L}^\infty$. Additionally, $\nu = \dot{\theta}_r - \mu \in\mathcal{L}^\infty$. Next, we consider constraint force. Since it holds that $\tilde{\lambda} = \lambda - \lambda_d \in\mathcal{L}^\infty$ when $(1+\alpha)I + \kappa\hat{M}$ is non-singular ($\hat{M}$ is a current estimate of M composed of the corresponding elements in $\hat{\phi}$) (Yuan (1997)), it follows that $\tilde{f} = J^T\tilde{\lambda}\in\mathcal{L}^\infty$, and that $f = J^T\lambda\in\mathcal{L}^\infty$. Then it is shown that $\tau\in\mathcal{L}^\infty$, and that $\dot{\sigma},\,\dot{\mu}\in\mathcal{L}^\infty$. It suggests that $\sigma,\,\mu\to 0$.

Then, we obtain the next theorem.

Theorem 1. The adaptive control system is uniformly bounded, if the following conditions (1), (2), (3) are satisfied.

(1) $(L^T L)^{-1}L^T \in\mathcal{L}^\infty$.
(2) $\theta^1\in\mathcal{L}^\infty$ implies $L\in\mathcal{L}^\infty$.
(3) $(1+\alpha)I + \kappa\hat{M}$ is non-singular.

Furthermore, $\tilde{\theta},\,\sigma,\,\mu$ converge to zero asymptotically.

$$\lim_{t\to\infty}\tilde{\theta}(t) = \lim_{t\to\infty}\sigma(t) = \lim_{t\to\infty}\mu(t) = 0. \tag{45}$$

5. NONLINEAR ADAPTIVE H_∞ CONTROL UNDER CONSTRAINT I

Next, based on the adaptive control scheme in Section 4, we construct the nonlinear adaptive H_∞ control systems, where estimation errors of tuning parameters $\tilde{\phi}$ and errors of constraint forces $\tilde{f}$ are regarded as external disturbances to the process. First, the control input is synthesized as follows:

$$\tau = -f_d + \alpha\tilde{f} + \Omega^T\hat{\phi} + v, \tag{46}$$

where v is a stabilizing signal derived from H_∞ control criterion. Then, the overall system is written by

$$\begin{aligned}\frac{d}{dt}\begin{bmatrix}\sigma\\ \mu\end{bmatrix} &= \begin{bmatrix}-M^{-1}C\sigma\\ -\kappa\mu\end{bmatrix} + \begin{bmatrix}M^{-1}(1+\alpha)\\ -\kappa I\end{bmatrix}\tilde{f} \\ &\quad + \begin{bmatrix}M^{-1}\Omega^T\\ 0\end{bmatrix}\tilde{\phi} + \begin{bmatrix}M^{-1}\\ 0\end{bmatrix}d + \begin{bmatrix}M^{-1}\\ 0\end{bmatrix}v,\end{aligned} \tag{47}$$

where an external disturbance $d\,(\in\mathcal{L}^2)$ is added for the sake of the problem formulation of H_∞ control. (47) is rewritten into the next form.

$$\frac{d}{dt}x = F(x) + g_{11}\tilde{f} + g_{12}\tilde{\phi} + g_{13}d + g_2 v, \tag{48}$$

$$x \equiv [\sigma^T,\,\mu^T]^T. \tag{49}$$

We are to stabilize the above system via a control input v by utilizing H_∞ criterion, where $\tilde{f}$, $\tilde{\phi}$, and d are regarded as external disturbances to the process (Miyasato (2000), Miyasato (2002), Miyasato (2007)). For that purpose, we introduce the following Hamilton-Jacobi-Isaacs (HJI) equation

$$\begin{aligned}&\frac{\partial}{\partial t}V + \mathcal{L}_F V \\ &+\frac{1}{4}\left\{\sum_{i=1}^{3}\frac{\|\mathcal{L}_{g_{1i}}V\|^2}{\gamma_i^2} - \mathcal{L}_{g_2}VR^{-1}\left(\mathcal{L}_{g_2}V\right)^T\right\} \\ &+q(x) \le 0,\end{aligned} \tag{50}$$

where the solution V is given by

$$V = \frac{1}{2}\sigma^T M(\theta)\sigma + \left(\frac{1+\alpha}{2\kappa}\right)\mu^T\mu. \tag{51}$$

