Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

# Exponential Stabilization of Linear Systems with Time-varying Sampling ⋆

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**Abstract:** This paper studies the problem of exponential stabilization of linear systems with time-varying sampling. The sampling rate varies from sample to sample with the given probability. By applying the input delay approach, the sampled-data system is transformed into a continuous time-delay system with stochastic parameter. A new exponential stability criterion is derived for the sampled-data system by using the Lyapunov functional approach. Based on this, the design procedure for stabilization controllers is presented by means of linear matrix inequalities (LMIs). An example shows the effectiveness of the proposed controller design methodology.

## 1. INTRODUCTION

In the past decades, the sampled-data control problems have been the subject of wide research owing to the reality that modern control systems are almost implemented in a digital computer of one form or another. By sampled-data systems, we refer to those systems containing both continuous-time and discrete-time signals and components. These hybrid systems frequently form an idealized model of computer control in a number of engineer applications. There have been a great number of research results concerning sampled-data systems scattered in the literature in the past several years. To mention a few, Chen and Francis presented a comprehensive study on the modern sampled-data systems in (Chen and Francis [1995]). In (Shi [1998]), the author investigated the $H_\infty$ filtering problem for a class of uncertain continuous-time systems under sampled measurements. In (Hagiwara et al. [2001]), the authors gave some methods to compute the upper and lower bounds of the norm of the frequency response operator of sampled-data systems. In (Toivonnen and Medvedev [2003]), optimal damping of harmonic disturbances of known frequencies was studied for sampled-data systems. Issues dealing with $H_\infty$ control and robustness of uncertain systems were investigated in (Fridman [2006], Shi and Nguang [2003], Tian et al. [2007]). In (Hu et al. [2007]), the authors considered the problem of analysis and synthesis for networked control systems that are modelled as sampled-data systems with time delay in their discrete-time systems.

The digital controllers are mostly designed under the condition of single sampling. However, the available data for signal processing and control are not always equidistant in some applications. A typical example of such systems can be found in networked control systems, where time delay caused by data transmission or packet dropout can occur because of the limitation of the network resource. As the network traffic load becomes heavy, which will increase the possibility of more communication time or data loss, it may improve the system performance that the sampling period gets large. Under such conditions, it could be expected that time-varying sampling can yield better performance than single sampling. Many existing works deal with the problems arising from time-varying sampling. The analysis and design of multirate control systems, where input updating and output sampling are performed with different rates, have been extensively investigated and many results have been obtained (Polushin and Marquez [2004], Tangirala et al. [2001], Wang et al. [2004]). In (Nagy [2000]), the author presented some basic design methods for the solution of the variable sampling interval linear stochastic control problem, where a single sampling interval model could be computed by an optimization method for I/O description.

In this paper, the problem of exponential stabilization is studied for linear systems with time-varying sampling. We assume that the sampling rate varies from sample to sample with the given probability. To make our idea more lucid and to avoid complicated notation, we consider the case in which only two sampling periods appear. By applying the input delay approach, the sampled-data system is transformed into a continuous time-delay system with stochastic parameter satisfying Bernoulli distribution. A new exponential stability criterion is derived for the sampled-data system by using the Lyapunov functional approach. Based on this, the design procedure for stabilization controllers is presented by means of linear matrix inequalities (LMIs). An example shows the effectiveness of the proposed controller design methodology.

*Notation*: The notation used throughout the paper is fairly standard. $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space and the notation $P > 0$ $(\geq 0)$ means that $P$ is real symmetric and positive definite (semi-definite). In symmetric block matrices or complex matrix expressions, we use an asterisk $(*)$ to represent

⋆ This work was partly supported by National Natural Science Foundation of China (60504008), Programme for New Century Excellent Talents in University of China, The Research Fund for the Doctoral Programme of Higher Education of China (20070213084), Key Laboratory of Integrated Automation for the Process Industry (Northeastern University), Ministry of Education of China and RGC HKU7031/07P.

a term that is induced by symmetry and diag$\{\cdots\}$ stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. If $A$ is a symmetric matrix, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the largest and smallest eigenvalue of $A$, respectively. $E\{x\}$ and $E\{x|y\}$ will, respectively, mean the expectation of $x$ and the expectation of $x$ conditional on $y$.

