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A LMI-Based Design of Dynamic Output Feedback Controller for T-S Fuzzy Systems

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Abstract: This paper presents a dynamic output feedback controller design for fuzzy dynamic systems based on the concept of dynamic parallel distributed compensation (DPDC). Three types of stabilizing controller design methods are proposed based on state feedback design methods. The controller design involves solving a set of linear matrix inequalities (LMIs), and the control laws are numerically tractable via LMI techniques. Moreover, performance of the fuzzy controller in terms of decay rate and constraint on the control input is studied and LMI conditions for these performance criteria are obtained. An example is given to illustrate validity of the proposed methods, and to compare their performance.

1. INTRODUCTION

Nonlinear control systems based on the Takagi-Sugeno (T-S) fuzzy models (Takagi & Sugeno, 1985) have received a great deal of attention over the last decade (Tanaka *et. al.*, 2007). To build a T-S fuzzy model, a number of linear time-invariant models, which approximate to the nonlinear plant in some regions of the state-space, are obtained, and then they are combined using nonlinear fuzzy membership functions (Takagi & Sugeno, 1985).

In most fuzzy control designs, it is assumed that the states of the systems are available, which is not true in many practical cases. On the other hand, output feedback controller approach in T-S fuzzy systems is considered in some papers, such as Han *et. al.* (2000) and Nguang & Shi (2003). Fig. 1 shows schematic of the fuzzy control system (T-S fuzzy model and dynamic PDC controller).

In this paper we will design stabilizing dynamic output feedback controllers for T-S fuzzy model, using stability conditions in Tanaka *et. al.* (1998) and Kim & Lee (2000). These conditions will be converted to LMIs using proposed method in Scherer *et. al.* (1997). Then we will apply these conditions to a mass-spring-damper system to compare their performance.

The paper is organized as follows. In Section-II, T-S fuzzy model and dynamic parallel distributed compensation (DPDC) are introduced. LMI-Based Design for the fuzzy control system is discussed in Section III. Simulation results are presented in Section IV. Finally, some concluding remarks are given in Section V.

2. T-S FUZZY MODEL AND DYNAMIC PDC

2.1 T-S Fuzzy Model

The T-S fuzzy dynamic model of a continuous-time nonlinear system is described by some fuzzy IF-THEN rules each of which represent a local linear input-output relation of the system. The overall fuzzy model is achieved by fuzzy aggregation of the linear models (Tanaka *et. al.*, 1998). The i-th rule of the T-S fuzzy model is of the following form:

Model Rule i:
IF $z_1(t)$ is M_{i1} and ... and $z_p(t)$ is M_{ip},

$$\text{THEN } \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \quad i = 1,2,\ldots,r . \tag{1}$$

where M_{ij} denotes the membership function associated with the i-th model rule and j-th premise variable component, r is the number of rules. $x(t) \in \Re^n$ is the state vector, $u(t) \in \Re^m$ is the control input vector, $y(t) \in \Re^q$ is the output vector, $A_i \in \Re^{n\times n}$, $B_i \in \Re^{n\times m}$, $C_i \in \Re^{q\times n}$ are local system matrices and $z_1(t) \sim z_p(t)$ are known premise variables that may be functions of the state variables, external disturbances, and/or time. Suppose that a pair of $(x(t), u(t))$ is given. The final output of the fuzzy system is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))\{A_i x(t) + B_i u(t)\}, \tag{2}$$

$$y(t) = \sum_{i=1}^{r} h_i(z(t))\, C_i x(t), \tag{3}$$

where for all t we have:

$$z(t) = [z_1(t)\;\; z_2(t) \ldots z_p\;(t)],$$
$$w_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t)), \tag{4}$$

$$h_i(z(t)) = w_i(z(t)) \Big/ \sum_{i=1}^{r} w_i(z(t)) . \tag{5}$$

2.2 Dynamic Parallel Distributed Compensation

PDC is a T-S fuzzy controller designed based on a T-S fuzzy model. In PDC, there is a controller rule for each rule of the

