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Robust Observer-based Output Tracking Control of Nonlinear Systems with Sensor Measurement Delays

Ya-Jun Pan

Department of Mechanical Engineering, Dalhousie University, Halifax, NS, B3J 2X4 Canada, Email: Yajun.Pan@Dal.Ca

Abstract: In this note, a practical issue related to the measurement delay is addressed for the output tracking control of a class of nonlinear uncertain systems. Observer based sliding mode control approach is proposed. The measurement delay is constant and bounded. The sliding mode control can handle matched L_∞ type system uncertainties with known bounding functions. However, the controller usually requires fully measurable and instantaneous states information without any delays. To deal with measurement time delay, a robust observer is constructed based on delayed output information from the sensor. Through designing the observer gain according to the Linear Matrix Inequality (LMI) techniques developed by Lyapunov Kravoskii method for time delay systems, the convergence of the estimation error with an uniform bound is ensured. Then the sliding mode control law is constructed based on estimated states. The convergence of the switching surface is ensured in finite time and the overall tracking error tends to be bounded due to the estimation error bound of the observer. Finally, a simulation example is presented to show the effectiveness of the proposed method.

Keywords: Nonlinear Systems, Observer, Time Delays, Linear Matrix Inequality (LMI), Sensor Measurement, Robust Control

1. INTRODUCTION

It is well known that sliding mode control has robustness to matched bounded uncertainties. SMC can handle matched $L_\infty[0, \infty)$ type system disturbance where the upper-bound knowledge is available (Utkin, 1992). Many of the conventional design approaches for sliding mode control systems assume that the states of the system are directly available for feedback control design (Chan, 1995) (Bartolini and Pydynowski, 1996) (Chung and Lin, 1998). Direct output feedback sliding mode control approaches are proposed as well by properly designing a sliding manifold based on the system output (Edwards *et al.*, 2000) (Edwards *et al.*, 2001). Observer-based approaches to estimate the states are desirable when the states are not measurable but observable (Zak and Hui, 1993)-(Rundell *et al.*, 1996). However, in nonlinear systems, the issue related to designing an appropriate nonlinear observer for the system should be carefully addressed.

Time delays including measurement delays from the sensor exist in many real systems (Dugard and Verriest, 1998) (Ramos and Pearson, 2000) (Niculescu, 2001) (Pan *et al.*, 2006*a*). When there are time delays involved in the states, (Niu *et al.*, 2004) addressed the observer based approach for the system with matched bounded uncertainties and state time delays. Though there are delays in the state, the controller design is based on the output $\mathbf{y}(t)$ instead of $\mathbf{y}(t-\tau)$ where τ is the time delay in the measurement channel. In (Gouaisbaut *et al.*, 2004), a class of nonlinear systems with time delays in the states is considered as well. The delay in the output causes significant effect on the sliding mode control design which requires instantaneous feedback from the measurable output. With partial measurable states, it is not straightforward to apply conventional SMC methods. With partial measurable states and delays in the output, it is even more difficult to deal with. Hence the main motivation of this work is to deal with such kind of problem when there are delays in the output channel - sensor measurement, for nonlinear uncertain systems.

The other consideration is that LMI techniques are now widely applied in dealing with time delay systems (Boyd *et al.*, 1994) (Mahmoud, 2000) (Gu *et al.*, 2003) (Pan *et al.*, 2006*b*). LMIs offer numerical methods to test the feasibility of a problem or to solve for an optimal solution. For linear systems with various types of time delays, LMIs are well applied in the literature.

In the proposed approach, an observer based on the delayed measurement output is firstly constructed. Then the robust controller is proposed to ensure the finite time convergence of the closed-loop systems. In this paper, the proper observer gain design is yielded through solving a delay-depended sufficient condition by LMI techniques, which is derived according to Lyapunov Krasovskii method. Instead of model transformation, zero equations are used by introducing additional slack matrix variables, which results in less conservative stability criteria (Pan *et al.*, 2006*b*). The estimation error of the observer re-

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sults in a bound. The main contributions of this work are that: (i) in the presence of uncertainties, the robust observer is designed by solving a Linear Matrix Inequality through achieving the delay dependent stability condition; (ii) bounded estimation error is achieved; (iii) the applied SMC can accomplish the tracking task in the existence of measurement time delays.

The paper is organized as follows. Section 2 presented the problem formulation as well as the assumptions applied. In Section 3, an observer based on delayed measurement output and less conservative delay dependent condition is designed. Section 4 proposes a nonlinear sliding mode control scheme for the whole system. A numerical example is demonstrated in Section 5 to show the effectiveness of the proposed scheme. Section 6 draws the conclusions. Notations: $\mathcal{R}^n$ denotes an n-dimension real vector space, $\|\cdot\|$ is the Euclidean norm and induced matrix norm, $L_\infty[0,\infty)$ is the space of uniformly bounded functions on $[0,\infty)$.

2. PROBLEM FORMULATION

Consider the following class of nonlinear system

$$\begin{cases} \dot{\mathbf{x}} = A\mathbf{x} + \boldsymbol{\psi}(t,\mathbf{x}) + B(t)\left[\mathbf{u} + \mathbf{d}(t,\mathbf{x})\right] \\ \mathbf{y}(t) = C_y\mathbf{x}(t-\tau), \end{cases} \tag{1}$$

where $\mathbf{x} \in \mathcal{R}^n$ is the state vector. $\mathbf{u} \in \mathcal{R}^n$ represents the control vector and $\mathbf{y} \in \Re^n$ is the output vector. τ is the delay due to the measurement of the sensor. $\mathbf{d}$ is the external disturbance. The system is required to track the desired trajectory $\mathbf{y} \to \mathbf{y}_d \in \mathcal{R}^n$ which is a smooth function. A is a known matrix.

