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Fault Detection of Networked Control Systems with Packet Dropout ⋆

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Abstract: Fault detection of networked control systems with packet dropouts in both sensor-to-controller link and controller-to-actuator link is discussed in this paper. Two types of packet dropouts are considered. One is packet dropout characterized by a Bernoulli process, the other is characterized by a Markov chain. According to the system configuration, packet dropout in the sensor-to-controller link is known to the fault detection system, therefore a time varying parity space based residual generator which is fully decoupled from this influence is designed. Robustness to packet dropout in the controller-to-actuator link which is unknown is achieved by an adaptive threshold. Due to the known probability properties of the residual signal in parity space approach, upper bounds of false alarm rates of the designed thresholds are also given, which are very difficult to obtain in existent observer based fault detection approaches of networked control systems with packet dropout. It is verified that the proposed fault detection approach can also be realized by diagnostic observers by employing the relationship between parity vectors and observers.

1. INTRODUCTION

Networked control systems (NCS) are feedback control systems wherein the control loops are closed via real-time networks (Zhang et al. [2001]). In an NCS, the plant output(s) and control input(s) are transmitted through communication networks. This new type of information transmission reduces system wiring, eases maintenance and diagnosis, and increases system agility, which makes NCS a promising structure for control (Zhang et al. [2001]). However, the introduction of networks also brings some new problems and challenges, such as network-induced delay (Liu et al. [2006]), packet dropout (Wang et al. [2007], Sahebsara et al. [2007]) and quantization problems (Goodwin et al. [2004]), etc.

Since fault detection technique is essential for improving the safety and reliability of networked control systems, recently, more and more attention has been paid to fault detection of NCS, such as Zhang et al. [2004], Ye and Ding [2004], Fang et al. [2006], Wang et al. [2006] and Wang et al. [2007]. However, due to the stochastic characteristics of packet dropout in NCS, fault detection of NCS with random packet dropout is still an open problem. In principle, an observer based fault detection can be divided into two stages: residual generation and residual evaluation. In the residual evaluation stage, a threshold should be provided to make a fault decision: if the generated residual signal is less than the given threshold, there is no fault, and if the residual signal surpasses the threshold, we say a fault occurs. In order to evaluate the performance of a certain threshold, its false alarm rate should be calculated, which is determined by the stochastic probability distribution of the residual signal. However, in existent observer based fault detection approaches of networked control systems with packet dropout, such as Zhang et al. [2004] and Fang et al. [2006], we can not get the probability distribution of the residual signal, which makes calculation of fault alarm rate impossible.

This paper is aimed at fault detection of NCS in the presence of stochastic packet dropouts in both sensor-to-controller and controller-to-actuator links. Problems of residual evaluation and false alarm rate calculation are circumvented by parity space approach, for under the condition that characteristics of packet dropout are governed by a Bernoulli process or a Markov chain, the probability distribution of residual signal is totally known in parity space approach, thus false alarm rate of a certain threshold can easily be obtained.

2. PROBLEM FORMULATION

As shown in Fig. 1, in this paper, we assume that the fault detection system and controller are located together at a remote place. Let dynamics of the plant after discretization be given by

$$\begin{aligned} x(k+1) &= Ax(k) + Bu_p(k) + E_d d(k) + E_f f(k) \\ y_p(k) &= Cx(k) + Du_p(k) + F_d d(k) + F_f f(k) \end{aligned} \quad (1)$$

where $x(k) \in \mathbb{R}^n$, $u_p(k) \in \mathbb{R}^p$, $y_p(k) \in \mathbb{R}^m$ denote the state, the actuator input, the measurement output of the plant, and $d(k) \in \mathbb{R}^{n_d}$, $f(k) \in \mathbb{R}^{n_f}$ denote the unknown disturbance and the latent fault to be detected in the plant. Matrices A, B, C, D, E_d, E_f, F_d, F_f are

