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Supply Network Dynamics and Delays; Performance, Synchronization, Stability ⋆

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Abstract: Recent research results in operations research specifically emphasize the critical role of delays in the functionality of supply networks. Delays arise due to the time needed for material deliveries, information flow and human perception towards adjusting to new decisions. Delays contaminate decision-making and lead to poor performance, synchronization problems and fluctuations in inventory levels resulting in major economical losses. This paper surveys continuous time deterministic mathematical models developed at system-level for supply network dynamics along with standard delay models pertaining to material deliveries, information flows and human perception. Next, the analogy between such delay models arising in supply networks and other real-life applications is pointed out. It is foreseen that complexity of the problem requires multi-disciplinary research bridging operations research, business, systems and control engineering, and mathematics. The paper concludes with an illustrative example and discussions of specific challenges anticipated in future research.

Keywords: modeling, supply networks, delay, inventory regulation, stability

1. INTRODUCTION

Supply networks, Forrester [1961], Simchi-Levi et al. [2003], Helbing et al. [2006], Helbing [2003], Sterman [2000] can be seen as interconnected dynamics of customers, suppliers, manufacturing units, companies and sources. While supplies flow along the directed links of these networks to satisfy the changing demand of customers, the information concerning the product orders flows in the opposite direction. One of the main objectives in supply network management is to control production rates and maintain steady inventories while responding to customer demands. Although this seems to be a simplistic proposition, supply network management is known to be a challenge, The Economist [2002].

There are numerous constraints inherent to the physics of the supply network. First of all, decision-making units (managers) tend to wait enough time before they order more/less supplies when demand changes Sterman [2000]. This wait time naturally occurs due to collection of necessary data to conclude a decision and perception of human behavior towards deciding a new command, Sterman [1989b]. Second, the adaptation of supplies and their transportation are not instantaneous, but need certain period of time, *only* after which supplies can meet with customers Riddalls et al. [2002a,b], Sipahi et al. [2008], Warburton, [2004].

The above arguments are equivalent to the following. What is currently occurring in the network is the *after-effects* of what has happened earlier. Consequently, any decision based only on what is currently observed in the network is likely to be unsuccessful as observations also represent effects of the past. The source of after-effects or *delays* is due to presence of **(a)** inventories, **(b)** transportation paths, **(c)** information flow and **(d)** decision-making in the network, Sterman [1989a], Riddalls et al. [2002a,b], Sipahi et al. [2008], Sterman [2000]. Undesirable effects of delays are very well known in operations research, business and control communities, Niculescu et al. [2004], Sterman [2000]. Delays lead to *oscillations, limit cycles, overshoot*, *excessive/depleted* inventories and *synchronization* problems across parallel-running processes, Agrawal et al. [2001], Ceroni et al. [2005]; and these effects may cost companies billions of dollars, CIO Magazine [2001], Agrawal et al. [2001].

Furthermore, depending on system structure and decision-making strategies, small delays may be the source of severe detrimental effects to system behavior, whereas large delays may stabilize a system. Although counter-intuitive, such behavioral classifications are known to exist, Niculescu et al. [2004], Hale [1993]. Clearly, one can conclude that "intuition alone" may be misleading to explain the effects of "large" and "small" delays in dynamical behavior of systems. This clearly justifies the five-decade research efforts in the field of *time delay systems.* In order to avoid the detrimental effects of delays, it becomes necessary to understand the dynamics of interactions between supply-demand points by developing mathematical models *considering delays.* In this paper, we survey deterministic continuous-time supply network models (Section 2), which are in the form of differential equations and have been broadly studied in the literature Sterman [2000], Riddalls et al. [2002a,b], Sipahi et al. [2008], Warburton, [2004]. Furthermore, various math-

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ematical models of delays depending on the physics of supply networks are covered (Section 3). It is foreseen that this survey will further motivate research in the area of supply network management "with delays". Moreover, the interconnection between these models and system-level approach particularly in the field of *time delay systems* (TDS) are discussed along with an illustrative example demonstrating the connection between system dynamics and supply networks management (Section 4). In Section 5, we conclude the paper by pointing out future research at the intersection of operations research, engineering and mathematics.

2. MATHEMATICAL MODELING OF SUPPLY NETWORKS WITHOUT DELAYS

In this section, we briefly present some ideas for mathematical modeling of supply networks developed in the literature, Helbing et al. [2004b], Sterman [1989a], Riddalls et al. [2002a]. The following main components of the network play role in the development of these models,

- inventories, communication medium, decision-making & human-in-the-loop dynamics, production/supplies, transportation medium,

where inventories and decision-making are the main components giving rise to mathematical models, as it is often the case in the literature. The remaining components, as we shall discuss in Section 3, will reveal further details on the supply network dynamics especially concerning the *delays*.