Assumption 1. The friction force $\mathbf{d}(t,\mathbf{x})$ is bounded as $\|\mathbf{d}(t,\mathbf{x})\| \le \beta_d$ where β_d is a constant.

Assumption 2. The time delay τ is assumed to be known.

Assumption 3. $\beta_{B1} \le \|B(t)\| \le \beta_{B2}$, where β_{B1} and β_{B2} are positive constants.

Assumption 4. The function $\boldsymbol{\psi}(t,\mathbf{x})$ is Lipschitz with respect to $\mathbf{x}$. Thus there exists $\rho > 0$ such that

$$\|\boldsymbol{\psi}(t,\mathbf{x}) - \boldsymbol{\psi}(t,\mathbf{x}_d)\| \le C_\psi\|\mathbf{x}-\mathbf{x}_d\|, \quad \forall \mathbf{x} \in \mathcal{R}^n.$$

Lemma 1. - Jensen Inequality (Gu *et al.*, 2003) For any constant matrix $E \in \mathcal{R}^{n\times n}$, $E = E^T > 0$, vector function $\boldsymbol{\omega} : [0,\tau] \to \mathcal{R}^n$ such that the integrations concerned are well defined, then,

$$\tau \int_0^\tau \boldsymbol{\omega}^T(s)E\boldsymbol{\omega}(s)ds \ge \left[\int_0^\tau \boldsymbol{\omega}(s)ds\right]^T E \left[\int_0^\tau \boldsymbol{\omega}(s)ds\right] . \tag{2}$$

In the following section, based on delay-dependent condition by using Lyapunov Krasovskii method, an observer is designed to estimate the state which is used to facilitate the robust controller design.

3. ROBUST OBSERVER DESIGN BASED ON DELAY-DEPENDENT CONDITION

According to (1), the observer based on delayed measurement is designed as

$$\begin{cases} \dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + \boldsymbol{\psi}(t,\hat{\mathbf{x}}) + B(t)\mathbf{u} - K_2[\mathbf{y}(t) - C_y\hat{\mathbf{x}}(t-\tau)] \\ \mathbf{y}(t) = C_y\mathbf{x}(t-\tau), \end{cases} \tag{3}$$

where $C_y \in \mathcal{R}^m$, and K_1 and $K_2 \in \mathcal{R}^{n\times n}$ are two designed gains. Denote $\tilde{\mathbf{x}}(t-\tau) \triangleq \mathbf{x}(t-\tau) - \hat{\mathbf{x}}(t-\tau)$ and $\hat{\mathbf{x}}(t)$, (3) can be rewritten as

$$\dot{\hat{\mathbf{x}}} = A\hat{\mathbf{x}} + \boldsymbol{\psi}(t,\hat{\mathbf{x}}) + B(t)\mathbf{u} - KC_y\tilde{\mathbf{x}}(t-\tau), \tag{4}$$

Compare (1) with (4), then the estimation error dynamics - $\tilde{\mathbf{x}}(t)$ becomes,

$$\dot{\tilde{\mathbf{x}}}(t) = A\tilde{\mathbf{x}}(t) + \tilde{\boldsymbol{\psi}}(t,\mathbf{x},\hat{\mathbf{x}}) + KC_y\tilde{\mathbf{x}}(t-\tau) + D(t)\mathbf{d}(t,\mathbf{x}), \tag{5}$$

where $\tilde{\boldsymbol{\psi}}(t,\mathbf{x},\hat{\mathbf{x}}) = \boldsymbol{\psi}(t,\mathbf{x}) - \boldsymbol{\psi}(t,\hat{\mathbf{x}})$ and $D(t) = B(t)$. Based on the delayed state signal available at the controller side, e.g. $\mathbf{x}(t-\tau)$, the observer (4) is designed to observe the state signal $\mathbf{x}(t)$. In the following theorem, K is designed according to the linear matrix inequality derived based on the Lyapunov Kravoskii method.

Theorem 1. Consider the estimation error dynamics (5), for a given time delay τ, if there exist symmetric positive definite matrices $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} > 0$, $Q = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22} \end{bmatrix} > 0$, $R = \begin{bmatrix} R_{11} & 0 \\ 0 & R_{22} \end{bmatrix} > 0$, matrices $K, M_i, N_i, i = 1, ..., 5$, with appropriate dimensions and a scalar $\varepsilon > 0$ such that the following inequality holds

$$\begin{bmatrix} \Xi & M \\ M^T & -\frac{\varepsilon}{2}I \end{bmatrix} < 0, \tag{6}$$

where $M = [M_1^T, M_2^T, M_3^T, M_4^T, M_5^T]^T$, and

$$\Xi = \begin{bmatrix} \Xi_{11} & * & * & * & * \\ \Xi_{21} & \Xi_{22} & * & * & * \\ \Xi_{31} & \Xi_{32} & \Xi_{33} & * & * \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} & * \\ \Xi_{51} & \Xi_{52} & \Xi_{53} & \Xi_{54} & \Xi_{55} \end{bmatrix}, \tag{7}$$