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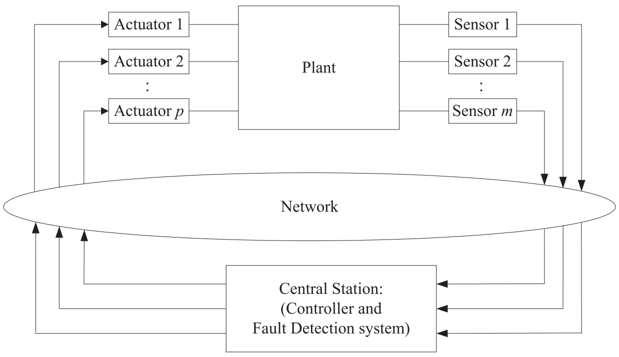

Fig. 1. Structure of networked control systems

of appropriate dimensions. Since the center station is located remotely, the plant input u_p is transmitted from the remote controller through the controller-to-actuator link. Similarly, the plant sensor measurement is transmitted to the controller through sensor-to-controller link. In practice, usually the network links are not reliable, control signals sent by the controller and plant measurements sent by the sensors may be lost or corrupted by noise during transmission, both of which are called packet dropout in this paper. In the following, we will use $u(k)$ to denote the control signal generated by the controller and $y(k)$ to denote the plant output measurement received by the controller. In the case that packet dropout occurs in the controller-to-actuator link, the following strategies can be adopted:

- use the last available control signal as the current control input: $u_p(k) = u_p(k-1)$;
- use 0 as the current control input (Zhang and Hristu [2006]): $u_p(k) = 0$.

Similarly, in the sensor-to-controller link, if the output information $y_p(k)$ of the plant is lost during transmission, the following strategies can be used for fault detection:

- use the last available sensor information as the current plant output (Zhang et al. [2004]) : $y(k) = y(k-1)$;
- use 0 as the plant output (Zhang and Hristu [2006]): $y(k) = 0$.

Note that instead of $u_p(k)$ and $y_p(k)$, $u(k)$ and $y(k)$ are the actual signals available for residual generation at the center monitoring system.

In this paper, both these two strategies can be used in the proposed fault detection approach. In fact, as will be illustrated later, they do not have any difference as far as fault detection system design is concerned.

3. MAIN RESULTS

In this section, residual is first generated by parity space approach, then thresholds as well as the corresponding false alarm rates are given for residual evaluation.

3.1 Residual generation

As mentioned before, in existing observer based fault detection of networked control systems with stochastic packet dropout, the probability distribution of residual signal is unknown, which leads to many problems on false alarm calculation. In this paper, we propose to circumvent this problem by using parity space approach (Chen and Patton [1999]), which can be formulated as

$$r(k) = v_s\big(y_s(k) - H_{u,s}u_s(k)\big) \tag{2}$$

where s is order of the parity space, v_s is the parity vector satisfying $v_s H_{0,s} = 0$, and

$$H_{0,s} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA_s \end{bmatrix}, H_{u,s} = \begin{bmatrix} D & 0 & \dots & 0 \\ CB & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}B & \dots & CB & D \end{bmatrix}$$

$$y_s(k) = \begin{bmatrix} y(k-s) \\ y(k-s+1) \\ \vdots \\ y(k) \end{bmatrix}, u_s(k) = \begin{bmatrix} u(k-s) \\ u(k-s+1) \\ \vdots \\ u(k) \end{bmatrix}$$

It is worthwhile noticing that signal $u(i)$ $(k-s \le i \le k)$ in $u_s(k)$ and $y(i)$ $(k-s \le i \le k)$ in $y_s(k)$ may not be the exact input and output information of the plant due to packet dropout. After rewriting (2) into

$$\begin{aligned} r(k) = v_s\Big(&\big(y_{p,s}(k) - y_{p,s}(k) + y_s(k)\big) \\ &-H_{u,s}\big(u_{p,s}(k) - u_{p,s}(k) + u_s(k)\big)\Big) \end{aligned} \tag{3}$$

we have the dynamics of (2) as follows according to (1)

$$\begin{aligned} r(k) &= v_s\Big(\big(y_{p,s}(k) - H_{u,s}u_{p,s}(k)\big) \\ &\quad -\big(y_{p,s}(k) - y_s(k)\big) + H_{u,s}\big(u_{p,s}(k) - u_s(k)\big)\Big) \\ &= v_s\Big(\big(H_{d,s}d_s(k) + H_{f,s}f_s(k) \\ &\quad -\big(y_{p,s}(k) - y_s(k)\big) + H_{u,s}\big(u_{p,s}(k) - u_s(k)\big)\Big) \end{aligned} \tag{4}$$

