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Nonlinear Control of PWM AC/DC Boost Rectifiers - Theoretical analysis of closed-loop performances

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Abstract: We are considering the problem of controlling AC/DC switched power converters of the Boost type. The control objectives are: (i) guaranteeing a regulated voltage for the supplied load, (ii) enforcing power factor correction (PFC) with respect to the main supply network. The considered problem is dealt with using a nonlinear controller that involves two loops in cascade. The inner-loop is designed, using the backstepping technique, to cope with the PFC issue. The outer-loop is designed to regulate the converter output voltage. Experimental tests show that the proposed controller actually meets the objectives it has designed for. While different controllers can be found in the relevant literature, it is the first time that a complete rigorous analysis of the controller performances is developed. Such a theoretical contribution is a major feature of this paper.

1. INTRODUCTION

From a dynamic viewpoint, an AC/DC converter is a nonlinear and hybrid system. Then, undesirable current harmonics may be generated when the converter is connected to an AC source. To avoid the above drawbacks, an AC/DC converter should be controlled bearing in mind, not only output voltage regulation, but also rejection of undesirable current harmonics. The last objective is referred to 'power factor correction'. It is only recently that output regulation and PFC have been simultaneously and explicitly accounted for in the control design, (Escobar and *al*., 2001-Karagiannis and al., 2003-Abouloifa and al., 2003-Giri and al., 2005). In (Escobar and *al*., 2001-Karagiannis and *al*., 2003) these objectives have been achieved for a second-order standard boost rectifier. However, the effect of the output voltage ripples, observed in closed-loop, has not been formally analyzed. In (Abouloifa and *al*., 2003-Giri and *al*., 2005), boost and buck-boost diode-based converters, containing input (LC or LCL) filter, have been considered. The obtained 4th order circuits have been controlled using the backstepping technique. A theoretical analysis of the output voltage ripples effect has been attempted in (Giri and *al*., 2005); however, the proposed analysis was not complete.

In the present paper, we consider the problem of controlling PWM AC/DC full-bridge converters (Fig.1). The complexity of the problem is twofold: the PFC requirement and the output regulation should be simultaneously achieved; the converters dynamics are 4th order, nonlinear and hybrid. The last feature is usually coped with basing the control design on average models. Based on such average model, we will develop a nonlinear cascade controller including two loops. The inner-loop is designed using the backstepping technique in such a way that the input current be sinusoidal and in phase with AC-voltage. The involved control issue is formulated as a problem of regulating the ratio current/voltage (at the converter input) to a desired value β by acting on the duty ratio α (control signal). The purpose of the outer-loop is specifically to tune β so that the output voltage x_4 coincides with its desired value despite the load changes. It is formally established that the nonlinear cascade controller thus constructed actually meets its objectives. It is the first time that the controller performances are so rigorously analyzed. Such a theoretical analysis is a major feature of this paper, compared to former works (e.g. (Escobar and *al*., 2001-Karagiannis and *al*., 2003-Abouloifa and *al*., 2003-Giri and *al*., 2005)).

The paper is organized as follows: the class of converters under study is presented and modelled in Section 2, the controller design and analysis are dealt with in Section 3, the control performances are experimentally illustrated in Section 4, a conclusion and a reference list end the paper.

2. CONVERTER DESCRIPTION AND MODELING

The full-bridge PWM boost rectifier under study is represented by Fig.1. It includes two main parts namely, a L_1CL_2-filter and a commutation-cell $(s_1 - s_4)$. The circuit operates according to the well known Pulse Width Modulation (PWM) principle, (Krein and *al*., 1990), (Andrieu and *al*., 1996, Tse and *al*., 2000, Erickson and *al*., 1990). Accordingly, time is shared in intervals of length *T*, also called cutting period. Within a given period, (s_1, s_4) are both ON while (s_2, s_3) are OFF during αT, for some $0 \le \alpha \le 1$. During the rest of time, i.e. $(1-\alpha)T$, (s_1, s_4) are OFF and (s_2, s_3) are ON. The value of α varies from a period to another and its time-variation law determines the trajectory of output voltage x_4. The variable α thus defined is called 'duty ratio' and serves as the control signal for the converter.