with

$$\begin{aligned}
\Xi_{11} &= Q_{11} + P_{12} + P_{12}^T + \tau R_{11} + N_1 + N_1^T - M_1A \\
&\quad -A^TM_1^T + \varepsilon C_\psi^2 I + I \\
\Xi_{21} &= -P_{12}^T + N_2 - M_2A - N_1^T - C_y^TK^TM_1^T, \\
\Xi_{22} &= -M_2KC_y - (M_2KC_y)^T - N_2 - N_2^T - Q_{11}, \\
\Xi_{31} &= P_{11} + N_3 - M_3A + M_1^T, \\
\Xi_{32} &= -N_3 + M_2^T - M_3KC_y, \\
\Xi_{33} &= Q_{22} + M_3 + M_3^T + \tau R_{22}, \\
\Xi_{41} &= P_{22} + N_4 - M_4A, \\
\Xi_{42} &= -P_{22} - N_4 - M_4KC_y, \quad \Xi_{43} = M_4 + P_{12}^T, \\
\Xi_{44} &= -\frac{R_{11}}{\tau}, \quad \Xi_{51} = N_5 - N_1^T - M_5A, \\
\Xi_{52} &= -M_5KC_y - N_5 - N_2^T, \quad \Xi_{53} = M_5 - N_3^T, \\
\Xi_{54} &= -N_4^T, \quad \Xi_{55} = -\frac{R_{22}}{\tau} - N_5 - N_5^T,
\end{aligned}$$

then the system (5) is stable, e.g. $\tilde{\mathbf{x}}(t)$ is globally uniformly bounded.

Proof: See Appendix A.

The LMI condition in (7) is non-convex and hence the following theorem is proposed to be the equivalent sufficient condition as in Theorem 1.

Theorem 2. For given scalars θ_i, $i = 1, \cdots, 5$, and a given time delay constant τ, if there exist symmetric positive definite matrices $\bar{P} = \begin{bmatrix} \bar{P}_{11} & \bar{P}_{12} \\ \bar{P}_{12}^T & \bar{P}_{22} \end{bmatrix} > 0$, $\bar{Q} = \begin{bmatrix} \bar{Q}_{11} & 0 \\ 0 & \bar{Q}_{22} \end{bmatrix} > 0$, $\bar{R} = \begin{bmatrix} \bar{R}_{11} & 0 \\ 0 & \bar{R}_{22} \end{bmatrix} > 0$, matrices Y, $\bar{N}_i$, $i = 1, \cdots, 5$, nonsingular matrix X with appropriate dimensions and constant $\varepsilon > 0$ such that the following inequality holds,

$$\begin{bmatrix} \bar{\Xi}_{11} & * & * & * & * & * & * \\ \bar{\Xi}_{21} & \bar{\Xi}_{22} & * & * & * & * & * \\ \bar{\Xi}_{31} & \bar{\Xi}_{32} & \bar{\Xi}_{33} & * & * & * & * \\ \bar{\Xi}_{41} & \bar{\Xi}_{42} & \bar{\Xi}_{43} & \bar{\Xi}_{44} & * & * & * \\ \bar{\Xi}_{51} & \bar{\Xi}_{52} & \bar{\Xi}_{53} & \bar{\Xi}_{54} & \bar{\Xi}_{55} & * & * \\ \theta_1 I & \theta_2 I & \theta_3 I & \theta_4 I & \theta_5 I & -\frac{\varepsilon I}{2} & 0 \\ (\varepsilon C_\psi^2 + 1)X^T & 0 & 0 & 0 & 0 & 0 & -\frac{\varepsilon I}{2} \end{bmatrix} < 0, \qquad (8)$$

where

$$\begin{aligned} \bar{\Xi}_{11} &= \bar{Q}_{11} + \bar{P}_{12} + \bar{P}_{12}^T + \tau \bar{R}_{11} + \bar{N}_1 + \bar{N}_1^T \\ &\quad - \theta_1 A X^T - \theta_1 X A^T \\ \bar{\Xi}_{21} &= -\bar{P}_{12}^T + \bar{N}_2 - \theta_2 A X^T - \bar{N}_1^T - \theta_1 Y^T, \\ \bar{\Xi}_{22} &= -\theta_2 Y - \theta_2 Y^T - \bar{N}_2 - \bar{N}_2^T - \bar{Q}_{11}, \\ \bar{\Xi}_{31} &= \bar{P}_{11} + \bar{N}_3 - \theta_3 A X^T + \theta_1 X, \\ \bar{\Xi}_{32} &= -\bar{N}_3 + \theta_2 X - \theta_3 Y, \\ \bar{\Xi}_{33} &= \bar{Q}_{22} + \theta_3 X + \theta_3 X^T + 2\tau \bar{R}_{22}, \\ \bar{\Xi}_{41} &= \bar{P}_{22} + \bar{N}_4 - \theta_4 A X^T, \\ \bar{\Xi}_{42} &= -\bar{P}_{22} - \bar{N}_4 - \theta_4 Y, \\ \bar{\Xi}_{43} &= \theta_4 X^T + \bar{P}_{12}^T, \quad \bar{\Xi}_{44} = -\frac{\bar{R}_{11}}{2\tau}, \\ \bar{\Xi}_{51} &= \bar{N}_5 - \bar{N}_1^T - \theta_5 A X^T, \\ \bar{\Xi}_{52} &= -\theta_5 Y - \bar{N}_5 - \bar{N}_2^T, \\ \bar{\Xi}_{53} &= \theta_5 X^T - \bar{N}_3^T, \\ \bar{\Xi}_{54} &= -\bar{N}_4^T, \quad \bar{\Xi}_{55} = -\frac{\bar{R}_{22}}{2\tau} - \bar{N}_5 - \bar{N}_5^T, \end{aligned}$$

then matrices K in Theorem 1 is obtained as

$$K = Y X^{-T} C_y^T \left(C_y C_y^T \right)^{-1}. \qquad (9)$$

As a result, the error dynamics (5) is stable, e.g. $\tilde{\mathbf{x}}(t)$ is globally uniformly bounded by $\sqrt{\varepsilon}\beta_{B2}\beta_d$.