$q(x)$ and R are a positive function and a positive definite matrix, respectively, and those are derived from HJI equation based on inverse optimality for the given solution V and the positive constants γ_i $(i = 1\sim 3)$. The substitution of the solution V (51) into HJI equation (50) yields

$$\begin{aligned}&-(1+\alpha)\|\mu\|^2 + \frac{1}{4\gamma_1^2}(1+\alpha)^2\|\sigma-\mu\|^2 \\ &\quad+\frac{1}{4\gamma_2^2}\sigma^T\Omega^T\Omega\sigma + \frac{1}{4\gamma_3^2}\|\sigma\|^2 - \frac{1}{4}\sigma^T R^{-1}\sigma \\ &\quad+q(x)\le 0.\end{aligned} \tag{52}$$

In order to obtain $q(x)$ and R, we consider the following relation (53) which is a sufficient condition for the above inequality (52).

$$\begin{aligned}&-(1+\alpha)\|\mu\|^2 + \frac{1}{2\gamma_1^2}(1+\alpha)^2(\|\sigma\|^2 + \|\mu\|^2) \\ &\quad+\frac{1}{4\gamma_2^2}\sigma^T\Omega^T\Omega\sigma + \frac{1}{4\gamma_3^2}\|\sigma\|^2 - \frac{1}{4}\sigma^T R^{-1}\sigma \\ &\quad+q(x)\le 0.\end{aligned} \tag{53}$$

Then, $q(x)$ and R satisfying (53) are given as follows:

$$q(x) = \frac{1}{4}\sigma^T K_R\sigma + (1+\alpha)\left(1 - \frac{1+\alpha}{2\gamma_1^2}\right)\|\mu\|^2, \tag{54}$$

$$R = \left\{\frac{2(1+\alpha)^2}{\gamma_1^2}I + \frac{\Omega^T\Omega}{\gamma_2^2} + \frac{1}{\gamma_3^2}I + K_R\right\}^{-1}, \tag{55}$$

$$K_R = K_R^T > 0. \tag{56}$$

In order that $q(x)$ is a positive function, α and γ_1 should satisfy the next relation.

$$\gamma_1^2 > \frac{1+\alpha}{2}. \tag{57}$$

By utilizing R, v is deduced as a solution for the corresponding H_∞ control problem.

$$\begin{aligned} v &= -\frac{1}{2}R^{-1}\left(\mathcal{L}_{g_2}V\right)^T = -\frac{1}{2}R^{-1}\sigma \\ &= -\frac{1}{2}\left\{\frac{2(1+\alpha)^2}{\gamma_1^2}I + \frac{\Omega^T\Omega}{\gamma_2^2} + \frac{1}{\gamma_3^2}I + K_R\right\}\sigma. \end{aligned} \tag{58}$$

By considering HJI equation, the time derivative of V is evaluated as follows:

$$\begin{aligned} \dot{V} &= (1+\alpha)(\sigma-\mu)^T\tilde{f} + \sigma^T\Omega^T\tilde{\phi} + \sigma^T d + \sigma^T v \\ &\quad -(1+\alpha)\mu^T\mu \\ &\leq -\frac{1}{4\gamma_1^2}(1+\alpha)^2\|\sigma-\mu\|^2 \\ &\quad -\frac{1}{4\gamma_2^2}\sigma^T\Omega^T\Omega\sigma - \frac{1}{4\gamma_3^2}\|\sigma\|^2 + \frac{1}{4}\sigma^T R^{-1}\sigma - q(x) \\ &\quad +(1+\alpha)(\sigma-\mu)^T\tilde{f} + \sigma^T\Omega^T\tilde{\phi} + \sigma^T d + \sigma^T v \\ &= \left(v + \frac{1}{2}R^{-1}\sigma\right)^T R \left(v + \frac{1}{2}R^{-1}\sigma\right) - v^T R v \\ &\quad -\gamma_1^2\left\|\tilde{f} - \frac{1+\alpha}{2\gamma_1^2}(\sigma-\mu)\right\|^2 + \gamma_1^2\|\tilde{f}\|^2 \\ &\quad -\gamma_2^2\left\|\tilde{\phi} - \frac{1}{2\gamma_2^2}\Omega\sigma\right\|^2 + \gamma_2^2\|\tilde{\phi}\|^2 \\ &\quad -\gamma_3^2\left\|d - \frac{1}{2\gamma_3^2}\sigma\right\|^2 + \gamma_3^2\|d\|^2 - q(x). \end{aligned} \tag{59}$$