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T-S fuzzy model (Tanaka & Wang, 2001). The Dynamic PDC (DPDC) is a PDC whose THEN parts are dynamical systems. The DPDC structure consists of a double index set of fuzzy rules (Li *et. al.*, 1999):

Dynamic Part, Rule ij:

IF $z_1(t)$ is M_{i1} and $z_1(t)$ is M_{j1} ...

and $z_p(t)$ is M_{ip} and $z_P(t)$ is M_{jp},

THEN $\dot{x}_c(t) = A_c^{ij} x_c(t) + B_c^j y(t)$.

Output Part, Rule ij:

IF $z_1(t)$ is M_{i1} and $z_1(t)$ is M_{j1} ...

and $z_p(t)$ is M_{ip} and $z_P(t)$ is M_{jp},

THEN $u(t) = C_c^j x_c(t) + D_c y(t)$,

which can be expressed as:

$$\dot{x}_c(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z) h_j(z)\, A_c^{ij} x_c(t) + \sum_{i=1}^{r} h_i(z)\, B_c^i y(t) \tag{6}$$

$$u(t) = \sum_{i=1}^{r} h_i(z)\, C_c^i x_c(t) + D_c y(t),$$

where $h_i(z) = h_i(z(t))$.

2.3 Resulting Closed-loop System

Now we are ready to obtain closed-loop system. By defining:

$$x_{cl}(t) = \begin{bmatrix} x(t)^T & x_c^T(t) \end{bmatrix}^T, \tag{7}$$

the resulting closed-loop dynamic equations are described by the equation:

$$\dot{x}_{cl}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t)) h_j(z(t))\, A_{cl}^{ij} x_{cl}(t), \tag{8}$$

where

$$A_{cl}^{ij} = \begin{pmatrix} A_i + B_i D_c C_j & B_i C_c^j \\ B_c^i C_j & A_c^{ij} \end{pmatrix}. \tag{9}$$

3. LMI_BASED DESIGN FOR THE CLOSED LOOP SYSTEM

The design of the stabilizing dynamic output feedback controller is to determine A_c^{ij}, B_c^i, C_c^j, and D_c in the consequent parts of the controller rules. In this section, we will derive the LMIs for stability, decay rate and constraint on the control input.

3.1 Stabilizing Controller Design

We can easily derive stability conditions for (8) using stability conditions in Tanaka *et. al.* (1998) and Kim & Lee (2000). Then by applying the proposed method in Scherer *et. al.* (1997), we derive stability conditions in the form of LMIs.

Theorem 1: The fuzzy control system of (2) and (3) is quadratically stabilizable via the DPDC controller (6) if there exist symmetric matrices X, Y and matrices $\hat{A}_c^{ij}, \hat{B}_c^i, \hat{C}_c^j, \hat{D}$ such that the following LMI conditions are feasible:

$$\begin{pmatrix} X & I \\ I & Y \end{pmatrix} > 0, \tag{10}$$

$$\Psi_{A_{ii}} + \Psi_{A_{ii}}^T < 0, \qquad i = 1,2,\ldots,r \tag{11}$$

$$\left(\frac{\Psi_{A_{ij}} + \Psi_{A_{ji}}}{2}\right)^T + \left(\frac{\Psi_{A_{ij}} + \Psi_{A_{ji}}}{2}\right) \le 0, \quad (1 \le i < j \le r, h_i(z(t)) h_j(z(t)) \ne 0) \tag{12}$$

where

$$\Psi_{A_{ij}} = \begin{pmatrix} A_i X + B_i \hat{C}_c^j & A_i + B_i \hat{D} C_j \\ \hat{A}_c^{ij} & Y A_i + \hat{B}_c^i C_j \end{pmatrix}. \tag{13}$$