Proof: In order to transform the nonconvex LMI in (7) into a solvable LMI, (7) could be represented as the following form by schur complement,

$$\begin{bmatrix} \Xi_{11} - (\varepsilon C_\psi^2 + 1)I & * & * & * & * & * & * \\ \Xi_{21} & \Xi_{22} & * & * & * & * & * \\ \Xi_{31} & \Xi_{32} & \Xi_{33} & * & * & * & * \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} & * & * & * \\ \Xi_{51} & \Xi_{52} & \Xi_{53} & \Xi_{54} & \Xi_{55} & * & * \\ M_1 & M_2 & M_3 & M_4 & M_5 & -\frac{\varepsilon I}{2} & 0 \\ (\varepsilon C_\psi^2 + 1)X^T & 0 & 0 & 0 & 0 & 0 & -(\varepsilon C_\psi^2 + 1)I \end{bmatrix} < 0, \qquad (10)$$

we assume that we have some relations in M_i's, $i = 1, \cdots, 5$. One possibility is that $M_i = \theta_i M_0$ where M_0 is nonsingular and θ_i is known and given. Define $X = M_0^{-1}$, $W = diag(X, X, X, X, X, I, I)$ and $Y = K C_y X^T$. Then by pre-multiplying the inequality in (10) by W and post-multiplying by W^T, we can obtain the inequality (8). Note that the inequality in (8) is only a sufficient condition for the solvability of (7) based on the derivation.

4. ROBUST NONLINEAR CONTROLLER DESIGN

4.1 Robust Controller Design

Define the tracking error as $\mathbf{e} = \hat{\mathbf{y}} - \mathbf{y}_d$. The switching surface here is usually chosen as

$$\boldsymbol{\sigma} = G\mathbf{e} = G(C_y \hat{\mathbf{x}} - \mathbf{y}_d), \qquad (11)$$

where $G > 0$ is a diagonal nonsingular matrix. The robust nonlinear controller is constructed as below

$$\mathbf{u} = \mathbf{u}_c + \Gamma(t)\mathbf{u}_s \qquad (12)$$

where

$$\mathbf{u}_c = -\Phi \left[C_y A \hat{\mathbf{x}} + C_y \boldsymbol{\psi}(t, \hat{\mathbf{x}}) - C_y K C_y \tilde{\mathbf{x}}(t - \tau) - \dot{\mathbf{y}}_d \right], \qquad (13)$$

is the designed nominal control,

$$\Phi = \left[(C_y B(t))^T C_y B(t) \right]^{-1} (C_y B(t))^T$$

and $\mathbf{u}_s = [u_{s1}, u_{s2}, \cdots, u_{sn}]^T$ is an n-vector switching quantity with

$$u_{si} = -sign(\sigma_i), \qquad i = 1, 2, \cdots, n. \qquad (14)$$

Furthermore, $\Gamma(t) = diag(\gamma_1(t), \cdots, \gamma_n(t)) > 0$ is a diagonal gain matrix.

4.2 Convergence Analysis

First construct a Lyapunov function $V(t) = \frac{1}{2}\boldsymbol{\sigma}^T G^{-1} \boldsymbol{\sigma} > 0$. The derivative of V becomes

$$\begin{aligned} \dot{V}(t) &= \boldsymbol{\sigma}^T G^{-1} \dot{\boldsymbol{\sigma}} = \boldsymbol{\sigma}^T \left(C_y \dot{\hat{\mathbf{x}}} - \dot{\mathbf{y}}_d \right) \\ &= \boldsymbol{\sigma}^T \left\{ C_y \left[A\hat{\mathbf{x}} + \boldsymbol{\psi}(t, \hat{\mathbf{x}}) + B(t)\mathbf{u}_c + B(t)\Gamma(t)\mathbf{u}_s \right] \right\} \\ &\quad - \boldsymbol{\sigma}^T \left[C_y K C_y \tilde{\mathbf{x}}(t - \tau) \right] - \boldsymbol{\sigma}^T \dot{\mathbf{y}}_d \\ &\le -\sum_{i=1}^{n} \beta_{B1} \gamma_i(t) \sigma_i^2 < 0. \end{aligned} \qquad (15)$$

Hence the system converges and it can reach the switching surface in finite time. As a result, the overall tracking error is bounded as well - see Remark 1.

Remark 1. Note that at the steady state, the estimation error $\tilde{\mathbf{x}}$ is bounded by $\beta_{\tilde{\mathbf{x}}} \overset{\triangle}{=} \sqrt{\varepsilon}\beta_{B_2}\beta_d$ and $\boldsymbol{\sigma} = 0 \Rightarrow C_y \hat{\mathbf{x}} - \mathbf{y}_d = 0$. The overall tracking error is

$$\mathbf{y} - \mathbf{y}_d = C_y \mathbf{x} - \mathbf{y}_d = \underbrace{C_y \hat{\mathbf{x}} - \mathbf{y}_d}_{0} + C_y(\mathbf{x} - \hat{\mathbf{x}}).$$

As a result, $\|\mathbf{y} - \mathbf{y}_d\| = \|C_y\| \beta_{\tilde{\mathbf{x}}}$, which shows that the overall tracking error is uniformly bounded.