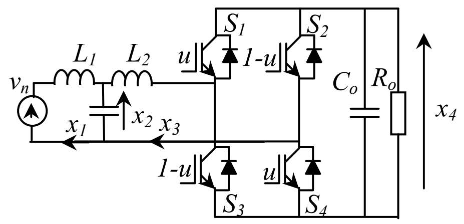

Fig. 1. PWM boost rectifier under study.

The average model thus obtained is described by the following equations, where $\bar{x}$ denotes the average value of x, over cutting periods (Abouloifa and *al*., 2003):

$$\frac{d\bar{x}_1}{dt} = -\frac{\bar{x}_2}{L_1} + \frac{v_n}{L_1} \tag{1}$$

$$\frac{d\bar{x}_2}{dt} = -\frac{\bar{x}_3}{C} + \frac{\bar{x}_1}{C} \tag{2}$$

$$\frac{d\bar{x}_3}{dt} = \frac{\bar{x}_2}{L_2} - \frac{(2\alpha - 1)}{L_2}\bar{x}_4 \tag{3}$$

$$\frac{d\bar{x}_4}{dt} = -\frac{\bar{x}_4}{R_o C_o} + \frac{(2\alpha - 1)}{C_o}\bar{x}_3 \tag{4}$$

3. CONTROLLER THEORETICAL DESIGN AND ANALYSIS

The controller synthesis is carried out in two major steps. First, a current inner loop is designed to cope with the PFC issue. In the second step, an outer loop is built-up to achieve voltage regulation.

3.1 Current inner loop design

The PFC objective means that the current entering the converter should be sinusoidal and in phase with the AC-voltage. We therefore seek a regulator that enforces the current $\bar{x}_1$ to track a reference signal of the form $x_1^* = \beta v_n$. At this point the quantity β is any positive real. The regulator will now be designed using the backstepping technique (Krstic and *al*., 1995), based on the partial model (1-3).

Step 1: Output regulation of subsystem (1)

Let us introduce the tracking error on the current:

$$z_1 = \bar{x}_1 - x_1^* \tag{5}$$

Using (1), time-derivation of (5) yields the following error dynamics:

$$\frac{dz_1}{dt} = -\frac{\bar{x}_2}{L_1} + \frac{v_n}{L_1} - \frac{dx_1^*}{dt} \tag{6}$$

In (6), $(-\bar{x}_2/L_1)$ stands as a virtual control variable. Then, z_1 can be regulated to zero if $(-\bar{x}_2/L_1) = \sigma_1$ where σ_1, called stabilizing function, is defined by:

$$\sigma_1 = -\frac{v_n}{L_1} + \frac{dx_1^*}{dt} - c_1 z_1 \tag{7}$$

where $c_1>0$ is a design parameter. Indeed, this choice would imply that: $\dot{z}_1 = -c_1 z_1$ which clearly establishes asymptotic stability of (1) with respect to the Lyapunov function:

$$W_1 = 0.5 z_1^2 \tag{8}$$

Then, time-derivation of W_1 would imply:

$$\dot{W}_1 = -c_1 z_1^2 < 0 \tag{9}$$

which is negative definite with respect to z_1. As $(-\bar{x}_2/L_1)$ is not the actual control input, a new error variable, denoted z_2, between the virtual control and its desired value (σ_1) is introduced:

$$z_2 = (-\bar{x}_2/L_1) - \sigma_1 \tag{10}$$

Then, equation (6) becomes, using (7) and (10):

$$\dot{z}_1 = -c_1 z_1 + z_2 \tag{11}$$

Also, the derivative of Lyapunov function (9) becomes:

$$\dot{W}_1 = -c_1 z_1^2 + z_1 z_2 \tag{12}$$

Step 2: Stabilization of the (z_1, z_2) - subsystem

Achieving the PFC objective amounts to enforcing the errors (z_1, z_2) to vanish. To this end, one needs the dynamics of z_2. Deriving (10), it follows from (2) that:

$$\frac{dz_2}{dt} = \frac{\bar{x}_3}{L_1 C} - \frac{\bar{x}_1}{L_1 C} - \frac{d\sigma_1}{dt} \tag{13}$$