where

$$y_{p,s}(k) = \begin{bmatrix} y_p(k-s) \\ y_p(k-s+1) \\ \vdots \\ y_p(k) \end{bmatrix}, u_{p,s}(k) = \begin{bmatrix} u_p(k-s) \\ u_p(k-s+1) \\ \vdots \\ u_p(k) \end{bmatrix}$$

$$H_{d,s} = \begin{bmatrix} F_d & 0 & \dots & 0 \\ CE_d & F_d & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}E_d & \dots & CE_d & F_d \end{bmatrix}$$

$$H_{f,s} = \begin{bmatrix} F_f & 0 & \dots & 0 \\ CE_f & F_f & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ CA^{s-1}E_f & \dots & CE_f & F_f \end{bmatrix}$$

$$d_s(k) = \begin{bmatrix} d(k-s) \\ d(k-s+1) \\ \vdots \\ d(k) \end{bmatrix}, f_s(k) = \begin{bmatrix} f(k-s) \\ f(k-s+1) \\ \vdots \\ f(k) \end{bmatrix}$$

$u_{p,s}(k) - u_s(k)$ and $y_{p,s}(k) - y_s(k)$ can be denoted by $u_{\Delta,s}(k)$ and $y_{\Delta,s}(k)$ respectively for short, and it can be verified that $y_{\Delta,s}(k) = I^{ind}_{k,s} y_{\Delta,s}(k)$, where $I^{ind}_{k,s}$ is a diagonal matrix composed of

$$I^{ind}_{k,s} = \begin{bmatrix} I^{ind}(y_\Delta(k-s)) & & \\ & \ddots & \\ & & I^{ind}(y_\Delta(k)) \end{bmatrix}$$

$$y_\Delta(i) = y_p(i) - y(i) \quad (k-s \le i \le k)$$

$$I^{ind}(x) = \begin{cases} I \text{ if } x \neq 0 \\ 0 \text{ if } x = 0 \end{cases}, \ I: \text{identity matrix}$$

So the dynamics of residual generator (2) can be reformulated as

$$r(k) = v_s\big(H_{d,s}d_s(k) + H_{f,s}f_s(k) - I^{ind}_{k,s} y_{\Delta,s} + H_{u,s}u_{\Delta,s}(k)\big) \tag{5}$$

From (5), we can see that residual signal $r(k)$ is influenced by the latent fault $f_s(k)$, the unknown input $d_s(k)$, packet dropout in the sensor-to-controller link $y_{\Delta,s}(k)$, and packet dropout in the controller-to-actuator link $u_{\Delta,s}(k)$. Since the fault detection system is located together with the controller, packet dropout in the sensor-to-controller link is totally known to fault detection, i.e. matrix $I^{ind}_{k,s}$ is a known matrix, influence of this dropout to residual signal can be fully decoupled by selecting a parity vector $v_{s,k}$ satisfying

$$v_{s,k}\left[\, H_{0,s} \ I^{ind}_{k,s} \,\right] = 0 \tag{6}$$

In fact, an optimal parity vector $v^{opt}_{s,k}$ can be chosen according to some performance index such as (Chen and Patton [1999]):

$$v^{opt}_{s,k} = \arg\left\{ \max_{v_{s,k},\, v_{s,k}[H_{0,s}\ I^{ind}_{k,s}]=0} \frac{v_{s,k}H_{f,s}H^T_{f,s}v^T_{s,k}}{v_{s,k}H_{d,s}H^T_{d,s}v^T_{s,k}} \right\} \tag{7}$$

where symbol T denotes matrix or vector transpose.

Remark 1: If all the sensor outputs $y_p(i)$ $(k-s \le i \le k)$ are lost during transmission, then (6) can not be satisfied. However, by selecting a proper order s, we can make this probability very small. What is more, even if this happens in practice, it is still not a problem, for we can stop generating residual signals and wait until the sensor information comes.

Remark 2: From (5), we can see that setting $u_p(k)$ and $y(k)$ as the last available information, i.e. $u_p(k-1)$ and $y(k-1)$, or 0 in the case of packet dropout does not influence the design of fault detection system.