5. ILLUSTRATIVE EXAMPLES

Consider the nonlinear system in (1) with

$$\begin{aligned}\boldsymbol{\psi}(t,\mathbf{x}) &= [0.1sin(x_1), 0.5cos(x_2)]^T,\\ B(t) &= [1+0.5cos(t), 0.3sin(t)]^T,\\ d(t,\mathbf{x}) &= sin(x_1(t)) + cos(x_2(t)),\end{aligned}$$

where $\mathbf{x} = [x_1, x_2]^T \in \mathcal{R}^4$ and $\tau = 1\ sec$. The initial condition is $\mathbf{x}(0) = [0.5, 0.9]^T$. $y(t) = x_1(t-\tau)$. $A = \begin{bmatrix} -1 & 1 \\ -2 & -3 \end{bmatrix}$, and $C_y = [\,1\ 0\,]$. It is obvious that $\boldsymbol{\psi}(t,\mathbf{x})$ satisfies the Lipschitz condition in Assumption 4. Furthermore, $C_yB(t)$ is nonsingular. The target trajectory is $y_d = x_{1d} = sin(\pi t)$. The initial condition for observer is $\hat{\mathbf{x}}(0) = [1,1]^T$.

By solving the LMI in Theorem 2, when $\tau = 1sec$, $K = \begin{bmatrix} 1.796 \\ -0.1828 \end{bmatrix}$; when $\tau = 0.1sec$, $K = \begin{bmatrix} 1.1302 \\ 0.3849 \end{bmatrix}$. The switching surface is designed according to (11) with $G = 1$. The control law is designed according to (12) with $\Gamma = 1$.

As shown in $Fig.1.(a)$, the system tracking error converge to a bound asymptotically. As well, the observation error in $Fig.1.(b)$ tends to a steady state within a certain bound due to the existence of the matched disturbance. The profile of the switching surface is as shown in $Fig.2$. We can observe that it reaches zero in finite time. It shows the evolution of the switching surface with smoothing scheme ($u_{si} = -\frac{\sigma_i}{|\sigma_i|+0.008}$).

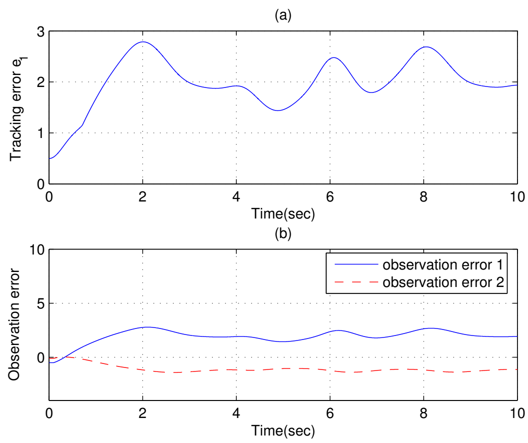

Fig. 1. (a) The profile of the tracking error e_1 when $\tau = 0.1\ sec$; (b) The profile of the observation error $\tilde{\mathbf{x}}$.

When we elongate the time delay to be 1 second, the corresponding results are as shown in the figures $Fig.3$ and $Fig.4$. Comparing the error magnitude with the one in $Fig.1.(a)$, the bound is larger due to the longer time delay.

6. CONCLUSIONS

In this paper, a robust observer-based control approach is proposed with rigorous proof of the convergence. The approach utilize the LMI techniques to facilitate the observer gain design for the error dynamics in the form of time delay systems. Delayed feedback is well applied due to the lack of instantaneous measurable output. For the uncertain nonlinear systems with partial linearity property, the proposed scheme ensures the boundedness of the closed loop system. The approach dealt with the case with known constant time delays. It would be interesting to further investigate

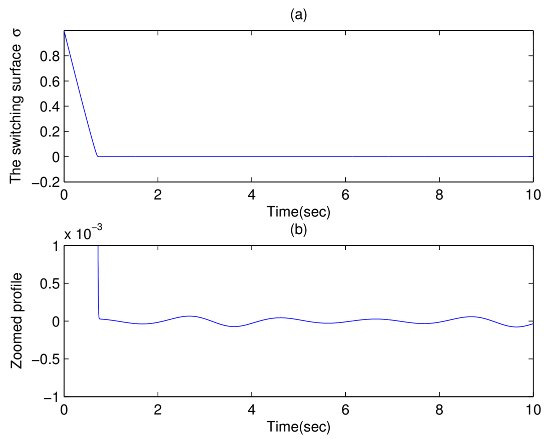

Fig. 2. The profile of the switching surface σ.

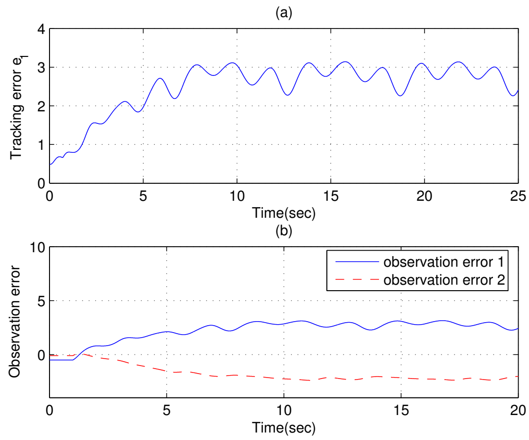

Fig. 3. (a) The profile of the tracking error e_1 when $\tau = 1\ sec$; (b) The profile of the observation error $\tilde{\mathbf{x}}$.