Proof: We know that (8) is stable if there exists a symmetric matrix P such that (Tanaka *et. al.*, 1998):

$$P > 0, \tag{14}$$

$$(A_{cl}^{ii})^T P + P A_{cl}^{ii} < 0, \qquad i = 1,2,\ldots,r \tag{15}$$

$$\left(\frac{A_{cl}^{ij} + A_{cl}^{ji}}{2}\right)^T P + P\left(\frac{A_{cl}^{ij} + A_{cl}^{ji}}{2}\right) \le 0, \quad (1 \le i < j \le r, h_i(z(t)) h_j(z(t)) \ne 0), \tag{16}$$

These conditions are not LMIs in P. In order to obtain these conditions in LMI form, we partition P and P^{-1} as:

$$P = \begin{pmatrix} Y & N \\ N^T & * \end{pmatrix}, \qquad P^{-1} = \begin{pmatrix} X & M \\ M^T & * \end{pmatrix}, \tag{17}$$

where X and Y are $n \times n$ and symmetric matrices (Scherer *et. al.*, 1997). Now we define the matrices Π_1 and Π_2 as:

$$\Pi_1 = \begin{pmatrix} X & I \\ M^T & 0 \end{pmatrix}, \qquad \Pi_2 = \begin{pmatrix} I & Y \\ 0 & N^T \end{pmatrix}, \tag{18}$$

and $P\Pi_1 = \Pi_2$. Note that here we have:

$$MN^T = I - XY. \tag{19}$$

Matrices M and N should be chosen such that (19) holds. Pre- and post-multiplying (14), (15) and (16) by Π_1^T and Π_1, we derive (10), (11) and (12), respectively, where:

$$\begin{cases} \hat{A}_c^{ij} = N A_c^{ij} M^T + N B_c^i C_j X + Y B_i C_c^j M^T + Y\left(A_i + B_i D_c C_j\right) X \\ \hat{B}_c^i = N B_c^i + Y B_i D_c \\ \hat{C}_c^i = C_c^i M^T + D_c C_i X \\ \hat{D} = D_c . \end{cases} \tag{20}$$

This completes the proof.□

Note that we can derive the controller matrices from:

$$\begin{cases} \hat{D} = D_c \\ C_c^i = \left(\hat{C}_c^i - D_c C_i X\right) M^{-T} \\ B_c^i = N^{-1}\left(\hat{B}_c^i - Y B_i D_c\right) \\ A_c^{ij} = N^{-1}\left(\hat{A}_c^{ij} - N B_c^i C_j X - Y B_i C_c^j M^T \right. \\ \quad \left. - Y\left(A_i + B_i D_c C_j\right) X\right) M^{-T}. \end{cases} \tag{21}$$

In Tanaka *et. al.* (1998), the authors derived relaxed stability conditions for a fuzzy control system with a state feedback PDC. Now we use these conditions for deriving new stability conditions for system (8).

Theorem 2: The fuzzy control system of (2) and (3) is quadratically stabilizable via the DPDC controller (6) if there exist symmetric matrices *X, Y, F* and matrices $\hat{A}_c^{ij}, \hat{B}_c^i, \hat{C}_c^j, \hat{D}$ such that (22), (23) and (24) are feasible:

$$\begin{pmatrix} X & I \\ I & Y \end{pmatrix} > 0, \qquad F \geq 0, \tag{22}$$

$$\Psi_{A_{ii}} + \Psi_{A_{ii}}^T + (s-1)F < 0, \qquad i = 1,2,\ldots,r \tag{23}$$

$$\left(\frac{\Psi_{A_{ij}} + \Psi_{A_{ji}}}{2}\right)^T + \left(\frac{\Psi_{A_{ij}} + \Psi_{A_{ji}}}{2}\right) - F \leq 0, \tag{24}$$
$$(1 \leq i < j \leq r, h_i(z(t))h_j(z(t)) \neq 0),$$

where *s* is the maximum of the number of fuzzy subsystems that are fired at an instant and $\Psi_{A_{ij}}$ is defined in (13).