The tuning law of $\hat{\phi}$ is the same as (38). Then, the positive function W (41) satisfies the next relation.

$$\begin{aligned} \dot{W} &= -\frac{1}{2}\sigma^T R^{-1}\sigma - (1+\alpha)\|\mu\|^2 \\ &= -\frac{1}{2}\sigma^T\left\{\frac{2(1+\alpha)^2}{\gamma_1^2}I + \frac{\Omega^T\Omega}{\gamma_2^2} + \frac{1}{\gamma_3^2}I + K_R\right\}\sigma \\ &\quad -(1+\alpha)\|\mu\|^2 \\ &\leq 0. \end{aligned} \tag{60}$$

From the evaluation of V and W (59), (60), we obtain the next theorem.

Theorem 2. The adaptive control system is uniformly bounded, if the conditions (1), (2), (3) (in Theorem 1) are satisfied. Furthermore, $\tilde{\theta}$, σ, μ converge to zero asymptotically (45). Also, v is an optimal control solution which minimizes the following cost functional.

$$\begin{aligned} J = \sup_{\tilde{\phi},\tilde{f},d\in\mathcal{L}^2} &\left\{ \int_0^t (q + v^T R v)d\tau + V(t) \right. \\ &\left. -\gamma_1^2\int_0^t \|\tilde{f}\|^2 d\tau - \gamma_2^2\int_0^t \|\tilde{\phi}\|^2 d\tau - \gamma_3^2\int_0^t \|d\|^2 d\tau \right\}. \end{aligned} \tag{61}$$

Additionally, the next inequality holds for any finite t.

$$\begin{aligned} &\int_0^t (q + v^T R v)d\tau + V(t) \leq \gamma_1^2\int_0^t \|\tilde{f}\|^2 d\tau \\ &\quad +\gamma_2^2\int_0^t \|\tilde{\phi}\|^2 d\tau + \gamma_3^2\int_0^t \|d\|^2 d\tau + V(0). \end{aligned} \tag{62}$$

Remark 3. Of course, J (61) is a fictitious cost functional, since $\tilde{\phi}$, $\tilde{f}$ are not actually external disturbances but errors of tuning parameters and constraint force, and since those are not generally included in $\mathcal{L}^2[0,\infty)$. Nevertheless, v, which is derived as a solution for that fictitious H_∞ control problem, attain the inequality (62), and it means that the $\mathcal{L}^2$ gains from the disturbances $\tilde{f}$, $\tilde{\theta}$, d to the generalized output $\sqrt{q + v^T R v}$ are prescribed by positive constants γ_1, γ_2, γ_3. However, $\mathcal{L}^2$ gain γ_1 is restricted by the control parameters α (57). Additionally, it should be noted that boundedness of $\tilde{\phi}$ and $\tilde{f}$ is assured in the stability analysis of the adaptive control systems (evaluation of $\dot{W}$ (60) and stability analysis similar to Theorem 1).

Remark 4. From the evaluation of $\dot{V}$ (59), it is seen that boundedness of the control system is assured even for non-adaptive and bounded $\hat{\phi}$. This is the robustness feature of the proposed control scheme. On the contrary, the adaptive $\hat{\phi}$ (38) attains asymptotic zero-tracking-error of $\tilde{\theta}$, σ, μ (60). That is an interplay of robust control and adaptive control.

6. NONLINEAR ADAPTIVE H_∞ CONTROL UNDER CONSTRAINT II

In the previous section, the proposed H_∞ control scheme is derived from the actual system description (47), where $\mathcal{L}^2$ gains from $\tilde{f}$, $\tilde{\phi}$, d to the generalized output $\sqrt{q + v^T R v}$ are prescribed by positive constants γ_i ($i = 1 \sim 3$). On the contrary, in the present section, the proposed H_∞ control strategy is deduced from the simplified description, where (15) is also considered.