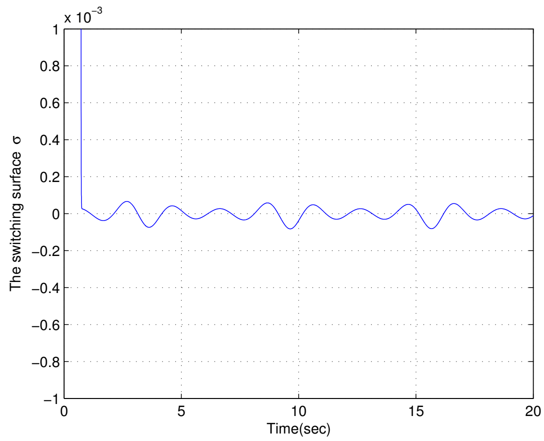

Fig. 4. The profile of the switching surface σ when using the smoothing scheme.

the general case with time varying delays and unmatched uncertainties.

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APPENDIX A: PROOF OF THEOREM 1

Consider the following Lyapunov Krasovskii functional candidate:

$$\begin{aligned}
V &= \tilde{\mathbf{x}}^T(t)P_{11}\tilde{\mathbf{x}}(t) + 2\tilde{\mathbf{x}}^T(t)P_{12}[\int_{t-\tau}^{t}\tilde{\mathbf{x}}(s)ds] \\
&+[\int_{t-\tau}^{t}\tilde{\mathbf{x}}(s)ds]^T P_{22}[\int_{t-\tau}^{t}\tilde{\mathbf{x}}(s)ds] \\
&+\int_{t-\tau}^{t}[\tilde{\mathbf{x}}^T(s)+\dot{\tilde{\mathbf{x}}}^T(s)]^T\, Q \begin{bmatrix}\tilde{\mathbf{x}}(s)\\ \dot{\tilde{\mathbf{x}}}(s)\end{bmatrix} ds \\
&+\int_{-\tau}^{0}\int_{t+\theta}^{t}[\tilde{\mathbf{x}}^T(s)\dot{\tilde{\mathbf{x}}}^T(s)]^T\, R \begin{bmatrix}\tilde{\mathbf{x}}(s)\\ \dot{\tilde{\mathbf{x}}}(s)\end{bmatrix} dsd\theta, \qquad (16)
\end{aligned}$$

where

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22}\end{bmatrix} > 0, \qquad Q = \begin{bmatrix} Q_{11} & 0 \\ 0 & Q_{22}\end{bmatrix} > 0,$$
$$R = \begin{bmatrix} R_{11} & 0 \\ 0 & R_{22}\end{bmatrix} > 0.$$

With appropriate dimensions, the following two zero equations hold:

$$\begin{aligned}
\Phi_1 &= 2\{\tilde{\mathbf{x}}^T(t)N_1 + \tilde{\mathbf{x}}^T(t-\tau)N_2 + \dot{\tilde{\mathbf{x}}}^T(t)N_3 \\
&\quad +[\int_{t-\tau}^{t}\tilde{\mathbf{x}}(s)ds]^T N_4 + [\int_{t-\tau}^{t}\dot{\tilde{\mathbf{x}}}(s)ds]^T N_5\} \\
&\quad [\tilde{\mathbf{x}}(t) - \int_{t-\tau}^{t}\dot{\tilde{\mathbf{x}}}(s)ds - \tilde{\mathbf{x}}(t-\tau)] = 0, \\
\Phi_2 &= 2\{\tilde{\mathbf{x}}^T(t)M_1 + \tilde{\mathbf{x}}^T(t-\tau)M_2 + \dot{\tilde{\mathbf{x}}}^T(t)M_3 \\
&\quad +[\int_{t-\tau}^{t}\tilde{\mathbf{x}}(s)ds]^T M_4 + [\int_{t-\tau}^{t}\dot{\tilde{\mathbf{x}}}(s)ds]^T M_5\}\{\dot{\tilde{\mathbf{x}}}(t) \\
&\quad -A\tilde{\mathbf{x}}(t) - \tilde{\boldsymbol{\psi}}(t,\mathbf{x},\hat{\mathbf{x}}) - KC_y\tilde{\mathbf{x}}(t-\tau) - D(\mathbf{x}_1)\mathbf{d}(t)\} \\
&\quad = 0.
\end{aligned}$$

Then the derivative of the Lyapunov function candidate is as follows,

$$\begin{aligned}
\dot{V} &= \dot{V} + \Phi_1 + \Phi_2 = \tilde{\mathbf{x}}^T(t)P_{11}\dot{\tilde{\mathbf{x}}}(t) + \dot{\tilde{\mathbf{x}}}^T(t)P_{11}\tilde{\mathbf{x}}(t) \\
&+2\dot{\tilde{\mathbf{x}}}^T(t)P_{12}[\int_{t-\tau}^{t}\tilde{\mathbf{x}}(s)ds] + 2\tilde{\mathbf{x}}^T(t)P_{12}[\tilde{\mathbf{x}}(t)-\tilde{\mathbf{x}}(t-\tau)] \\
&+[\tilde{\mathbf{x}}(t)-\tilde{\mathbf{x}}(t-\tau)]^T P_{22}\int_{t-\tau}^{t}\tilde{\mathbf{x}}(s)ds \\
&+[\int_{t-\tau}^{t}\tilde{\mathbf{x}}(s)ds]^T P_{22}[\tilde{\mathbf{x}}(t)-\tilde{\mathbf{x}}(t-\tau)]
\end{aligned}$$