3.2 Residual evaluation

In practical fault detection, a threshold $J_{th,k}$ should be provided to make a decision:

$$\begin{cases} \|r(k)\|_2 < J_{th,k} \Rightarrow \quad \text{no fault} \\ \|r(k)\|_2 \ge J_{th,k} \Rightarrow \text{alarm for fault} \end{cases} \tag{8}$$

Usually, there is a tradeoff in the selection of the threshold: if $J_{th,k}$ is set too large, a lot of small faults can not be detected, in contrast, if $J_{th,k}$ is set too small, false alarms caused by unknown disturbance input may become intolerable. In this section, we will give a systematic threshold design method together with the false alarm rate analysis.

In order to construct a threshold for fault decision, we reconsider (5) in the absence of fault, i.e. $f_s(k)$ is zero, then

$$r(k) = v^{opt}_{s,k}\big(H_{d,s}d_s(k) + H_{u,s}u_{\Delta,s}(k)\big) \tag{9}$$

note that influence of $y_{\Delta,s}(k)$ has been decoupled from $r(k)$ via the selection of $v^{opt}_{s,k}$ according to (7).

Therefore we have

$$\begin{aligned} \|r(k)\|_2 &= \|v^{opt}_{s,k}\big(H_{d,s}d_s(k) + H_{u,s}u_{\Delta,s}(k)\big)\|_2 \\ &\le \bar\sigma(v^{opt}_{s,k}H_{d,s})\|d_s(k)\|_2 + \bar\sigma(v^{opt}_{s,k}H_{u,s})\|u_{\Delta,s}(k)\|_2 \end{aligned} \tag{10}$$

where $\bar\sigma(\bullet)$ denotes the maximum singular value of matrix $(\bullet)$. In (10), the probability distribution of $u_{\Delta,s}(k)$ can be obtained since the probability distribution of $u_\Delta(k) \triangleq u_p(k) - u(k) = 0$ is known, which further determines the probability distribution of r_k. When the probability distribution of r_k is obtained, its corresponding false alarm rate can also be obtained.

In this paper, we assume that $\|d(k)\|_2 \le \sup_k \sqrt{d(k)^T d(k)} \triangleq \delta_d$, $\|u_\Delta(k)\|_2 \le \sup_k \sqrt{u_\Delta(k)^T u_\Delta(k)} \triangleq \delta_u$, then the threshold can be set as

$$J_{th,k} = \bar\sigma(v^{opt}_{s,k}H_{d,s})\sqrt{s+1}\,\delta_d + \bar\sigma(v^{opt}_{s,k}H_{u,s})\|u_{\Delta,s}(k)\|_2 \tag{11}$$

where $\|u_{\Delta,s}(k)\|_2$ will be determined later according to its probability distribution. Next, we will give thresholds and their corresponding false alarm rates under different stochastic characteristics of packet dropout of $u(k)$ in the controller-to-actuator link.

Packet dropout is characterized by a Bernoulli process We first consider the case that packet dropout in the controller-to-actuator link is characterized by a Bernoulli process (Babak [2003], Wang et al. [2003]). $\gamma(k)$ is used to represent the status of packet transmission: if $u(k)$ is received by the plant actuator successfully, i.e. $u_p(k) = u(k)$, $\gamma(k) = 1$, otherwise $\gamma(k) = 2$, and $P\{\gamma(k) = 1\} = \alpha$, $P\{\gamma(k) = 2\} = 1 - \alpha$. Now we consider the $s+1$ elements in $u_{\Delta,s}(k)$, i.e. $u_\Delta(i) = u_p(i) - u(i)$ $(k-s \le i \le k)$. Since the probability of $u_\Delta(i) = 0$ $(k-s \le i \le k)$ is known, we have the probability of $u_{\Delta,s}(k)$, which is shown in table 1. So if we set the threshold as

$$J_{th,k} = \bar\sigma(v^{opt}_{s,k}H_{d,s})\sqrt{s+1}\,\delta_d + \bar\sigma(v^{opt}_{s,k}H_{u,s})\sqrt{i}\,\delta_u \tag{12}$$

where $0 \le i \le s+1$, then we have the upper bounds of false alarm rates (FAR) given in Table 2.