First, the control input τ is synthesized by (46). Since (15) holds, $\dot{V}$ is written as follows:

$$\dot{V} = \sigma^T\Omega^T\tilde{\theta} + \sigma^T d + \sigma^T v - (1+\alpha)\mu^T\mu, \tag{63}$$

where an external disturbance $d\,(\in \mathcal{L}^2)$ is added. From the above relation, we introduce the following virtual system.

$$\begin{aligned} \frac{d}{dt}\begin{bmatrix}\sigma \\ \mu\end{bmatrix} &= \begin{bmatrix}-M^{-1}C\sigma \\ -\kappa\mu\end{bmatrix} + \begin{bmatrix}M^{-1}\Omega^T \\ 0\end{bmatrix}\tilde{\phi} \\ &\quad + \begin{bmatrix}M^{-1} \\ 0\end{bmatrix}d + \begin{bmatrix}M^{-1} \\ 0\end{bmatrix}v. \end{aligned} \tag{64}$$

The virtual process is rewritten into the next form.

$$\frac{d}{dt}x = F(x) + g_{11}\tilde{\phi} + g_{12}d + g_2 v. \tag{65}$$

We are to stabilize the virtual system via the control input v by utilizing H_∞ control criterion, where $\tilde{\phi}$ and d are regarded as external disturbances to the process. Similarly to the previous section, for HJI equation

$$\begin{aligned} &\mathcal{L}_F V + \frac{1}{4}\left\{\sum_{i=1}^{2}\frac{\|\mathcal{L}_{g_{1i}}V\|^2}{\gamma_i^2} - \mathcal{L}_{g_2}V R^{-1}\left(\mathcal{L}_{g_2}V\right)^T\right\} \\ &\quad +q(x) \leq 0, \end{aligned} \tag{66}$$

together with the solution V (51), or for the following equivalent relation

$$-(1+\alpha)\|\mu\|^2 + \frac{1}{4\gamma_1^2}\sigma^T\Omega^T\Omega\sigma + \frac{1}{4\gamma_2^2}\|\sigma\|^2 - \frac{1}{4}\sigma^T R^{-1}\sigma + q(x) \le 0, \quad (67)$$

$q(x)$, R and the optimal solution v are given as follows:

$$q(x) = \frac{1}{4}\sigma^T K_R \sigma + (1+\alpha)\|\mu\|^2, \quad (68)$$

$$R = \left\{ \frac{\Omega^T\Omega}{\gamma_1^2} + \frac{1}{\gamma_2^2} I + K_R \right\}^{-1}, \quad (69)$$

$$v = -\frac{1}{2}R^{-1}\left(\mathcal{L}_{g_2}V\right)^T = -\frac{1}{2}R^{-1}\sigma = -\frac{1}{2}\left\{ \frac{\Omega^T\Omega}{\gamma_1^2} + \frac{1}{\gamma_2^2} I + K_R \right\}\sigma, \quad (70)$$

$$K_R = K_R^T > 0. \quad (71)$$

Theorem 5. The adaptive control system is uniformly bounded, if the conditions (1), (2), (3) (in Theorem 1 and Theorem 2) are satisfied. Also, $\tilde{\theta}$, σ, μ converge to zero asymptotically (45). Furthermore, v is an optimal control solution which minimizes the following cost functional.

$$J = \sup_{\tilde{\phi}, d \in \mathcal{L}^2} \left\{ \int_0^t (q + v^T R v) d\tau + V(t) - \gamma_1^2 \int_0^t \|\tilde{\phi}\|^2 d\tau - \gamma_2^2 \int_0^t \|d\|^2 d\tau \right\}. \quad (72)$$

Additionally, the next inequality holds for any finite t.

$$\int_0^t (q + v^T R v) d\tau + V(t) \le \gamma_1^2 \int_0^t \|\tilde{\phi}\|^2 d\tau + \gamma_2^2 \int_0^t \|d\|^2 d\tau + V(0). \quad (73)$$

Remark 6. The $\mathcal{L}^2$ gains from the disturbances $\tilde{\phi}$ and d to the generalized output $\sqrt{q + v^T R v}$ are prescribed by positive constants γ_1, γ_2. However, $\mathcal{L}^2$ gain from $\tilde{f}$ to the generalize output $\sqrt{q + v^T R v}$ is not prescribed in the present control scheme.