2.1 Helbing's model, Helbing et al. [2004b]

This model considers a supply network of n suppliers i delivering products to other suppliers μ or to costumers, Figure 1. The rate at which supplier i delivers products to and consumes product from supplier μ is given by $d_{\mu i}X_i(t)$ and $c_{i\mu}X_i(t)$, respectively, where $X_i(t) > 0$ denotes the production rate. The coefficients $c_{i\mu}$ define an input matrix $\mathbf{C}$ and $d_{i\mu}$ an output matrix $\mathbf{D}$ with $0 \leq d_{i\mu}, c_{i\mu} \leq 1$.

Inventories. The inventory level $N_i(t)$ represented by a bowl at supplier i changes at the rate

$$\frac{d}{dt}N_i = \sum_{\mu=1}^{n_1} (d_{i\mu} - c_{i\mu})\, X_\mu(t) + Y_i(t), \quad i = 1, \ldots, n_2, \quad (1)$$

where the external demand is denoted by $Y_i(t)$. In order to keep the inventory at some desired level $\overline{N}_i$, any changes in the demand $Y_i(t)$ require an *adaptation* of the production rates $X_i(t)$.

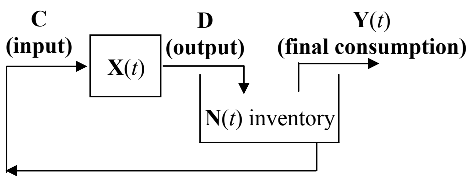

Fig. 1. Supply network model proposed by Helbing et al. [2004b].

Decision-making. The adaptation is represented by the time constant T_μ, which defines the measure of speed of actual production rate $X_\mu(t)$ converging to a desired one $W_\mu(t)$

$$\frac{dX_\mu(t)}{dt} = \frac{1}{T_\mu}\left(W_\mu(\{N_i(t)\}, \{dN_i(t)/dt\}) - X_\mu(t)\right), \quad (2)$$

which concludes the foundation of Helbing's model (1)-(2).

2.2 Sterman's model, Sterman [1989a]

Among various models of Sterman, a fundamental one arising in a stock acquisition system is given below. Different than Helbing's model, Sterman [1] utilizes two *sequential* bowls representing the supply line SL and inventory (or stock), S, respectively. The rate at which supplies being delivered from SL to S is the acquisition rate, A, and the rate supplies leaving S is called loss rate, L.

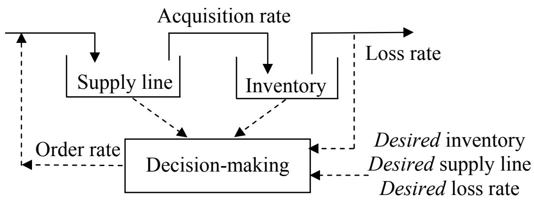

Fig. 2. Stock (inventory) acquisition model proposed by Sterman [1989a].

Inventories. The equations defining the dynamics of the inventory and supply line form as follows

$$S(t) = \int_{t_0}^{t} (A(\kappa) - L(\kappa))d\kappa + S(t_0), \quad (3)$$

$$SL(t) = \int_{t_0}^{t} (O(\kappa) - A(\kappa))d\kappa + SL(t_0). \quad (4)$$

where $O(\kappa)$ is the order rate.

Decision-making. The decision-making utilizes the information concerning related to SL, S and L. Furthermore, desired supply line, SL^*, desired inventory S^* and expected loss rate $\hat{L}$, which can be constant or time-varying, are used for comparison with SL, S and L, respectively. This comparison is necessary to re-formulate the order rate O to correct the actual SL, S and L towards maintaining SL^*, S^* and $\hat{L}$. The order rate strategy is formulated as,

$$O(t) = \max(0, IO(t)), \quad (5)$$

$$IO(t) = \hat{L} + AS(t) + ASL(t), \quad (6)$$

$$AS(t) = \alpha_S(S^*(t) - S(t)), \quad (7)$$

$$ASL(t) = \alpha_{SL}(SL^*(t) - SL(t)), \quad (8)$$

$$\hat{L} = L^*, \quad (9)$$

where $IO(t)$ is the indicated order rate, 'maximum' between zero and $IO(t)$ assures that $O(t)$ is nonnegative, α_S is the stock adjustment parameter, α_{SL} is the fractional

[1] We also note that delays are a part of Sterman's model, however, to maintain the coherency of this section, delays will be discussed in the following section.

adjustment rate for the supply line, AS is the actual stock and ASL is the actual supply line.