$$
\begin{aligned}
&+ [\tilde{\mathbf{x}}^T(t)\ \dot{\tilde{\mathbf{x}}}^T(t)]^T Q \begin{bmatrix} \tilde{\mathbf{x}}(t) \\ \dot{\tilde{\mathbf{x}}}(t) \end{bmatrix} \\
&-[\tilde{\mathbf{x}}^T(t-\tau)\ \ \dot{\tilde{\mathbf{x}}}^T(t-\tau)]^T Q \begin{bmatrix} \tilde{\mathbf{x}}(t-\tau) \\ \dot{\tilde{\mathbf{x}}}(t-\tau) \end{bmatrix} \\
&+\tau[\tilde{\mathbf{x}}^T(t)\ \dot{\tilde{\mathbf{x}}}^T(t)]^T R \begin{bmatrix} \tilde{\mathbf{x}}(t) \\ \dot{\tilde{\mathbf{x}}}(t) \end{bmatrix} \\
&-\int_{t-\tau}^{t} [\tilde{\mathbf{x}}^T(s)\ \ \dot{\tilde{\mathbf{x}}}^T(s)]^T R \begin{bmatrix} \tilde{\mathbf{x}}(s) \\ \dot{\tilde{\mathbf{x}}}(s) \end{bmatrix} ds \\
&+2\mathbf{z}^T N[\tilde{\mathbf{x}}(t) - \int_{t-\tau}^{t} \dot{\tilde{\mathbf{x}}}(s)ds - \tilde{\mathbf{x}}(t-\tau)] \\
&+2\mathbf{z}^T M[\dot{\tilde{\mathbf{x}}}(t) - A\tilde{\mathbf{x}}(t) - \tilde{\boldsymbol{\psi}}(t,\mathbf{x},\hat{\mathbf{x}}) \\
&-KC_y\tilde{\mathbf{x}}(t-\tau) - D(\mathbf{x}_1)\mathbf{d}(t)],
\end{aligned} \tag{17}
$$

where

$$
\mathbf{z} = \left[\tilde{\mathbf{x}}^T(t), \tilde{\mathbf{x}}^T(t-\tau), \dot{\tilde{\mathbf{x}}}^T(t), [\int_{t-\tau}^{t} \tilde{\mathbf{x}}(s)ds]^T, [\int_{t-\tau}^{t} \dot{\tilde{\mathbf{x}}}(s)ds]^T\right]^T
$$

$$
\begin{aligned}
N &= [N_1^T\ \ N_2^T\ \ N_3^T\ \ N_4^T\ \ N_5^T]^T, \\
M &= [M_1^T\ \ M_2^T\ \ M_3^T\ \ M_4^T\ \ M_5^T]^T.
\end{aligned}
$$

Furthermore, we have

$$
\begin{aligned}
-2\mathbf{z}^T M\tilde{\boldsymbol{\psi}} &\le \varepsilon^{-1}(\mathbf{z}^T MM^T\mathbf{z}) + \varepsilon\tilde{\boldsymbol{\psi}}^T\tilde{\boldsymbol{\psi}} \\
&\le \varepsilon^{-1}(\mathbf{z}^T MM^T\mathbf{z}) + \varepsilon C_\psi^2\tilde{\mathbf{x}}^T(t)\tilde{\mathbf{x}}(t), \quad (18) \\
-2\mathbf{z}^T MD\mathbf{d} &\le \varepsilon^{-1}(\mathbf{z}^T MM^T\mathbf{z}) + \varepsilon(D\mathbf{d})^T D\mathbf{d} \\
&\le \varepsilon^{-1}(\mathbf{z}^T MM^T\mathbf{z}) + \varepsilon\beta_{B2}^2\beta_d^2. \quad (19)
\end{aligned}
$$

where ε and γ are positive constants. Using (17), (19) and the Jensen inequality in (2),