In the above equation, the quantity $(\bar{x}_3/L_1 C)$ stands as a new virtual control input. We now need to select a Lyapunov function W_2 for the (z_1, z_2)-system. As the objective is to drive its states (z_1, z_2) to zero, it is natural to choose the following function:

$$W_2 = W_1 + 0.5 z_2^2 \tag{14}$$

Using (12)-(14), one gets the following derivative:

$$\dot{W}_2 = -c_1 z_1^2 + z_2(z_1 + \dot{z}_2) \tag{15}$$

For the (z_1, z_2)-system to be globally asymptotically stable, it is sufficient to choose the virtual control input so that $\dot{W}_2 = -c_1 z_1^2 - c_2 z_2^2$ (for any $c_2>0$). If this holds, (15) implies:

$$\dot{z}_2 = -z_1 - c_2 z_2 \tag{16}$$

Comparing (16) and (13) and solving with respect to $(\bar{x}_3/L_1 C)$, yields $(\bar{x}_3/L_1 C) = \sigma_2$ with:

$$\sigma_2 = \frac{\bar{x}_1}{L_1 C} + \frac{d\sigma_1}{dt} - z_1 - c_2 z_2 \tag{17}$$

As $(\bar{x}_3/L_1C)$ is not the actual control input, a new error variable z_3 between the above virtual control and the stabilizing function σ_2 is introduced:

$$z_3 = (\bar{x}_3/L_1C) - \sigma_2 \tag{18}$$

Then, equation (13) becomes, using (17) and (18):

$$\dot{z}_2 = -z_1 - c_2 z_2 + z_3 \tag{19}$$

Also, the Lyapunov function derivative (15) becomes:

$$\dot{W}_2 = -c_1 z_1^2 - c_2 z_2^2 + z_2 z_3 \tag{20}$$

Step 3: Stabilization of the (z_1, z_2, z_3)- subsystem

Time-derivation of z_3 gives, using (18) and (3):

$$\frac{dz_3}{dt} = \frac{\bar{x}_2}{L_1 L_2 C} - \frac{2\alpha - 1}{L_1 L_2 C}\bar{x}_4 - \frac{d\sigma_2}{dt} \tag{21}$$

The actual control variable, namely α, appears for the first time in equation (21). An appropriate control law for generating α has now to be found for the system (11), (16) and (21) whose state vector is (z_1, z_2, z_3). Let us consider the Lyapunov function W_3:

$$W_3 = W_2 + 0.5 z_3^2 \tag{22}$$

Using (30), the time-derivative of W3 can be rewritten as:

$$\dot{W}_3 = -c_1 z_1^2 - c_2 z_2^2 + z_3(z_2 + \dot{z}_3) \tag{23}$$

This shows that, for the (z_1, z_2, z_3)-system to be globally asymptotically stable, it is sufficient to choose the control α so that $\dot{W}_3 = -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2$ which, due to (23), amounts to ensuring that:

$$\dot{z}_3 = -z_2 - c_3 z_3 \tag{24}$$

Comparing (24) and (21) yields the following backstepping control law:

$$\alpha = \frac{1}{2} + \frac{1}{\bar{x}_4}\left[\bar{x}_2 + L_1 L_2 C\left(z_2 + c_3 z_3 - \frac{d\sigma_2}{dt}\right)\right] \tag{25}$$

Since the above control law involves the reference signal $x_1^* = \beta v_n$, it follows from (17) and (7) that the ratio β and its three first derivatives should be available. The results thus established are summarized in the following proposition.