2.3 Riddalls's model, Riddalls et al. [2002a]

The work in Riddalls explicitly incorporates pure time delays in the model. In this work, the inventory dynamics is taken as in (3), while assuming $A(t) = O(t)$ thus disregarding (4). In modeling decision-making, Riddalls utilizes (5)-(8), but modifies (9) by proposing a short term forecast/trend detector [2]

$$\hat{L} = \frac{1}{T}\int_{t-T}^{t} L(\kappa)d\kappa, \tag{10}$$

where T is a period of time. The above equation suggests that the expectation is the average of the integration of loss rate over the period T. In order to maintain a consistent flow here, we will discuss the presence of time delays in Riddalls's formulation later in a devoted section. Riddalls's model, *without delays*, becomes

$$\frac{dO(t)}{dt} = \alpha_S(L(t) - O(t)), \tag{11}$$

It is important to note that the work in Warburton, [2004] also utilizes Riddalls's model with a single delay in analyzing inventory dynamics. For more detailed models, see also Helbing [2003], Helbing et al. [2004a], Helbing [2003], Nagatani et al. [2004], Riddalls et al. [2002b], Sterman [2000] and the references therein.

3. MATHEMATICAL MODELING OF DELAYS IN SUPPLY NETWORKS

For accurate understanding of supply network dynamics, it is crucial to model the delays based on the physics they originate from, see Sipahi et al. [2008], Sterman [2000, 1989a], Riddalls et al. [2002a,b], Warburton, [2004]. Delays originate due to transportation of materials/supplies, flow/distribution of energy, communication with technological constraints, lead times for machine set-up and human behavior. As we explain in the sequel, the underlying physics of these components and ultimately their delay modeling are different. It is critical to state that various dynamical systems with different delay models are also studied in the field of *time delay systems*, Niculescu et al. [2004], Richard [2003], Stepan [1989]. Therefore, we also aim to point out the interconnection between supply networks and time delay systems community.

3.1 Constant delay model:

This model assumes that delay $\tau > 0$ is constant. In operations research, it is also known as *pipeline delay*, while in time delay systems it is also called *discrete delay*. An inflow $i(t)$ through a constant delay model will create an outflow $o(t)$, where $o(t) = i(t-\tau)$, Figure 3. In supply networks, this class of delay model represents

- human as decision-maker: waiting the trends in the network before a new decision, updating of beliefs, adjusting towards a new decision, communication, data collection and measurement times, machine set-up lead times, material flow in assembly lines.

[2] In order to enable easier comparison, we adopt Sterman's notation to express Riddalls work.

Discrete delay is also used in Riddalls model where $O(t)$ is replaced by $O(t-\tau)$ in (11). This type of delays is also widely seen in traffic flow behavior, Treiber et al. [2006]; machine tool chatter, Stepan [1989]; multi-agent consensus/synchronization problems, Ren et al. [2005]; teleoperation, active vibration suppression, Niculescu et al. [2004]. Note however that constant delay only models FIFO (first in, first out) type behavior in supply networks, but it does not consider any *mixing*, which might be needed in biological systems and chemical processes. Incorporating the effects of mixing requires the utilization of distribution functions, as we explain below.

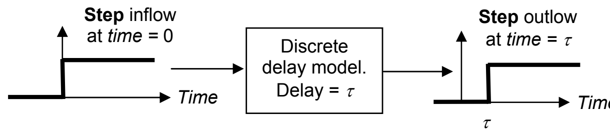

Fig. 3. Discrete delay modeling and its effects between an input and an output.

3.2 Distributed delay model:

In many cases, this type of delay models is used for

- material delivery delays, mixing of materials, diffusion in social networks, chemical and biological systems, energy flow delays.

"Distributed" indicates that materials being delivered do not arrive to their destination all at once, but rather in a distributed fashion along the time. Some examples are exponential, gamma (γ) and Erlang distributions, which also arise in in biology, Kuang [1993]; machine tool chatter, Stepan [1989]; traffic flow, Sipahi et al. [2007] and in chemical process control, Niculescu [2001]. An example supply network with distributed delays is mass mailing, which corresponds to a pulse, while delivery of these mails to various destinations will not be at the same time, thus they will exhibit a distribution with respect to delivery time, Sterman [2000], similar to those given in Figure 4. Furthermore, notice in this figure that distribution functions are depicted with a dead-time h, which is nothing but a discrete delay after which deliveries start to arrive at their destinations. We notice that when dead-time is zero, one will recover those distributed delay models presented in the cited references. For instance, n^{th} order Erlang distribution, which is closely related to a gamma distribution, is given by

$$p(\kappa) = \frac{(n/D)^n}{(n-1)!}\kappa^{n-1}e^{-\frac{n}{D}\kappa}, \tag{12}$$

where $\kappa > 0$ is the delivery time and D is the outflow average (mean of the distribution) in Figure 4; and when $n = 1$, Eq.(12) becomes an exponential distribution.