Proof: Using the same method given in the proof of Theorem 1 and relaxed stability conditions in Tanaka *et. al.* (1998), we can derive these conditions. □

More relaxed conditions than those described in the Theorems 1 and 2 were proposed in Kim & Lee (2000); thus, we use these conditions to derive more relaxed stability conditions for system (8) in Theorem 3.

Theorem 3: The fuzzy system given by (2) and (3) is quadratically stabilizable via DPDC controller (6) if there exist symmetric matrices *X, Y,* H_{ij}s $(1 \leq i, j \leq r)$ and matrices $\hat{A}_c^{ij}, \hat{B}_c^i, \hat{C}_c^j, \hat{D}$ such that the following LMI conditions are feasible:

$$\begin{pmatrix} X & I \\ I & Y \end{pmatrix} > 0, \tag{25}$$

$$\Lambda_{ii}^T + \Lambda_{ii} + H_{ii} < 0, \qquad i = 1,2,\ldots,r, \tag{26}$$

$$\Lambda_{ij}^T + \Lambda_{ij} + H_{ij} \leq 0, \qquad (1 \leq i < j \leq r), \tag{27}$$

$$\widetilde{H} = \begin{pmatrix} H_{11} & H_{12} & \cdots & H_{1r} \\ H_{12} & H_{22} & \cdots & H_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ H_{1r} & H_{2r} & \cdots & H_{rr} \end{pmatrix} > 0, \tag{28}$$

where $\Lambda_{ij} = ((\Psi_{A_{ij}} + \Psi_{A_{ji}})/2)$ and $\Psi_{A_{ij}}$ is defined in (13).

Proof: Using the same method in the proof of Theorem 1 and stability conditions in Kim & Lee (2000), we can derive these LMIs. □

3.2 Decay Rate

The speed of close-loop system response is related to decay rate, which is the largest Lyapunov exponent (Tanaka *et. al.*, 1998). Considering $\alpha > 0$ as the decay rate, we have:

$$\dot{V}(x(t)) \leq -2\alpha V(x(t)), \tag{29}$$

where $V(x(t))$ is the Lyapunov function.

Lemma 1: The fuzzy control system of (2) and (3) is quadratically stabilizable via the DPDC controller (6) with the decay rate of *α*, if there exist symmetric matrices *X, Y,* H_{ij}s and matrices $\hat{A}_c^{ij}, \hat{B}_c^i, \hat{C}_c^j, \hat{D}$ such that LMI conditions (25), (28), (30) and (31) are feasible:

$$\Lambda_{ii}^T + \Lambda_{ii} + H_{ii} + 2\alpha\begin{pmatrix} X & I \\ I & Y \end{pmatrix} < 0, \quad i = 1,2,\ldots,r, \tag{30}$$

$$\Lambda_{ij}^T + \Lambda_{ij} + H_{ij} + 2\alpha\begin{pmatrix} X & I \\ I & Y \end{pmatrix} \leq 0, (1 \leq i < j \leq r), \tag{31}$$

where $\Lambda_{ij} = ((\Psi_{A_{ij}} + \Psi_{A_{ji}})/2)$ and $\Psi_{A_{ij}}$ is defined in (13).

Proof: Applying (29) to (26) and (27), we have (30) and (31). Note that (25) and (28) do not change by decay rate condition. □

Note that the controller matrices can be obtained by (21).