Proposition 1. Consider the system, next called inner closed-loop, consisting of the subsystem (1)-(3) and the control law (25). If the ratio β and its three first derivatives are available, then the inner closed-loop system undergoes, in the (z_1, z_2, z_3)-coordinates, the following equation:

$$\frac{d}{dt}\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} = \begin{pmatrix} -c_1 & 1 & 0 \\ -1 & -c_2 & 1 \\ 0 & -1 & -c_3 \end{pmatrix}\begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \tag{26}$$

Furthermore, (26) is globally asymptotically stable with respect to the Lyapunov function $W_3 = 0.5(z_1^2 + z_2^2 + z_3^2)$. Finally, as (26) is linear, the error vector (z_1, z_2, z_3) converges exponentially fast to zero.

3.2 Voltage outer loop design

The aim of the outer loop is to generate a tuning law for the ratio β in such a way that the output voltage $\bar{x}_4$ be regulated to a given reference value x_4^*.

<u>Relation between β and $\bar{x}_4$</u>

The first step in designing such a loop is to establish the relation between the ratio β (control input) and the output voltage $\bar{x}_4$. This is carried out in the following proposition.

Proposition 2. Consider the power converter described by (1)-(4) augmented with the inner control law defined by (25). Under the same assumptions as in Proposition 1, one has:

1) The output voltage $\bar{x}_4$ varies in response to the tuning ratio β according to the following equation:

$$\frac{d\bar{x}_4}{dt} = -\frac{\bar{x}_4}{R_o C_o} + \frac{\hat{v}_n^2}{2C_o \bar{x}_4}(1 - cos(2\omega_n t))\beta + \frac{v_n z_1}{C_o \bar{x}_4} \tag{27}$$

where $\hat{v}_n$ denotes the amplitude of the network (sinusoidal) voltage v_n.

2) Then, the squared-voltage $y = \bar{x}_4^{\,2}$ varies, in response to the tuning ratio β, according to the following first-order time-varying linear equation:

$$\frac{dy}{dt} = -ay + k_o(1 - cos(2\omega_n t))\beta + \frac{2v_n z_1}{C_o} \tag{28}$$

With $a = 2/(R_o C_o)$, $k_o = \hat{v}_n^2/C_o$

Proof. 1) The first step consists in replacing the circuit part above the set (C_o, R_o), by an equivalent current generator, as shown by Fig. 2. In view of equation (4), the underlying current value i_{equ} coincides with $(2\alpha - 1)\bar{x}_3$. So, (4) becomes:

$$\frac{d\bar{x}_4}{dt} = -\frac{\bar{x}_4}{R_o C_o} + \frac{i_{equ}}{C_o} \tag{29}$$

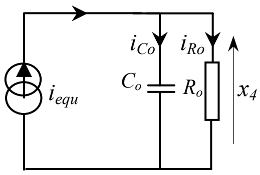

Fig 2. Equivalent current

The equivalent current i_{equ} will now be expressed in function of the tuning ratio β, using power conservation arguments. From (5) one has $\bar{x}_1 = \beta v_n + z_1$. Then, the instantaneous power entering the converter turns out to be the following:

$$p_n = \beta (v_n)^2 + v_n z_1 = \frac{\beta \hat{v}_n^2}{2}(1 - cos(2\omega_n t)) + \hat{v}_n \, sin(\omega_n t) \, z_1 \tag{30}$$

On the other hand, the power that is actually transmitted to the load is $p_{load} = \bar{x}_4 i_{equ}$. But, the entering power is integrally transmitted to the load (which is the only dissipative element). Then, the quantity P_{load} does coincide with p_n.This yields: $i_{equ} = (\beta.\hat{v}_n^2/2\bar{x}_4)(1 - cos(2\omega_n t)) + v_n z_1/\bar{x}_4$, which together with (29) establishes (27).