We finally wish to remark that, when $h = 0$, the outflow *average* in Figure 4 is also called as pipeline delay, which is what this distribution converges to as its variance becomes zero.

3.3 Other delay models:

Other delay models comprise time-varying and state-dependent delays. Time-varying delay $\tau = \tau(t)$ creates

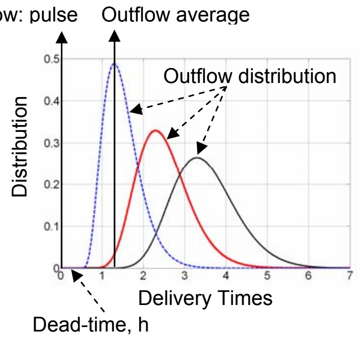

Fig. 4. Distributed delay modeling and its effects between an input pulse and output.

an outflow $o(t) = i(t - \tau(t))$ for an inflow $i(t)$. The time dependency in the delay may take into account the uncertainties in delivery times, possible changes in delivery routes and interruption of deliveries, Riddalls et al. [2002a]. State-dependent delays can be seen as delays associated with what the inventories, acquisition rates, loss rates are at an instant. For instance, it takes time to load materials on a truck, which will add delay in the process, however, this delay is dependent on the loss rate of the inventory and the instantaneous available capacity for loading. Finally, we state that delays may also act as multipliers of system states. For instance, the desired supply line $SL^*(t)$ in (8) requires an adaptation, which depends on the desired throughput Φ^* and an expected delay $\hat{\tau}$ as $SL^*(t) = \Phi^*\hat{\tau}$. This indicates that if a retailer wants to receive 100 items of a product per day and delivery takes 5 days, then 500 items should always be on order so that the retailer does not experience any interruption of deliveries at the desired rate, Sterman [1989a].

4. CONNECTION WITH SYSTEM-LEVEL APPROACH

Although the first sight might think that operations research and business fields independently progress from control theory, this is not the case. One can find numerous successful "systems thinking" approaches for understanding supply network dynamics, see Dejonckheere et al. [2002], Simchi-Levi et al. [2003], Sterman [2000, 1989a], Helbing et al. [2004b], Riddalls et al. [2002a], Warburton, [2004] and the references therein. At system-level, one can consider the supply-network dynamics as a connection of block diagrams representing suppliers (feed-forward line) along with transportation lines for material deliveries and information flow forming the *feedback* line, Figure 5.

4.1 Equilibrium Dynamics

As it is often implicitly needed in queuing theory, the tendency of supply network dynamics around an an equilibrium state is of interest, Dejonckheere et al. [2002], Helbing et al. [2004b], Riddalls et al. [2002a,b], Sipahi et al. [2008], Warburton, [2004]. The linearized dynamics, obtained from the non-linear one, carries rich information as to the fate of the inventory levels and how managerial decisions might be appropriately given in order to main-

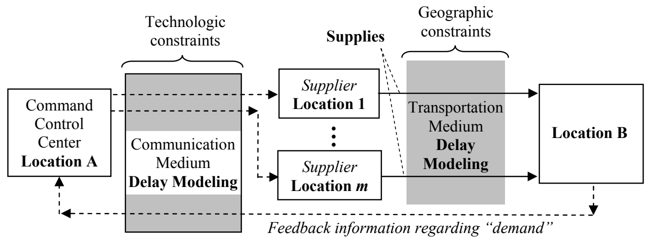

Fig. 5. Supply network with feed-forward and feedback lines. Interpretation from system-level perspective.

tain the inventories at steady levels. The insight to this can be extracted by analyzing the stability.

Stability The stability analysis requires the study of a class of differential equations that contains delays. This analysis can be quite cumbersome due to presence of delays and this explains why research along this line is still active. Details on this topic is beyond the scope of this paper. We direct interested readers to Gu [2005], Michiels [2002], Olgac et al. [2002], Richard [2003], Sipahi et al. [2005], Stepan [1989]. The main challenge in stability analysis even in the linear case arises due to need for analyzing *infinitely many* dynamic modes even if supply network dynamics possesses finite degrees of freedom. This, in summary, means that one needs to study the locations of infinitely many eigenvalues on the complex plane for the delays, τ, in order to assess whether or not the network is stable or unstable.