Packet dropout is characterized by a Markov chain If the packet dropout in the controller-to-actuator link is characterized by a homogenous Markov chain (Zhang et al. [2004], Xiong and Lam [2007]), we can also given proper thresholds and corresponding false alarm rates in the framework of parity space approach.

We use $\gamma(k) = \{1, 2\}$ to denote the transmission status of control signal $u(k)$ in the controller-to-actuator link. If $u(k)$ arrives at the actuator successfully, $\gamma(k) = 1$, otherwise, $\gamma(k) = 2$. $P = (p_{ij})$ is the transition probability matrix with elements $p_{ij} = P\{\gamma(k+1) = j | \gamma(k) = i\}$

Table 1. Probability distribution of $u_{\Delta,s}(k)$–Bernoulli case

$Num\left(u_{\Delta,s}(k)\right)$	0	1	...	s	$s+1$
Maximum $\|u_{\Delta,s}(k)\|_2$	0	δ_u	...	$\sqrt{s}\delta_u$	$\sqrt{s+1}\delta_u$
Probability	$p^0=\alpha^{s+1}$	$p^1=C^1_{s+1}\alpha^s(1-\alpha)$	...	$p^s=C^s_{s+1}\alpha(1-\alpha)^s$	$p^{s+1}=(1-\alpha)^{s+1}$

$Num\left(u_{\Delta,s}(k)\right)$ denotes number of nonzero elements in $u_{\Delta,s}(k)$;
C^m_n denotes binomial coefficient.

Table 2. Thresholds and corresponding false alarm rates–Bernoulli case

i in Equation (12)	0	1	...	s	$s+1$
Maximum FAR	$1-p^0$	$1-p^0-p^1$	...	$1-p^0-p^1-\ldots-p^s$	$1-p^0-p^1-\ldots-p^{s+1}=0$

$i,j\in\{1,2\}$. The initial probability distribution is assumed to be $\pi(0)=\left[\,P\{\gamma(0)=1\}\;P\{\gamma(0)=2\}\,\right]=\left[\,\alpha\;1-\alpha\,\right]$.

According to the properties of Markov chain, we can get the probability of i ($0\le i\le s+1$) packet dropout during time instants $k-s\le i\le k$:

- no packet is dropped out during $k-s\le i\le k$

$$\begin{aligned}p^0_k &= P\{\text{no packet dropout during}\,k-s\le i\le k\}\\ &= P\{\gamma(k-s)=\gamma(k-s+1)=\ldots=\gamma(k)=1\}\\ &= P\{\gamma(k-s)=1\}p_{1,1}p_{1,1}\ldots p_{1,1} \qquad (13)\end{aligned}$$

- 1 packet is dropped out during $k-s\le i\le k$

$$\begin{aligned}p^1_k &= P\{1\text{ packet dropout during}\,k-s\le i\le k\}\\ &= P\{\gamma(k-s)=2,\gamma(k-s+1)=\gamma(k-s+2)=\\ &\quad\ldots=\gamma(k)=1\}\\ &\quad+P\{\gamma(k-s+1)=2,\gamma(k-s)=\gamma(k-s+2)=\\ &\quad\ldots=\gamma(k)=1\}\\ &\quad+\ldots\\ &\quad+P\{\gamma(k)=2,\gamma(k-s)=\gamma(k-s+1)=\\ &\quad\ldots=\gamma(k-1)=1\}\\ &= P\{\gamma(k-s)=2\}p_{2,1}p_{1,1}p_{1,1}\ldots p_{1,1}\\ &\quad+P\{\gamma(k-s)=1\}p_{1,2}p_{2,1}p_{1,1}\ldots p_{1,1}\\ &\quad+\ldots\\ &\quad+P\{\gamma(k-s)=1\}p_{1,1}p_{1,1}\ldots p_{1,1}p_{1,2} \qquad (14)\end{aligned}$$

- ...
- all $s+1$ packets are dropped out during $k-s\le i\le k$

$$\begin{aligned}p^{s+1}_k &= P\{s+1\,\text{packet dropouts during}\,k-s\le i\le k\}\\ &= P\{\gamma(k-s)=\gamma(k-s+1)=\ldots=\gamma(k)=2\}\\ &= P\{\gamma(k-s)=2\}p_{2,2}p_{2,2}\ldots p_{2,2} \qquad (15)\end{aligned}$$

where

$$\begin{aligned}&\left[\,P\{\gamma(k-s)=1\}\;P\{\gamma(k-s)=2\}\,\right]\\ &=\pi(k-s)=\pi(0)P^{k-s} \qquad (16)\end{aligned}$$