## 2. MAIN RESULTS

### 2.1 Problem Formulation

Consider the following linear system:

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; $u(t) \in \mathbb{R}^p$ is the control input; $A$ and $B$ are system matrices. For the state-feedback sampled-data control with zero-order hold (ZOH), the controller takes the following form:

$$u(t) = u_d(t_k) = Kx(t_k), \quad t_k \le t < t_{k+1}, \tag{2}$$

where $u_d$ is a discrete-time control signal; $t_k$ denotes the sampling instant. Under control law (2), the closed-loop system is given by

$$\dot{x}(t) = Ax(t) + BKx(t_k), \quad t_k \le t < t_{k+1}. \tag{3}$$

It is assumed that we have two sampling periods, denoted as $c_1$ and $c_2$ with $0 < c_1 < c_2$, and the probability of the occurrence of each is known, that is,

$$\begin{aligned} &\text{Prob}\{t_{k+1} - t_k = c_1\} = \beta, \\ &\text{Prob}\{t_{k+1} - t_k = c_2\} = 1 - \beta. \end{aligned}$$

Now, for $t_k \le t < t_{k+1}$ we denote

$$t_k = t - (t - t_k) \triangleq t - d(t),$$

where $d(t)$ is a time-varying delay, which is less than a sampling period. Then, the closed-loop sampled-data system in (3) can be transformed into the following continuous-time system with time delay:

$$\dot{x}(t) = Ax(t) + BKx(t - d(t)), \quad t_k \le t < t_{k+1}.$$

Now, introduce two time-varying delays $\tau_1(t)$ and $\tau_2(t)$, which satisfy

$$\begin{aligned} &0 \le \tau_i(t) < c_i, \;\; \tau_i(t_k) = 0, \\ &\dot{\tau}_i(t) = 1, \quad t \ne t_k, \quad i = 1, 2. \end{aligned}$$

Thus we have

$$\begin{aligned} &\text{Prob}\{d(t) = \tau_1(t)\} = \beta, \\ &\text{Prob}\{d(t) = \tau_2(t)\} = 1 - \beta, \; t_k \le t < t_{k+1}, \end{aligned} \tag{4}$$

which leads to

$$\begin{aligned} \text{Prob}\{0 \le d(t) < c_1\} &= \beta + \frac{c_1}{c_2}(1 - \beta), \\ \text{Prob}\{c_1 \le d(t) < c_2\} &= \frac{c_2 - c_1}{c_2}(1 - \beta), \\ & \qquad t_k \le t < t_{k+1}. \end{aligned}$$

Now introduce the following stochastic variable

$$\alpha(t) = \begin{cases} 1 & 0 \le d(t) < c_1, \\ 0 & c_1 \le d(t) < c_2. \end{cases}$$

Then, we have

$$\begin{aligned} \text{Prob}\{\alpha(t) = 1\} &= \text{Prob}\{0 \le d(t) < c_1\} \\ &= \beta + \frac{c_1}{c_2}(1 - \beta) \triangleq \alpha, \\ \text{Prob}\{\alpha(t) = 0\} &= \text{Prob}\{c_1 \le d(t) < c_2\} \\ &= \frac{c_2 - c_1}{c_2}(1 - \beta) \triangleq 1 - \alpha. \end{aligned} \tag{5}$$

From (5), we can obtain

$$E\{\alpha(t) = \alpha, \;\; E\{(\alpha(t) - \alpha)^2\} = \alpha(1 - \alpha).$$