$$
\begin{aligned}
\dot{V} \le\ & \tilde{\mathbf{x}}^T(t)P_{11}\dot{\tilde{\mathbf{x}}}(t) + \dot{\tilde{\mathbf{x}}}^T(t)P_{11}\tilde{\mathbf{x}}(t) \\
&+2\dot{\tilde{\mathbf{x}}}^T(t)P_{12}[\int_{t-\tau}^{t} \tilde{\mathbf{x}}(s)ds] + 2\tilde{\mathbf{x}}^T(t)P_{12}[\tilde{\mathbf{x}}(t) - \tilde{\mathbf{x}}(t-\tau)] \\
&+[\tilde{\mathbf{x}}(t) - \tilde{\mathbf{x}}(t-\tau)]^T P_{22}\int_{t-\tau}^{t} \tilde{\mathbf{x}}(s)ds \\
&+[\int_{t-\tau}^{t} \tilde{\mathbf{x}}(s)ds]^T P_{22}[\tilde{\mathbf{x}}(t) - \tilde{\mathbf{x}}(t-\tau)] \\
&+ [\tilde{\mathbf{x}}^T(t)\ \ \dot{\tilde{\mathbf{x}}}^T(t)]^T Q \begin{bmatrix} \tilde{\mathbf{x}}(t) \\ \dot{\tilde{\mathbf{x}}}(t) \end{bmatrix} \\
&-[\tilde{\mathbf{x}}^T(t-\tau)\ \ \dot{\tilde{\mathbf{x}}}^T(t-\tau)]^T Q \begin{bmatrix} \tilde{\mathbf{x}}(t-\tau) \\ \dot{\tilde{\mathbf{x}}}(t-\tau) \end{bmatrix} \\
&+\tau[\tilde{\mathbf{x}}^T(t)\ \ \dot{\tilde{\mathbf{x}}}^T(t)]^T R \begin{bmatrix} \tilde{\mathbf{x}}(t) \\ \dot{\tilde{\mathbf{x}}}(t) \end{bmatrix} \\
&-\int_{t-\tau}^{t} [\tilde{\mathbf{x}}^T(s)\ \ \dot{\tilde{\mathbf{x}}}^T(s)]^T R \begin{bmatrix} \tilde{\mathbf{x}}(s) \\ \dot{\tilde{\mathbf{x}}}(s) \end{bmatrix} ds \\
&+2Z^T N[\tilde{\mathbf{x}}(t) - \int_{t-\tau}^{t} \dot{\tilde{\mathbf{x}}}(s)ds - \tilde{\mathbf{x}}(t-\tau)] \\
&+2\mathbf{z}^T M[\dot{\tilde{\mathbf{x}}}(t) - A\tilde{\mathbf{x}}(t) - KC_y\tilde{\mathbf{x}}(t-\tau)] \\
&+2\varepsilon^{-1}(\mathbf{z}^T MM^T\mathbf{z}) + \varepsilon C_\psi^2\tilde{\mathbf{x}}^T(t)\tilde{\mathbf{x}}(t) + \varepsilon\beta_{B2}^2\beta_d^2 \\
\le\ & \tilde{\mathbf{x}}^T(t)P_{11}\dot{\tilde{\mathbf{x}}}(t) + \dot{\tilde{\mathbf{x}}}^T(t)P_{11}\tilde{\mathbf{x}}(t) \\
&+2\dot{\tilde{\mathbf{x}}}^T(t)P_{12}[\int_{t-\tau}^{t} \tilde{\mathbf{x}}(s)ds] \\
&+2\tilde{\mathbf{x}}^T(t)P_{12}[\tilde{\mathbf{x}}(t) - \tilde{\mathbf{x}}(t-\tau)] \\
&+[\tilde{\mathbf{x}}(t) - \tilde{\mathbf{x}}(t-\tau)]^T P_{22}\int_{t-\tau}^{t} \tilde{\mathbf{x}}(s)ds \\
&+[\int_{t-\tau}^{t} \tilde{\mathbf{x}}(s)ds]^T P_{22}[\tilde{\mathbf{x}}(t) - \tilde{\mathbf{x}}(t-\tau)] \\
&+ [\tilde{\mathbf{x}}^T(t)\ \ \dot{\tilde{\mathbf{x}}}^T(t)]^T Q \begin{bmatrix} \tilde{\mathbf{x}}(t) \\ \dot{\tilde{\mathbf{x}}}(t) \end{bmatrix} \\
&-\tilde{\mathbf{x}}^T(t-\tau)Q_{11}\tilde{\mathbf{x}}(t-\tau) \\
&+\tau[\tilde{\mathbf{x}}^T(t)\ \ \dot{\tilde{\mathbf{x}}}^T(t)]^T R \begin{bmatrix} \tilde{\mathbf{x}}(t) \\ \dot{\tilde{\mathbf{x}}}(t) \end{bmatrix} \\
&-\frac{1}{\tau}\left\{\int_{t-\tau}^{t} [\tilde{\mathbf{x}}^T(s)\ \ \dot{\tilde{\mathbf{x}}}^T(s)]^T ds\right\} R \left\{\int_{t-\tau}^{t} \begin{bmatrix} \tilde{\mathbf{x}}(s) \\ \dot{\tilde{\mathbf{x}}}(s) \end{bmatrix}\right\} ds \\
&+2Z^T N[\tilde{\mathbf{x}}(t) - \int_{t-\tau}^{t} \dot{\tilde{\mathbf{x}}}(s)ds - \tilde{\mathbf{x}}(t-\tau)] \\
&+2\mathbf{z}^T M[\dot{\tilde{\mathbf{x}}}(t) - A\tilde{\mathbf{x}}(t) - KC_y\tilde{\mathbf{x}}(t-\tau)] \\
&+2\varepsilon^{-1}(\mathbf{z}^T MM^T\mathbf{z}) + \varepsilon C_\psi^2\tilde{\mathbf{x}}^T(t)\tilde{\mathbf{x}}(t) + \varepsilon\beta_{B2}^2\beta_d^2 \\
\le\ & -\tilde{\mathbf{x}}^T(t)\tilde{\mathbf{x}}(t) + \mathbf{z}^T\Xi\mathbf{z} + 2\varepsilon^{-1}(\mathbf{z}^T MM^T\mathbf{z}) + \varepsilon\beta_{B2}^2\beta_d^2,
\end{aligned} \tag{20}
$$

where Ξ is as shown in (7). The inequality (20) is equivalent to

$$
\dot{V} \le -\tilde{\mathbf{x}}^T(t)\tilde{\mathbf{x}}(t) + \varepsilon\beta_{B2}^2\beta_d^2 + \mathbf{z}^T \begin{bmatrix} \Xi & M \\ M^T & -\frac{\varepsilon}{2}I \end{bmatrix} \mathbf{z}. \tag{21}
$$

If there exist symmetric positive definite matrices $P > 0, Q > 0, R > 0$, matrices $K, M_i, N_i, i = 1,...5$, with appropriate dimensions and a scalar $\varepsilon > 0$ such that the inequality (6) holds, then from (21) we have

$$
\dot{V} \le -\|\tilde{\mathbf{x}}(t)\|^2 + \varepsilon\beta_{B2}^2\beta_d^2. \tag{22}
$$

From the Lyapunov stability theory, the system (5) is stable and $\tilde{\mathbf{x}}(t)$ is globally uniformly bounded, i.e., $\|\tilde{\mathbf{x}}\| \le \sqrt{\varepsilon}\beta_{B2}\beta_d$.