2) Deriving y with respect to time and using (27), yields the first-order differential equation (28) and completes the proof of Proposition 2

<u>Squared output voltage control</u>

The ratio β stands as a control input in the first-order system (38). The problem at hand is to design for β a tuning law so that the squared voltage $y = \bar{x}_4^{\,2}$ tracks a given reference signal $y^* = (x_4^*)^2$. Ignoring the linear time-varying feature of the first-order system, a PI control is invoked. Bearing in mind the fact that β and its three first derivatives should be available (Proposition 1), a filtered PI control law is resorted to, namely:

$$\beta = (b/(b+s))^3 (k_p e_1 + k_i e_2) \tag{31a}$$

$$e_1 = y^* - y, \quad e_2 = \int_0^t e_1 dt \tag{31b}$$

where s may denote as well the Laplace variable or the derivative operator $(s = d/dt)$. At this point, the regulator parameters (b, k_p, k_i) are any positive real constants. The next analysis will make it clear how these should be chosen for the control objectives to be achieved.

3.3 Control system analysis

In the following Theorem, it is shown that, for a specific class of reference signals, the control objectives are achieved (in the mean) with an accuracy that depends on the network frequency ω_n. The following notations are needed to formulate the results:

$$\begin{array}{lll} \varepsilon = 1/2\omega_n & a_o = k_o k_i b^3 & a_1 = (a + k_o k_p) b^3 \\ a_2 = 3ab^2 + b^3 & a_3 = 3ab + 3b^2 & a_4 = (a + 3b) \end{array} \tag{32}$$

Theorem 1 (main result). Consider the AC/DC Boost power converter shown by Fig.1, represented by its average model (1)-(4), together with the controller consisting of the inner-loop regulator (25) and the outer-loop regulator (31a-b). Then, the closed-loop system has the following properties:

1) The error $z_1 = \bar{x}_1 - x_1^*$ vanishes exponentially fast (where $x_1^* = \beta \, v_n$),

2) Let the reference signal y^* be nonnegative and periodic with period $N\pi/\omega_n$, where N is any positive integer. Let the positive regulator parameters (b, k_p, k_i) satisfy the following inequalities:

$$(a_4 a_3 - a_2) > 0 \tag{33a}$$

$$(a_4 a_3 - a_2) a_2 - (a_4 a_1 - a_o) a_4 > 0 \tag{33b}$$

$$[(a_4 a_3 - a_2) a_2 - (a_4 a_1 - a_o) a_4](a_4 a_1 - a_o) - (a_4 a_3 - a_2)^2 a_o > 0 \tag{33c}$$

Then, there exists a positive real ε^* such that if $\varepsilon \le \varepsilon^*$, one has:

a) the tracking error is a harmonic signal that continuously depends on ε, i.e. $e_1 = e_1(t, \varepsilon)$,

b) $\lim_{\varepsilon \to 0} e_1(t, \varepsilon) = 0$.

Proof. **1**) Equation (31a) guarantees that β and its derivatives (up to the third order) are available. Then, Part 1 of the Theorem follows directly from Proposition 1. This also guarantees that equation (28), in Proposition 2, holds. In order to prove the second part of Theorem l, let us introduce the following state variables:

$$\begin{array}{l} x_{0,1} = e_1; \quad x_{0,2} = e_2; \quad x_{0,3} = \dfrac{b}{b+s}(k_p e_1 + k_i e_2); \\ x_{0,4} = \dfrac{b}{b+s} x_{0,3}; \quad x_{0,5} = \dfrac{b}{b+s} x_{0,4}; \quad x_{0,6} = z_1; \\ x_{0,7} = z_2; \quad x_{0,8} = z_3 \end{array} \tag{34b}$$