Now, introduce two time-varying delays

$$0 \le d_1(t) < c_1, \quad 0 \le d_2(t) < c_2 - c_1.$$

Thus, the closed-loop system in (3) can be expressed as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \alpha(t)BKx(t - d_1(t)) \\ &\quad + (1 - \alpha(t))BKx\big(t - \tilde{d}_2(t)\big), \quad t_k \le t < t_{k+1}, \end{aligned} \tag{6}$$

where

$$\tilde{d}_2(t) = c_1 + d_2(t).$$

We can rewrite (6) as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \alpha BKx(t - d_1(t)) + (1 - \alpha)BKx\big(t - \tilde{d}_2(t)\big) \\ &\quad + (\alpha(t) - \alpha)[BKx(t - d_1(t)) - BKx(t - \tilde{d}_2(t))], \\ &\qquad t_k \le t < t_{k+1}. \end{aligned} \tag{7}$$

Now, we give the following definition of exponential stability which will be used in the exponential stability analysis.

*Definition 1.* System (6) is said to be exponentially stable in the mean square if there exist constants $\mu > 0$ and $\delta > 0$ such that

$$E\{\|x(t)\|^2\} \le \mu e^{-\delta t} \sup_{-2c_2 \le \theta \le 0} E\{\|\phi(\theta)\|^2\},$$

where $x(t) = \phi(t), t \in [-2c_2, 0]$ is the initial condition and $\phi$ is a continuous function.

### 2.2 Exponential Stability Analysis

This subsection is concerned with the problem of exponential stability analysis.

*Theorem 1.* System (6) is exponentially stable in the mean square if there exist matrices $P > 0$, $Q_1 \ge 0$, $Q_2 \ge 0$, $R_1 > 0$, $R_2 > 0$, and $S$, $W$, $U$, $V$ satisfying

$$\begin{bmatrix} \Xi_1 + \Xi_2 + \Xi_2^T + \Xi_3 & \Xi_4 \\ * & \Xi_5 \end{bmatrix} < 0, \tag{8}$$

*where*

$$\Xi_1 = \begin{bmatrix} PA + A^TP + Q_1 & 0 & \alpha PBK & 0 & (1-\alpha)PBK \\ * & Q_2 - Q_1 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & -Q_2 & 0 \\ * & * & * & * & 0 \end{bmatrix},$$

$$\begin{aligned} \Xi_2 &= [\, S \;\; U - W \;\; W - S \;\; -V \;\; V - U \,], \\ \Xi_3 &= \Psi_1^T Z \Psi_1 + \Psi_2^T Z \Psi_2, \\ \Xi_4 &= [\, \sqrt{c_1}S \;\; \sqrt{c_1}W \;\; gU \;\; gV \,], \\ \Xi_5 &= \text{diag}\{-R_1, -R_1, -R_2, -R_2\,\}, \\ \Psi_1 &= [\, A \;\; 0 \;\; \alpha BK \;\; 0 \;\; (1-\alpha)BK \,], \end{aligned}$$

$$\begin{aligned} \Psi_2 &= [\,0 \;\, 0 \;\, fBK \;\, 0 \;\, -fBK\,], \\ Z &= c_1R_1 + (c_2 - c_1)R_2, \\ f &= \sqrt{\alpha(1-\alpha)}, \\ g &= \sqrt{c_2 - c_1}. \end{aligned} \tag{9}$$

**Proof.** Define the following Lyapunov-Krasovskii functional:

$$\begin{aligned} V(t) &= x^T(t)Px(t) + \int_{t-c_1}^{t} x^T(s)Q_1x(s)ds \\ &+ \int_{t-c_2}^{t-c_1} x^T(s)Q_2x(s)ds + \int_{t-c_1}^{t}\int_{s}^{t} \dot{x}^T(\theta)R_1\dot{x}(\theta)d\theta ds \\ &+ \int_{t-c_2}^{t-c_1}\int_{s}^{t} \dot{x}^T(\theta)R_2\dot{x}(\theta)d\theta ds, \end{aligned} \tag{10}$$

where $P > 0$, $Q_1 \geq 0$, $Q_2 \geq 0$, $R_1 > 0$, $R_2 > 0$ are matrices to be determined. The infinitesimal operator $\mathscr{L}$ of $V(t)$ is defined as

$$\mathscr{L}V(t) \triangleq \lim_{\triangle \to 0^+} \frac{1}{\triangle}\{E\{\, V(t+\triangle)|\, t\} - V(t)\}. \tag{11}$$