3.3 Constraint on the Control Input

Lemma 2: The fuzzy control system of (2) and (3) with the DPDC controller (6) satisfies the condition $\|u(t)\| \leq \zeta e^{-\alpha t}$, $\forall t \geq 0$, if there exist symmetric matrices *X, Y*, a symmetric positive semi-definite matrix *F* and matrices $\hat{A}_c^{ij}, \hat{B}_c^i, \hat{C}_c^j$, $\hat{D}, N$ such that LMI conditions (25), (32), (33), (34) and (35) are feasible:

$$\Psi_{A_{ii}} + \Psi_{A_{ii}}^T + (s-1)F + 2\alpha\begin{pmatrix} X & I \\ I & Y \end{pmatrix} < 0, \tag{32}$$

$$\left(\frac{\Psi_{A_{ij}} + \Psi_{A_{ji}}}{2}\right)^T + \left(\frac{\Psi_{A_{ij}} + \Psi_{A_{ji}}}{2}\right) - F + 2\alpha\begin{pmatrix} X & I \\ I & Y \end{pmatrix} \leq 0, \tag{33}$$

$$\begin{pmatrix} X & I & x(0) \\ I & Y & Yx(0) + Nx_c(0) \\ x^T(0) & x^T(0)Y + x_c^T(0)N^T & \zeta I \end{pmatrix} > 0, \tag{34}$$

$$\begin{pmatrix} X & I & \hat{C}_c^{iT} \\ I & Y & C_i^T \hat{D}^T \\ \hat{C}_c^i & \hat{D}C_i & \zeta I \end{pmatrix} > 0 \quad, i = 1,\ldots,r, \tag{35}$$

where $\Psi_{A_{ij}}$ is defined in (13) and the initial condition is $x_{cl}(0) = \begin{bmatrix} x^T(0) & x_c^T(0) \end{bmatrix}^T$.

Proof: Applying (29) to (23) and (24), we can derive (32) and (33), respectively. Two other conditions for obtaining constraint on the control input $u(t) = \sum_{i=1}^{r} h_i(z) K_i x_{cl}(t)$ is that (Tanaka & Wang, 2001):

$$\begin{pmatrix} P & Px_{cl}(0) \\ x_{cl}^T(0)P & \zeta I \end{pmatrix} > 0, \quad \begin{pmatrix} P & K_i^T \\ K_i & \zeta I \end{pmatrix} > 0, \qquad (36)$$

where $K_i = [D_c C_i \; C_c^i]$.Pre- and post-multiplying these conditions by $diag(\Pi_1^T, I)$ and $diag(\Pi_1, I)$, we will derive (34) and (35), respectively. This completes the proof.□

4. NUMERICAL EXAMPLE

Nonlinear mass-spring-damper system:

Consider a nonlinear mass-spring-damper system with a nonlinear spring (Huang & Nguang, 2007):

$$\begin{aligned} \dot{x}_1(t) &= -0.1125\, x_1(t) - 0.02\, x_2(t) - 0.67\, x_2^3(t) + u(t), \\ \dot{x}_2(t) &= x_1(t), \qquad y(t) = x_2(t), \end{aligned} \qquad (37)$$

where $x_2(t)$ is the spring's displacement. We can represent (37) exactly by a 2-rules T-S fuzzy model under the assumption on bounds of the state variable $x_2(t) \in [-2, 2.5]$ with the following subsystem matrices:

$$A_1 = \begin{bmatrix} -0.1125 & 1.32 \\ 1 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -0.1125 & -1.695 \\ 1 & 0 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix},$$

and the following membership functions:

$$h_1(x_2(t)) = -\frac{x_2^2(t) - 2}{4.5}, h_2(x_2(t)) = \frac{x_2^2(t) + 2.5}{4.5}.$$

The controller's order must be at least the same as the system's order. Thus, here, the controller's order is $n_c=2$. In order to solve LMIs, we use Yalmip (Löfberg, 2004) as the parser and LMILAB (Gahinet *et. al.*, 1994) as the solver.