The results are concluded in Table 3. If we set the threshold as

$$J_{th,k}=\bar{\sigma}(v^{opt}_{s,k}H_{d,s})\sqrt{s+1}\delta_d+\bar{\sigma}(v^{opt}_{s,k}H_{u,s})\sqrt{i}\delta_u \qquad (17)$$

where $0\le i\le(s+1)$, then the corresponding maximum false alarm rates can easily be obtained, which are given in Table 4.

Remark 3: It is worthwhile noticing that the false alarm rate of a certain threshold is time varying since $\pi(k-s)$ in (13)-(16) is time varying.

If we suppose $0<p_{ij}<1$ (no equal mark), it is easy to obtain that this Makrov chain is irreducible and aperiodic (Meyn and Tweedie [1993]), then we have Lemma 1.

Lemma 1 (Meyn and Tweedie [1993]): If a Markov chain with finite states is irreducible and aperiodic, then there is a unique stationary distribution π^*, i.e. $\pi^*P=\pi^*$. In addition, P^k converges to a rank-one matrix in which each row is the stationary distribution π^*, that is

$$\lim_{k\to\infty}P^k=\begin{bmatrix}1\\1\end{bmatrix}\pi^*,\quad \lim_{k\to\infty}\pi(k)=\pi(0)\lim_{k\to\infty}P^k=\pi^* \qquad (18)$$

Lemma 1 means that as time goes by, the Markov chain forgets where it began (its initial distribution) and converges to its stationary distribution. Thus it can be concluded that the stochastic probabilities in (13)-(16) converge to constant values as time goes by.

4. CONSTRUCTION OF AN ADAPTIVE DIAGNOSTIC OBSERVER

All the discussions above are given in the framework of parity space approach, in this section, we will show that a diagnostic observer which has more design freedom can also be constructed to achieve fault detection of networked control systems with packet dropout.

The diagnostic observer is given by (Gertler [1998], Chen and Patton [1999])

$$\begin{aligned}z(k+1)&=G_kz(k)+H_ku(k)+L_ky(k)\\ r(k)&=-w_kz(k)-q_ku(k)+p_ky(k)\end{aligned} \qquad (19)$$

where $r(k)\in\mathbb{R}$, $z(k)\in\mathbb{R}^s$ is an observation of $T_kx(k)$. T_k, G_k, H_k, L_k, w_k , q_k, and p_k are parameters to be designed. In order to be used for residual generation, the following conditions should be satisfied (Gertler [1998], Chen and Patton [1999]):

$$T_{k+1}A_k-G_kT_k=L_kC_k \qquad (20)$$

$$w_kT_k=p_kC \qquad (21)$$

Set $e(k)=T_kx(k)-z(k)$, the dynamics of (19) are given by

$$\begin{aligned}e(k+1)&=G_ke(k)+(T_{k+1}E_f-L_kF_f)f(k)\\ &\quad+(T_{k+1}E_d-L_kF_d)d(k)+\end{aligned}$$

Table 3. Probability distribution of $u_{\Delta,s}(k)$–Markov case

$Num\left(u_{\Delta,s}(k)\right)$	0	1	...	s	$s+1$
Maximum $\|u_{\Delta,s}(k)\|_2$	0	δ_u	...	$\sqrt{s}\delta_u$	$\sqrt{s+1}\delta_u$
Probability	p_k^0	p_k^1	...	p_k^s	p_k^{s+1}

$Num\left(u_{\Delta,s}(k)\right)$ denotes number of nonzero elements in $u_{\Delta,s}(k)$

Table 4. Thresholds and corresponding false alarm rates–Markov case

i in Equation (17)	0	1	...	s	$s+1$
Maximum FAR	$1-p_k^0$	$1-p_k^0-p_k^1$	...	$1-p_k^0-p_k^1-\ldots-p_k^s$	$1-p_k^0-p_k^1-\ldots-p_k^{s+1}=0$

$$\begin{aligned}&(T_{k+1}B-L_kD)u_\Delta(k)+L_ky_\Delta(k)\\ r(k)&=w_ke(k)+p_kF_ff(k)+p_kF_dd(k)\\ &+p_kDu_\Delta(k)-p_ky_\Delta(k)\end{aligned}\tag{22}$$