Then, from (10) and (11) and taking expectation, we can obtain

$$E\{\mathscr{L}V(t)\} \leq E\{\varsigma^T(t)[\Xi_1 + \Xi_2 + \Xi_2^T + \Xi_3 + \Xi_6]\varsigma(t) + \sum_{i=7}^{10} \Xi_i\}, \tag{12}$$

where

$$\begin{aligned} \varsigma^T(t) &= \big[\, x^T(t) \;\, x^T(t-c_1) \;\, x^T(t-d_1(t)) \;\, x^T(t-c_2) \\ &\qquad x^T(t-\tilde{d}_2(t))\,\big], \\ \Xi_6 &= c_1SR_1^{-1}S^T + c_1WR_1^{-1}W^T + (c_2-c_1)UR_2^{-1}U^T \\ &\quad + (c_2-c_1)VR_2^{-1}V^T, \\ \Xi_7 &= -\int_{t-d_1(t)}^{t} [\varsigma^T(t)S + \dot{x}^T(s)R_1]R_1^{-1}[S^T\varsigma(t) + R_1\dot{x}(s)]ds, \\ \Xi_8 &= -\int_{t-c_1}^{t-d_1(t)} [\varsigma^T(t)W + \dot{x}^T(s)R_1]R_1^{-1}[W^T\varsigma(t) + R_1\dot{x}(s)]ds, \\ \Xi_9 &= -\int_{t-\tilde{d}_2(t)}^{t-c_1} [\varsigma^T(t)U + \dot{x}^T(s)R_2]R_2^{-1}[U^T\varsigma(t) + R_2\dot{x}(s)]ds, \\ \Xi_{10} &= -\int_{t-c_2}^{t-\tilde{d}_2(t)} [\varsigma^T(t)V + \dot{x}^T(s)R_2]R_2^{-1}[V^T\varsigma(t) + R_2\dot{x}(s)]ds. \end{aligned} \tag{13}$$

Note that $R_i > 0$, $i = 1,2$, thus $\Xi_i$, $i = 7,\ldots,10$, are all non-positive. By Schur complement, (8) guarantees

$$\Xi_1 + \Xi_2 + \Xi_2^T + \Xi_3 + \Xi_6 < 0. \tag{14}$$

We proceed to prove that system (6) is exponentially stable in the mean square. Under condition (8), we obtain that there exists a sufficiently small constant $\lambda > 0$ such that the left of (14) is less than $-\lambda I$. Therefore, from (12), it is easy to show that

$$E\{\mathscr{L}V(t)\} \leq -\lambda E\{\varsigma^T(t)\varsigma(t)\}. \tag{15}$$

By Itô's formula Mao et al. [1998], we obtain that

$$\begin{aligned} &E\left\{e^{\varepsilon T}V(T)\right\} \\ &= E\left\{e^{\varepsilon 0}V(0)\right\} + \int_0^T \varepsilon e^{\varepsilon t}E\{V(t)\}dt + \int_0^T e^{\varepsilon t}E\{\mathscr{L}V(t)\}dt \\ &\leq G \sup_{-2c_2 \leq \theta \leq 0} E\{\|\phi(\theta)\|^2\} + \int_0^T e^{\varepsilon t}E\{\varsigma^T(t)\Lambda\varsigma(t)\}dt, \end{aligned} \tag{16}$$

where

$$\begin{aligned} H &= \max(\lambda_{\max}(P), \lambda_{\max}(Q_1), \lambda_{\max}(Q_2), \lambda_{\max}(R_1), \lambda_{\max}(R_2)), \\ F &= 2\varepsilon c_2 e^{\varepsilon c_2}H, \\ M &= \max(\lambda_{\max}(\|A\|^2), \lambda_{\max}(\|BK\|^2)), \\ G &= H(1 + 2c_2 + 4c_2^2M) + c_2F(1 + 2c_2M), \\ \Lambda &= \mathrm{diag}\{\varepsilon H + F + c_2FM - \lambda, -\lambda, \alpha c_2FM - \lambda, -\lambda, \\ &\qquad (1-\alpha)c_2FM - \lambda\}. \end{aligned}$$