Then, it follows from (26), (28) and (31a-b) that the state vector $X_o = (x_{0,1} \; \ldots \; x_{0,8})^T$ undergoes the following state equation:

$$\dot{X}_o = f(t, X_o, y^*) \tag{35}$$

where:

$$f(t, X_o, y^*) = \begin{pmatrix} \delta \\ x_{0,1} \\ b(-x_{0,3} + k_p x_{0,1} + k_i x_{0,2}) \\ b(-x_{0,4} + x_{0,3}) \\ b(-x_{0,5} + x_{0,4}) \\ -c_1 x_{0,6} + x_{0,7} \\ -x_{0,6} - c_2 x_{0,7} + x_{0,8} \\ -x_{0,7} - c_3 x_{0,8} \end{pmatrix} \tag{36}$$

with:

$$\delta = -a x_{0,1} - k_o (1 - cos(2\omega_n t)) x_{0,5} + a y^* + \frac{dy^*}{dt} - \frac{2\hat{v}_n \, sin(\omega_n t)}{C_o} x_{0,6}$$

Stability of the above system will now be dealt with using averaging. As y^* is periodic with period $N\pi/\omega_n$, it will prove to be useful introducing the following auxiliary reference function:

$$y^r(t) = y^*(Nt/2\omega_n) \tag{37}$$

This readily implies that $y^*(t) = y^r(2\omega_n t/N)$ and that y^r is periodic, with period 2π. Let us now introduce the time-scale change $\tau = 2\omega_n t$. Then, the term containing y^* in (36) becomes:

$$ay^*(t) + \frac{dy^*(t)}{dt} = ay^r(\tau/N) + 2\omega_n \frac{dy^r(\tau/N)}{d\tau} \tag{38}$$

It also follows from (35)-(36) that X_o undergoes the differential equation:

$$\dot{X}_o = \varepsilon\, g(\tau, X_o, \varepsilon) \tag{39}$$

with:

$$g(\tau, X_o, \varepsilon) = \begin{pmatrix} \delta_\tau \\ x_{0,1} \\ b(-x_{0,3} + k_p x_{0,1} + k_i x_{0,2}) \\ b(-x_{0,4} + x_{0,3}) \\ b(-x_{0,5} + x_{0,4}) \\ -c_1 x_{0,6} + x_{0,7} \\ -x_{0,6} - c_2 x_{0,7} + x_{0,8} \\ -x_{0,7} - c_3 x_{0,8} \end{pmatrix} \tag{40}$$

with:

$$\delta_\tau = -ax_{10} - k_o(1 - cos(\tau))x_{50} + ay^r(\tau/N) + 2\omega_n \frac{dy^r(\tau/N)}{dt} - \frac{2\hat{v}_n\, sin(\tau/2)}{C_o} x_{0,6}$$

Now, let us introduce the average function $G(X_o) = \lim_{\varepsilon \to 0} \frac{1}{2\pi N}\int_0^{2\pi N} g(\tau, X_o, \varepsilon) d\tau$. It follows from (40) that:

$$G(X_o) = \begin{pmatrix} -ax_{10} - k_o x_{50} + a\bar{y}^r \\ x_{0,1} \\ b(-x_{0,3} + k_p x_{0,1} + k_i x_{0,2}) \\ b(-x_{0,4} + x_{0,3}) \\ b(-x_{0,5} + x_{0,4}) \\ -c_1 x_{0,6} + x_{0,7} \\ -x_{0,6} - c_2 x_{0,7} + x_{0,8} \\ -x_{0,7} - c_3 x_{0,8} \end{pmatrix} \tag{41}$$

where $\bar{y}^r$ denotes the mean value of y^r (which is the same as for y^*). Note that the mean value over $[0, 2N\pi]$, of the derivative in the first line of (36) is zero because y^r is periodic with period 2π. In order to get stability results regarding the system of interest (35)-(36), it is sufficient (thanks to averaging theory) to analyze the following averaged system:

$$\dot{Z} = \varepsilon\, G(Z) \tag{42}$$

To this end, notice that (42) has a unique equilibrium at:

$$Z^* = \begin{bmatrix} 0 & \frac{-a\bar{y}^r}{k_o k_i} & \frac{-a\bar{y}^r}{k_o} & \frac{-a\bar{y}^r}{k_o} & \frac{-a\bar{y}^r}{k_o} \end{bmatrix}^T \tag{43}$$