Similar with Ding et al. [1998] and Zhang and Ding [2007], diagnostic observer (19) can be parameterized by

$$G_k=\begin{bmatrix}G_0 & g_k\end{bmatrix}\tag{23}$$

$$G_0=\begin{bmatrix}0&0&\ldots&0\\1&0&\ldots&0\\ \vdots&\ddots&\ddots&\vdots\\0&\ldots&1&0\\0&\ldots&0&1\end{bmatrix},g_k=\begin{bmatrix}g_{k,1}\\g_{k,2}\\ \vdots\\g_{k,s-1}\\g_{k,s}\end{bmatrix}\tag{24}$$

$$w_k=\begin{bmatrix}0&0&\ldots&0&1\end{bmatrix}\tag{25}$$

$$L_k=-\begin{bmatrix}v_{s,k+s}^0\\v_{s,k+s-1}^1\\ \vdots\\v_{s,k+1}^{s-1}\end{bmatrix}-g_kv_{s,k}^s\tag{26}$$

$$T_k=\begin{bmatrix}v_{s,k+s-1}^1&\cdots&v_{s,k+s-1}^{s-1}&v_{s,k+s-1}^s\\v_{s,k+s-2}^2&\cdots&v_{s,k+s-2}^s&0\\ \vdots&\cdot^{\cdot^{\cdot}}&\cdot^{\cdot^{\cdot}}&\vdots\\v_{s,k}^s&0&\ldots&0\end{bmatrix}\begin{bmatrix}C\\CA\\ \vdots\\CA^{s-1}\end{bmatrix}\tag{27}$$

$$p_k=v_{s,k}^s\tag{28}$$

$$H_k=T_{k+1}B-L_kD\tag{29}$$

$$q_k=p_kD\tag{30}$$

where s is order of the diagnostic observer, $v_{s,k}=\begin{bmatrix}v_{s,k}^0 & v_{s,k}^1 & \ldots & v_{s,k}^s\end{bmatrix}$ satisfies (6), and g_k can be freely chosen on condition that (19) is stable.

Following Zhang and Ding [2007], (22) can be rewritten as

$$\begin{aligned}e(k+1)&=\bar{G}_0e(k)+(T_{k+1}E_f-\bar{L}_kF_f)f(k)\\ &+(T_{k+1}E_d-\bar{L}_kF_d)d(k)+(T_{k+1}B-\bar{L}_kD)u_\Delta(k)\\ &+\bar{L}_ky_\Delta(k)+g_kr(k)\\ r(k)&=w_ke(k)+p_kF_ff(k)+p_kF_dd(k)\\ &+p_kDu_\Delta(k)-p_ky_\Delta(k)\end{aligned}\tag{31}$$

where

$$\bar{G}_0=\begin{bmatrix}G_0 & 0\end{bmatrix},\quad \bar{L}_k=-\begin{bmatrix}v_{s,k+s}^0\\v_{s,k+s-1}^1\\ \vdots\\v_{s,k+1}^{s-1}\end{bmatrix}$$

(31) can be expressed in the non-iterative form:

$$\begin{aligned}r(k)&=v_{s,k}\big(H_{f,s}f_s(k)+H_{d,s}d_s(k)+H_{u,s}u_{\Delta,s}(k)\\ &-I_{k,s}^{ind}y_{\Delta,s}(k)\big)+g_{k-1,s}r(k-1)\\ &+g_{k-2,s-1}r(k-2)+\ldots+g_{k-s,1}r(k-s)\end{aligned}\tag{32}$$

where $H_{f,s}$, $H_{d,s}$, $H_{u,s}$, and $I_{k,s}^{ind}$ are the same with the ones in Section 3. If the residual signal to be evaluated is set as