Using the results of Theorems 1, 2 and 3, we can derive the stabilizing controller matrices. For instance, using Theorem 2 with s=2 yields the following dynamic output feedback controller matrices:

$$D_c = 0, C_c^1 = \begin{bmatrix} -0.034 & -0.168 \end{bmatrix},$$

$$C_c^2 = \begin{bmatrix} -0.025 & -0.041 \end{bmatrix}, B_c^1 = \begin{bmatrix} -45.21 \\ 44.98 \end{bmatrix}, B_c^2 = \begin{bmatrix} 35.31 \\ 7.74 \end{bmatrix},$$

$$A_c^{11} = \begin{bmatrix} 0.5506 & 5.9620 \\ -1.1916 & -2.8488 \end{bmatrix}, A_c^{12} = \begin{bmatrix} -1.0140 & 2.5651 \\ -0.4674 & -1.2830 \end{bmatrix},$$

$$A_c^{21} = \begin{bmatrix} 0.5508 & 5.9568 \\ -1.1906 & -2.8500 \end{bmatrix}, A_c^{22} = \begin{bmatrix} -1.0138 & 2.5599 \\ -0.4664 & -1.2843 \end{bmatrix}.$$

Theorem 1 yields the following dynamic output feedback controller matrices:

$$D_c = 0, C_c^1 = \begin{bmatrix} -0.015 & -0.064 \end{bmatrix},$$

$$C_c^2 = \begin{bmatrix} 0.011 & -0.012 \end{bmatrix}, B_c^1 = \begin{bmatrix} -105.05 \\ 104.73 \end{bmatrix}, B_c^2 = \begin{bmatrix} 77.97 \\ 13.86 \end{bmatrix},$$

$$A_c^{11} = \begin{bmatrix} 0.5870 & 5.3263 \\ -1.2433 & -2.6285 \end{bmatrix}, A_c^{12} = \begin{bmatrix} -0.9737 & 2.1806 \\ -0.4683 & -1.0676 \end{bmatrix},$$

$$A_c^{21} = \begin{bmatrix} 0.5870 & 5.3255 \\ -1.2431 & -2.6287 \end{bmatrix}, A_c^{22} = \begin{bmatrix} -0.9737 & 2.1798 \\ -0.4681 & -1.0678 \end{bmatrix},$$

and Theorem 3 these controller matrices:

$$D_c = 0, C_c^1 = \begin{bmatrix} -0.023 & -0.0307 \end{bmatrix},$$

$$C_c^2 = \begin{bmatrix} 0.023 & -0.156 \end{bmatrix}, B_c^1 = \begin{bmatrix} -51.17 \\ 53.51 \end{bmatrix}, B_c^2 = \begin{bmatrix} 53.43 \\ 21.75 \end{bmatrix},$$

$$A_c^{11} = \begin{bmatrix} 0.3299 & 12.5077 \\ -0.9795 & -4.7818 \end{bmatrix}, A_c^{12} = \begin{bmatrix} -1.2499 & 7.2719 \\ -0.4995 & -3.2000 \end{bmatrix},$$

$$A_c^{21} = \begin{bmatrix} 0.3300 & 12.4996 \\ -0.9789 & -4.7839 \end{bmatrix}, A_c^{22} = \begin{bmatrix} -1.2498 & 7.2638 \\ -0.4989 & -3.2021 \end{bmatrix}.$$

Fig. 2 depicts the output $y= x_2(t)$ for the different results obtained by Theorems 1, 2 and 3. Here we assume that the initial condition is $x(0)$=[0.1,-0.5].

Using Lemma 1 with α=1, we will obtain a dynamic controller which yields system outputs as shown in Fig. 3. We see that system response is very fast when we insert decay rate condition; however, control signal increases in all three cases. More specifically, we have $max_t \, ||u(t)||$= 1.3051 in Theorem 3, but $max_t \, ||u(t)||$= 6.5709 in Lemma 1. Thus, we should use constraint on the control input condition to decrease $max_t \, ||u(t)||$. Using constraints on the control input (for example Lemma 2) with α=1 and ζ =10, we will obtain a dynamic controller which yields system outputs as shown in Fig. 4. In this case, we assume that initial condition for the controller is: $x_c(0)$=[0,0].