On the other hand, as (42) is linear, the stability properties of its equilibrium are fully determined by the state-matrix:

$$A = \begin{bmatrix} A_1 & O \\ O & A_2 \end{bmatrix}$$

where O denotes null matrices of appropriate dimensions and:

$$A_1 = \begin{pmatrix} -a & 0 & 0 & 0 & -k_o \\ 1 & 0 & 0 & 0 & 0 \\ bk_p & bk_i & -b & 0 & 0 \\ 0 & 0 & b & -b & 0 \\ 0 & 0 & 0 & b & -b \end{pmatrix};\ A_2 = \begin{pmatrix} -c_1 & 1 & 0 \\ -1 & -c_2 & 1 \\ 0 & -1 & -c_3 \end{pmatrix} \tag{44}$$

More specifically, the equilibrium Z^* will be globally asymptotically stable if the matrix A is Hurwitz. It has already noted (see Proposition 1) that A_2 is Hurwitz. So, it is sufficient to check that A_1 is also Hurwitz. To this end, note that its eigenvalues are the zeros of the following polynomial:

$$det(\lambda I - A_1) = \lambda^5 + a_4\lambda^4 + a_3\lambda^3 + a_2\lambda^2 + a_1\lambda^1 + a_o \tag{45}$$

where the a_i*'s* are defined by (32). Applying for instance the well known Routh's algebraic criteria, it follows that all zeros of the polynomial (45) have negative real parts if the coefficients (a_0 to a_4) satisfy (33a-c). Now, invoking averaging theory, e.g. Theorem 4.1 in (Zhi-Fen and al., 1992), one concludes that there exists a $\varepsilon^* > 0$ such that for $\varepsilon < \varepsilon^*$, the differential equation (35)-(36) has a harmonic solution $X_o = X_o(t, \varepsilon)$ that continuously depends on ε. Moreover, one has $\lim_{\varepsilon \to 0} X_o(t, \varepsilon) = Z^*$. This, together with (39), yields in particular that $\lim_{\varepsilon \to 0} e_1(t, \varepsilon) = 0$. The Theorem is thus established

4. EXPERIMENTAL EVALUATION

4.1. Experimental setup.

The performances of the proposed controller are now experimentally evaluated using a real PWM rectifier with the following characteristics: L_1=1,5mH, L_2=1mH, C=7μF, *Co=4.5mF, Ro=40Ω, network:* $50V/50Hz$, DSP-card with sampling frequency of $20\,Khz$ .Inner-loop parameters: $c_1 = c_2 = c_3 = 10^4$, Outer-loop design parameters: $k_p = 2.73\,10^{-5}$, $k_i = 3.20\,10^{-4}$ and $b = 10^3$.

4.2. Experimental protocol.

The experiments aim at illustrating the behavior of the controller in response to step changes on both the voltage reference x_4^* and the load resistance R_o . More specifically, the voltage reference goes from $100\,V$ to $120\,V$ and then back to $100\,V$. The load resistance steps from its nominal value $(40\,\Omega)$ up to infinity (unload converter) and then down to its nominal value.

4.3. Experimental results.

The controller performances are illustrated by figures 3 to 6. As expected by theorem 1, the output voltage x_4 converges in the mean to its reference value (Fig. 3). Furthermore, it is observed that voltage ripple oscillates at the frequency $2\omega_n$ with amplitude that is much smaller than the average value of the signals. Finally, Fig 4 shows that the input current x_1 and the network voltage v_n are in phase. Hence, the converter connection to the supply network is made with a unitary power factor.