7. NUMERICAL EXAMPLE

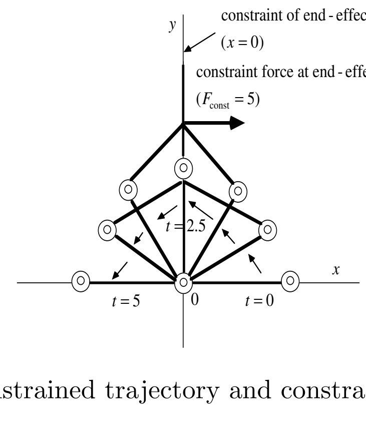

Fig. 1. Constrained trajectory and constraint force.

Numerical simulation studies are performed. A SICE-DD arm (the standard manipulator model in SICE) with two-degree of freedom ($n = 2$) is considered. Physical parameters are written in Table 1.

Table 1 Physical parameters.

Link (i)	1	2
m_i (kg)	12.27	2.083
I_i (kg · m^2)	0.1149	0.0144
l_i (m)	0.2	0.2
r_i (m)	0.063	0.080

The output $\theta = [\theta_1, \theta_2]^T$ is subject to the following geometric constraint

$$\Phi(\theta) = 2\theta_1 + \theta_2 - \pi = 0,$$

which corresponds to the situation where the end-effector of the manipulator is constrained to $y-$axis ($x = 0$) in Fig.1. Since $m = 1$, θ_1 can correspond to θ^1, and θ_2 to θ^2, respectively.

$$J(\theta) = \frac{\partial\Phi}{\partial\theta} = \begin{bmatrix} \frac{\partial\Phi}{\partial\theta_1} & \frac{\partial\Phi}{\partial\theta_2} \end{bmatrix} = [2 \quad 1], \quad (74)$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} = L\dot{\theta}_1, \quad \left(L = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \right), \quad (75)$$

$$f = J^T\lambda = \begin{bmatrix} 2 \\ 1 \end{bmatrix}\lambda.$$

Then, $(L^T L)^{-1} L^T = \begin{bmatrix} \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$, and it follows that $(L^T L)^{-1} L^T \in \mathcal{L}^\infty$, $L \in \mathcal{L}^\infty$.

For that controlled process, we apply the proposed design method in Section 6 (Theorem 5). K_R is determined such as $K_R = \mathrm{diag.}(k_{R1}, k_{R2})$, and the design parameters are chosen as follows:

$$\lambda = 1, \quad \kappa = 1, \quad \alpha = 1, \quad \Gamma = 10I,$$
$$\gamma_1 = 0.1, \quad \frac{1}{2}\left(\frac{1}{\gamma_2^2} + k_{R1}\right) = \frac{1}{2}\left(\frac{1}{\gamma_2^2} + k_{R2}\right) = 1.$$

For comparison, the adaptive control scheme in Section 4 (Yuan (1997)) is also applied with the same conditions of design parameters (λ, κ, α, Γ), where K is determined such as $K = \mathrm{diag.}(k_1, k_2)$ ($k_1 = k_2 = 1$). It should be noted that $k_1 = \frac{1}{2}\left(\frac{1}{\gamma_2^2} + k_{R1}\right) = 1$ and $k_2 = \frac{1}{2}\left(\frac{1}{\gamma_2^2} + k_{R2}\right) = 1$; the same linear feedback gains are utilized in the proposed and conventional adaptive control schemes.