Fig. 5 shows $u(t)$ for the stabilizing controller design. Fig 6 depicts $u(t)$ when we use decay rate condition. Fig. 7 shows $u(t)$ when we use constraint on the input and decay rate condition. We see that all control signals satisfy the condition $\|u(t)\| \le \zeta\, e^{-\alpha t}$ in Fig. 7. Note that here we have $max_t \, ||u(t)||$= 2.9654.

5. CONCLUSIONS

In this paper, we presented three methods for designing dynamic output feedback controller for T-S fuzzy systems with LMI formulation. The constraints for stabilizing dynamic output feedback based on DPDC approach were obtained in each case in the form of LMIs. Then, by incorporating some control performances, such as decay rate and constraint on the control input in the design, their corresponding conditions were obtained. Numerical example was given to illustrate validity of the proposed methods, and to compare their performance.

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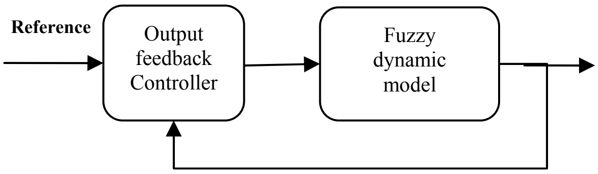

Fig. 1. Schematic of the fuzzy control system.

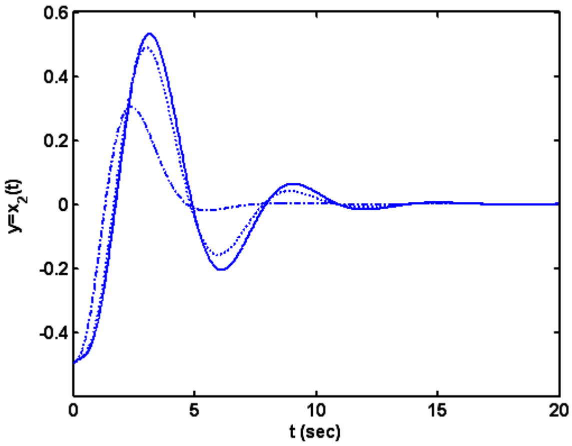

Fig. 2: Closed loop system output y=$x_2(t)$ for the design given in Theorem 1 (solid lines), Theorem 2 (dotted lines) and Theorem 3 (dash-dot lines).

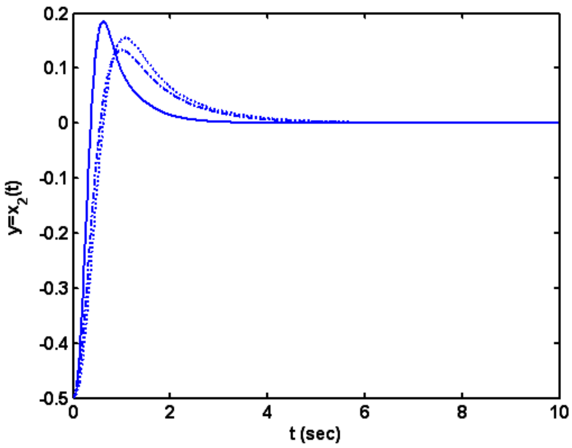

Fig. 3: Closed loop system output y=$x_2(t)$ with the decay rate condition (α=1) for the design given in Theorem 1 (solid lines), Theorem 2 (dotted lines) and Lemma1 (dash-dot lines).