$$\begin{aligned}\bar{r}(k)&=r(k)-g_{k-1,s}r(k-1)-\ldots-g_{k-s,1}r(k-s)\\ &=v_{s,k}\big(H_{f,s}f_s(k)+H_{d,s}d_s(k)+H_{u,s}u_{\Delta,s}(k)\\ &-I_{k,s}^{ind}y_{\Delta,s}(k)\big)\end{aligned}\tag{33}$$

since $v_{s,k}I_{k,s}^{ind}=0$ according to (6), we have

$$\begin{aligned}\bar{r}(k)&=r(k)-g_{k-1,s}r(k-1)-\ldots-g_{k-s,1}r(k-s)\\ &=v_{s,k}\big(H_{f,s}f_s(k)+H_{d,s}d_s(k)+H_{u,s}u_{\Delta,s}(k)\big)\end{aligned}\tag{34}$$

So far, the residual evaluation method in Section 3 can be used to get the threshold and the corresponding false alarm rate.

Remark 4: The adaptive diagnostic observer obtained here is time varying but not necessarily periodic, which is different from Zhang and Ding [2007].

5. SIMULATION EXAMPLE

In this section, a simple example is given to illustrate the results obtained. The continuous plant is given by

$$\begin{aligned}\dot{x}(t)&=\begin{bmatrix}-3&0&1\\0&-4&1\\0&0&-2\end{bmatrix}x(t)+\begin{bmatrix}1&0&0\\2&0&1\\0&2&1\end{bmatrix}u_p(t)\\ &+\begin{bmatrix}1&0.7&1\\0.3&1&0.3\\1&1&0\end{bmatrix}d(t)+\begin{bmatrix}1\\0\\1\end{bmatrix}f(t)\\ y_p(t)&=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}x(t)+\begin{bmatrix}1&0&0\\0&0.5&1\\0&1&0\end{bmatrix}d(t)\end{aligned}$$

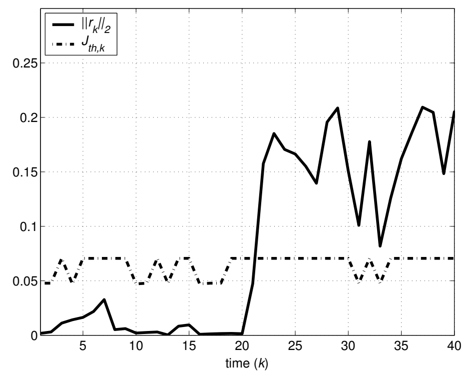

Fig. 2. Residual signal and threshold

and the sampling period is 1 sec. Packet dropouts in both sensor-to-controller and controller-to-actuator links are characterized by Bernoulli processes, and the probability of successful transmission are supposed to be 0.8 in both links. If packet dropout occurs in the sensor-to-controller link, the last available measurement is used for fault detection, i.e. $y(k) = y(k-1)$. In the controller-to-actuator link, if the control information sent by the controller is lost, we assume the old control input is used by the actuator, i.e. $u_p(k) = u_p(k-1)$ and $\delta_u = 0.1$. Suppose a step fault $f(t) = 0.3$ occurs at $t = 20$ sec and the disturbance $d(t)$ is an uniformly distributed random signal with $\delta_d = 0.05$. A residual generator is constructed by parity space approach with order 3, and threshold is designed according to the worst case with all $u(i)$ $(k-s \leq i \leq k)$ during $k-s$ and k being lost, which makes a zero false alarm rate. The results are given in Fig. 2. It can be seen that the designed residual generator is robust to packet dropout and can detect the fault efficiently.

6. CONCLUSION

Fault detection of networked control systems with packet dropouts in both sensor-to-controller link and controller-to-actuator link are considered in this paper. One of the main problems with existent fault detection approaches for networked control systems in the presence of packet dropout is residual evaluation: in existent observer based approaches, it is very difficult to get the false alarm rate of a certain threshold, for the probability distribution of residual signal is unknown. In this paper, this problem is circumvented by a parity space based residual generator, whose output, i.e. residual signal has a known probability distribution. Thresholds as well as corresponding upper bounds of false alarm rates are given. The proposed fault detection approach can be used in scenarios where packet dropout is characterized by a Bernoulli process or a Markov chain. By employing the relationship between parity vectors and observer based residual generators, diagnostic observers can be used to realize the proposed fault detection approach as well. An simple example is also given to illustrate the results obtained.

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