The desired trajectory (θ_{d1}, θ_{d2}) (rad) and the constraint force (f_{d1}, f_{d2}) (N·m) are given below:

$$\theta_{d1}(t) = \frac{\pi}{2} \cdot \frac{6}{125} \cdot \left(\frac{5}{2}t^2 - \frac{1}{3}t^3\right), \quad \theta_{d2}(t) = \pi - 2\theta_{d1}(t),$$
$$f_{d1} = -2\sin\theta_1, \quad f_{d2} = -\sin\theta_1, \qquad (0 \le t \le 5),$$

where $\Phi(\theta_d) = 2\theta_{d1} + \theta_{d2} - \pi = 0$. The constraint force ($f_{d1}$, f_{d2}) at each joints corresponds to the interaction force $F_{\rm const} = 5$ (N) generated at the end-effector (Fig.1).

In the following, Fig.2 ∼ Fig.5 show simulation results. The first two figures (Fig.2 and Fig.3) are given by the conventional approach (Yuan (1997)), and the last two figures (Fig.4 and Fig.5) are deduced from the proposed control scheme (Theorem 5). It is seen that good convergence and transient properties are attained in the proposed methodology (Fig.4 and Fig.5) compared with the conventional approach (Fig.2 and Fig.3).

Remark 7. The proposed control strategy in the numerical studies, can be also considered as the method in Section 5 (Theorem 2), where $\gamma_1 \sim \gamma_3$ are chosen such that

$$\frac{1}{2}\left\{\frac{2(1+\alpha)^2}{\gamma_1^2}+\frac{1}{\gamma_3^2}I+k_{Ri}\right\}=1, \quad (i=1,2),$$
$$\gamma_2 = 0.05.$$

This is because the control structures of $v(t)$ (58) and (70) are essentially the same as each others.

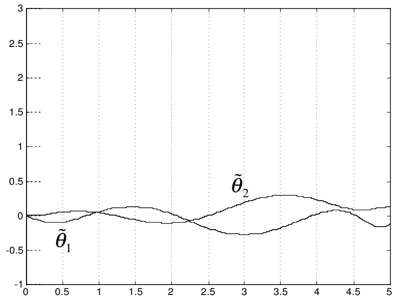

Fig. 2. Conventional approach ($\tilde{\theta}_1$, $\tilde{\theta}_2$ versus time).

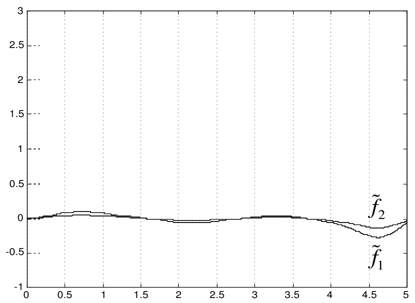

Fig. 3. Conventional approach ($\tilde{f}_1$, $\tilde{f}_2$ versus time).

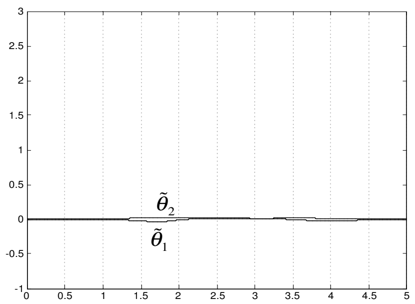

Fig. 4. Proposed approach ($\tilde{\theta}_1$, $\tilde{\theta}_2$ versus time).

8. CONCLUDING REMARKS

Design methodologies of nonlinear adaptive H_∞ control for constrained robotic manipulators are proposed, where tracking control of constrained trajectories and control of constraint forces are considered. The resulting control strategies are derived as solutions of corresponding H_∞ control problems, where estimation errors of tuning parameters and errors of constrained forces are regarded as external disturbances to the process. Two approaches

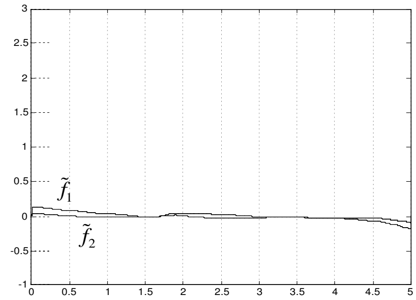

Fig. 5. Proposed approach ($\tilde{f}_1$, $\tilde{f}_2$ versus time).

are deduced based on that policy, and it is shown that $\mathcal{L}^2$ gains from those disturbances to generalized outputs are prescribed by several design parameters, explicitly. Extensions to more practical constraints including friction effects will be important topics in the future study.

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