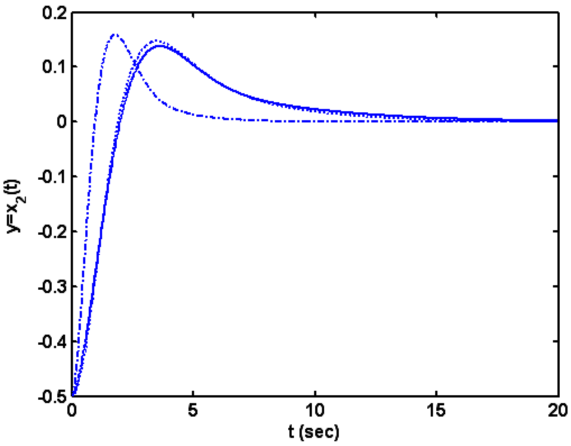

Fig. 4: Closed loop system output $y=x_2(t)$ for decay rate condition and constraint on input (α=1, ζ=10) for the design given in Theorem 1 (solid lines), Theorem 3 (dash-dot lines) and Lemma 2 (dotted lines).

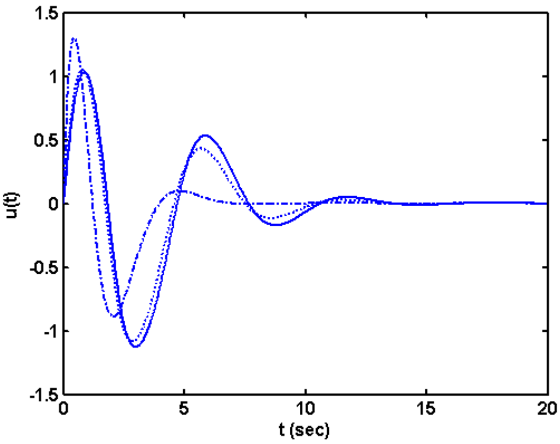

Fig. 5: Control signal for the design given in Theorem 1 (solid lines), (dotted lines) and Theorem 3 (dash-dot lines).

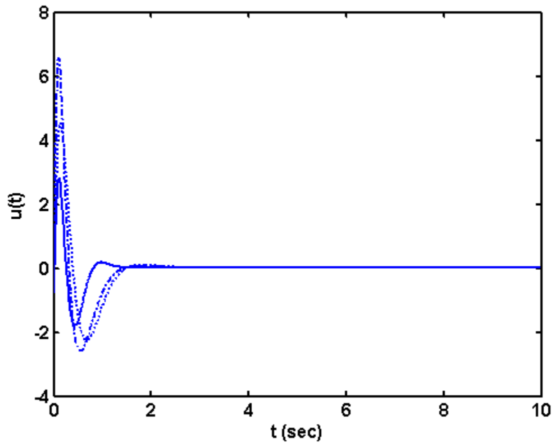

Fig. 6: Control signal with the decay rate condition (α=1) for the design given in Theorem 1 (solid lines), Theorem 2 (dotted lines) and Lemma1 (dash-dot lines).

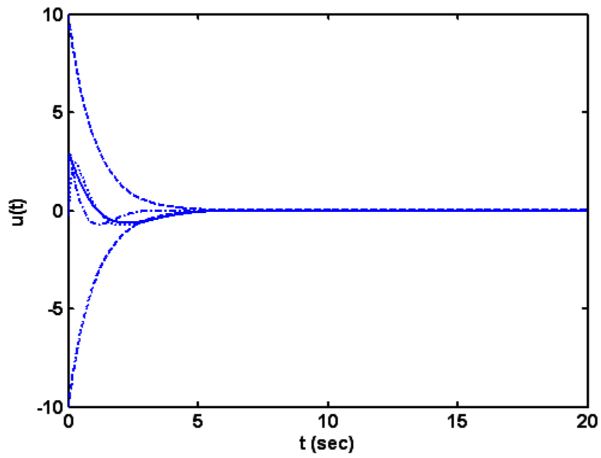

Fig. 7: Control signal for decay rate condition and constraint on input (α=1, ζ=10) for Theorem 1 (solid lines), Theorem 3 (dash-dot lines) and Lemma 2 (dotted lines). The dashed lines stand for $|\zeta e^{-\alpha t}|$.