Then, by choosing $\varepsilon > 0$ such that $\varepsilon H + F + c_2FM - \lambda \leq 0$, we obtain that

$$E\{V(T)\} \leq Ge^{-\varepsilon t} \sup_{-2c_2 \leq \theta \leq 0} E\{\|\phi(\theta)\|^2\}. \tag{17}$$

Since $V(T) \geq \lambda_{\min}(P)x^T(T)x(T)$, it can be shown from (17) that

$$E\{x^T(T)x(T)\} \leq \bar{G}e^{-\varepsilon T} \sup_{-2c_2 \leq \theta \leq 0} E\{\|\phi(\theta)\|^2\}, \tag{18}$$

where

$$\bar{G} = \frac{G}{\lambda_{\min}(P)}.$$

Therefore, by Definition 1, system (6) is exponentially stable in the mean square. The proof is completed. □

### 2.3 *Stabilization Controller Design*

This subsection is devoted to solving the problem of state-feedback controller design based on Theorem 1.

*Theorem 2.* There exists a state-feedback controller such that the closed-loop system in (6) is exponentially stabilizable in the mean square if there exist matrices $\bar{P} > 0$, $\bar{Q}_1 \geq 0$, $\bar{Q}_2 \geq 0$, $\bar{R}_1 > 0$, $\bar{R}_2 > 0$, and $\bar{K}$, $\bar{S}$, $\bar{W}$, $\bar{U}$, $\bar{V}$, satisfying

$$\begin{bmatrix} \Pi_1 + \Pi_2 + \Pi_2^T & \Pi_4 & \Pi_3 & \Pi_7 \\ * & \Pi_5 & 0 & 0 \\ * & * & \Pi_6 & 0 \\ * & * & * & \Pi_6 \end{bmatrix} < 0, \tag{19}$$

wherewhere

$$\begin{aligned} \Pi_1 &= \begin{bmatrix} A\bar{P} + \bar{P}A^T + \bar{Q}_1 & 0 & \alpha B\bar{K} & 0 & (1-\alpha)B\bar{K} \\ * & \bar{Q}_2 - \bar{Q}_1 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & -\bar{Q}_2 & 0 \\ * & * & * & * & 0 \end{bmatrix}, \\ \Pi_2 &= \big[\, \bar{S} \;\, \bar{U} - \bar{W} \;\, \bar{W} - \bar{S} \;\, -\bar{V} \;\, \bar{V} - \bar{U}\,\big], \\ \Pi_3 &= \begin{bmatrix} \sqrt{c_1}\bar{P}A^T & g\bar{P}A^T \\ 0 & 0 \\ \alpha\sqrt{c_1}\bar{K}^TB^T & \alpha g\bar{K}^TB^T \\ 0 & 0 \\ (1-\alpha)\sqrt{c_1}\bar{K}^TB^T & (1-\alpha)g\bar{K}^TB^T \end{bmatrix}, \\ \Pi_4 &= \big[\, \sqrt{c_1}\bar{S} \;\, \sqrt{c_1}\bar{W} \;\, g\bar{U} \;\, g\bar{V}\,\big], \\ \Pi_5 &= \mathrm{diag}\{\bar{R}_1 - 2\bar{P}, \bar{R}_1 - 2\bar{P}, \bar{R}_2 - 2\bar{P}, \bar{R}_2 - 2\bar{P}\}, \\ \Pi_6 &= \mathrm{diag}\{-\bar{R}_1, -\bar{R}_2\}, \\ \Pi_7 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ f\sqrt{c_1}\bar{K}^TB^T & fg\bar{K}^TB^T \\ 0 & 0 \\ -f\sqrt{c_1}\bar{K}^TB^T & -fg\bar{K}^TB^T \end{bmatrix}. \end{aligned} \tag{20}$$

Moreover, if the above condition is feasible, a desired controller gain matrix is given by

$$K = \bar{K}\bar{P}^{-1}. \tag{21}$$

*Remark 1.* Theorem 2 presents a sufficient condition for the state-feedback controller design which guarantees the exponentially stability in system (6). By using a LMI approach, the sufficient condition is derived by a congruence transformation and some changes of matrix variables. The details of the proof can be found in the full version of the paper.