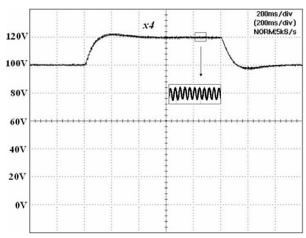

Fig. 3. Response of the output voltage

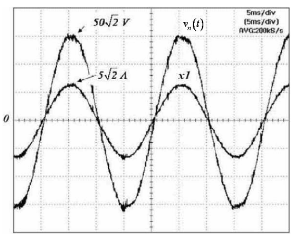

Fig. 4. Unity power factor

Figs 5 and 6 illustrate the variation of the output voltage and the input current in response to load changes. The other converter characteristics are kept unchanged. It is seen by Fig 5 that the disturbing effect due to load changes is well compensated by the controller. Furthermore, Fig 6 shows the correlation of the current amplitude with the output voltage. Robustness of the proposed controller with respect to load changes is thus established.

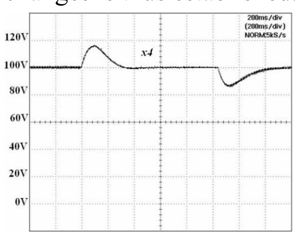

Fig. 5. Voltage transient

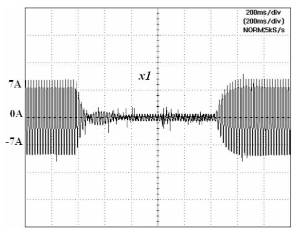

Fig. 6. Current x_1

5. CONCLUSION

In this paper we have considered the problem of controlling a full-bridge rectifier of boost type. The control objectives are power factor correction and voltage regulation. The converter dynamics have been described by the 4th order nonlinear state-space averaging (1)-(4). Based on such a model, a cascade structure nonlinear controller has been designed. It has been formally established, using averaging theory, that the obtained controller meets its objectives. These results have been confirmed by an experimental study which, further, showed robustness of controller performances with respect to significant load changes. It is the first time that an averaging analysis is formally carried out to describe the performances of the global closed-loop system.

REFERENCES

Krein P.T., J. Bentsman, R.M. Bass and B. Lesieutre (1990). On the use of averaging for analysis of power electronic system. *IEEE Transaction on Power Electronics*, **Vol. 5**, n° 2, pp. 182-190.

Krstic, M., I. Kanellakopoulos and P.V. Kokotovic (1995). *Nonlinear and adaptive control design*. John Willy & Sons, NY.

Zhi-Fen Z., D. Tong-Ren, H. Wen-Zao and D. Zhen-Xi (1992). *Qualitative theory of differential equations*. American Mathematical Society, Rhode Island, USA.

Escobar, G., D. Chevreau, R. Ortega and E. Mendes (2001). An adaptive passivity-based controller for a unity power factor rectifier. *IEEE Trans. Contr. Syst. Technol*., **Vol. 9**, pp. 637–644.

Andrieu, C., J. P. Ferrieux and M. Rocher (1996). The ac/dc stage: A survey of structures and chopper control modes for power factor correction. *EPE J*., **vol**. 5, pp. 17–22.

Tse, C. K. and M. H. L. Chow (2000). Theoretical study of switching converters with power factor correction and output regulation. *IEEE Trans. Circuits Syst. I*, **vol. 47**, pp. 1047–55.

Erickson, R., M. Madigan and S. Singer (1990) Design of simple high power factor rectifier based on the flyback converter. Proceedings of IEEE Applied Power Electronics Conference and Exposition.

Karagiannis, D. E. Mendes, A. Astolfi, and R. Ortega (2003). An experimental comparison of several PWM controllers for a single-phase AC-DC converter. *IEEE Trans. Contr. Syst. Technol.*, **vol. 11**, pp. 940–947.

Abouloifa, A., F. Giri and I. Lachkar (2003). Nonlinear control of a PWM rectifier output voltage regulation and power factor correction. Proceedings of the 5th Int. Symp. On Advanced Electromechanical Motion Systems (ELECTROMOTION), pp. 2-7.

Giri, F., A. Abouloifa and I. Lachkar (2005). Nonlinear control of Boost AC/DC converters. IFAC World Congress, Prague, Czech Republic, July 4-8.