## 3. ILLUSTRATIVE EXAMPLE

In this section, an example is provided to illustrate the results developed above.

**Example 1**. Consider a satellite system with parameters as follows:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.3 & 0.3 & -0.004 & 0.004 \\ 0.3 & -0.3 & 0.004 & -0.004 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

The eigenvalues of $A$ are $-0.004+0.7746j$, $-0.004-0.7746j$, 0, 0; thus the above system is not stable. It is assumed that we have two sampling periods, with probabilities given by

$$\text{Prob}\{c_1 = 0.1 \text{ s}\} = 0.9, \quad \text{Prob}\{c_2 = 1 \text{ s}\} = 0.1. \tag{22}$$

By using Theorem 2, we obtain the following matrices:

$$\bar{P} = \begin{bmatrix} 3.2253 & 2.1707 & -0.6695 & -0.9542 \\ 2.1707 & 4.7945 & 0.8466 & -0.3548 \\ -0.6695 & 0.8466 & 1.4630 & -0.0296 \\ -0.9542 & -0.3548 & -0.0296 & 0.8621 \end{bmatrix},$$

$$\bar{K} = [\, -0.1964 \ \ -1.0056 \ \ -1.4743 \ \ 0.3194 \,].$$

Therefore, according to (21), the gain matrix for the state-feedback controller is given by

$$K = [\, -0.7572 \ \ 0.3861 \ \ -1.5850 \ \ -0.3631 \,].$$

We illustrate that the closed-loop system is exponentially stable in the mean square under the above obtained controller. The initial condition is assumed to be $[\, -0.4 \ \ 0.1 \ \ -0.7 \ \ 0.5 \,]^T$. The state responses are depicted in Fig. 1, from which we can see that all four state components converge to zero, showing the effectiveness of the controller design.

By calculation according to Theorem 1, this system is exponentially stable under any single sampling period less than 0.8 s. In the following, we will show that when the probability is taken into consideration, the maximum value $c_2$ could be much larger such that the system is exponentially stabilizable. Firstly, we assume that $c_1 = 0.1$, and we are interested in finding the maximum value of $c_2$ by Theorem 1, for different values of $\beta$, such that the system is exponentially stabilizable. The results are listed in Table 1.

| $\beta$ | 0.9 | 0.8 | 0.7 | 0.6 | 0.5 | 0.4 |
|---|---|---|---|---|---|---|
| $c_2$ | 2 | 1.6 | 1.4 | 1.2 | 1.1 | 1.0 |

Table 1. The upper bounds of $c_2$ for different $\beta$ when $c_1 = 0.1$

Then, we assume that the probability is fixed with $\beta = 0.9$, and we will use Theorem 1 to calculate the maximum value of $c_2$ for different values of $c_1$, such that the system is exponentially stabilizable. The results are listed in Table 2.

| $c_1$ | 0.01 | 0.05 | 0.1 | 0.2 | 0.4 | 0.7 |
|---|---|---|---|---|---|---|
| $c_2$ | 2.2 | 2.1 | 2 | 2 | 1.5 | 1 |

Table 2. The upper bounds of $c_2$ for different $c_1$ when $\beta = 0.9$

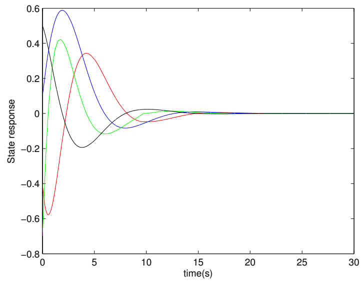


Fig. 1. State response

From these tables, we can see that when the probability is taken into consideration, the maximum sampling period may be larger compared with the single sampling case.

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