Performance. If the local stability holds, the minimum of the distances of the eigenvalues to the imaginary axis, $\min(|\Re(\lambda_i)|)$, is a measure of disturbance rejection speed of the dynamics. Smaller $\min(|\Re(\lambda_i)|)$ is, longer it takes to damp out the effects of disturbances, Michiels [2002].

Bullwhip Effects. Assume that a production unit receives periodic orders with a frequency ω and an amplitude $v_d > 0$, $v_d \sin(\omega t)$. For linear dynamics, the output of the production unit is a supply response in the form of $v_s \sin(\omega t + \phi)$ where $v_s > 0$ is the amplitude of the supply variation, and ϕ is the relative phase difference between what is demanded and what is supplied. If $v_s > v_d$ then a combination of similar production units creates supplies with increasing amplitudes. This phenomenon is known as *Bullwhip Effects* [3]. Obviously, this undesirable behavior should be avoided by appropriately designing the network so that $v_s < v_d$ holds in a certain excitation frequency range and with respect to production rates. See Sipahi et al. [2008] for the effects of delays to bullwhip effects.

Sequential/Parallel Running Processes For the particular lay out of the supply network, some processes run sequentially, while some others run in parallel. For instance, the supply line and the inventory in Sterman's model are sequential whereas some processes need to run in parallel for synchronization, Ceroni et al. [2005]. From analyzing the stability point of view, this makes a major difference in the way the delays in these processes couple with each other. In the case of sequentially connected

[3] In the literature, bullwhip phenomenon is called as 'slinky effects' in control theory, Niculescu [2001]; 'chain/string stability' in traffic flow research, Treiber et al. [2006].

delayed processes, delays between the beginning and the end of a chain of processes are equal to the summation of delays in each process. However, in parallel connected processes, the delays cross-talk with each other and lead to synchronization problems.

Remark. Network Induced Instability: Around an equilibrium, each member of the network can be designed to exhibit stable over-damped response characteristics. However, it is recently shown that overall network behavior with these members may behave dramatically differently, e.g. in a low-damped, oscillatory or unstable regime, Helbing et al. [2004b].

4.2 Case Study

In order to present the outcomes of utilizing system-level approach, we borrow the mathematical modeling from Riddalls et al. [2002a]. Recall that this model takes into account discrete delay h which arises due to production lead times. We modify this model slightly by considering that acquisition rate $A(t)$ [4] does not instantaneously affect the inventory. Such an assumption takes into account the transportation times, τ, that will arise between the geographic locations of production and inventory. This time is the delay; what is observed about inventory levels exhibits the effects of acquisition rate that occurred τ time units ago. Our aim is to analyze the robust stability of the inventory level against such transportation delay, τ. Although τ is in the feed-forward path of the supply chain, it will induce instability since the information regarding the inventory will be fed-back, coupling with the decision-making. Consequently, the governing dynamics comprise the equations (3), (6)-(8), (11) and additionally the acquisition rate $A(t)$ in (3) will be modified as $A(t-\tau)$.

In summary, we wish to perform stability analysis with respect to delays, which is a part of the analysis in the work of Riddalls et al. [2002a]. The main difference here is the additional parameter τ that will complicate the stability analysis. As opposed to a single delay h, the stability analysis here should be performed in a two-dimensional delay parameter space h versus τ. Without going into details, we give in the following the homogenous part of the governing dynamics (delay differential equation) over which the stability should be studied with respect to h and τ,

$$\frac{dO(t)}{dt} = -\alpha_S O(t-\tau-h) - \alpha_{SL}(O(t) - O(t-h)). \quad (13)$$

From (13), the characteristic equation is obtained as,

$$f(s,h,\tau) = s + \alpha_S e^{-(\tau+h)s} + \alpha_{SL}(1 - e^{-hs}) = 0. \quad (14)$$

We remark that the complete stability analysis of the characteristic function above is not trivial due to presence of two delays τ and h. Starting from 1989s, various analytical techniques corresponding to necessary and sufficient conditions of stability in the delay parameter space have been developed, Stepan [1989], Hale et al. [1993], Gu [2005], Sipahi et al. [2005]. We by-pass the details on these techniques and direct the readers to the cited references. Utilizing the ideas given in the work of Sipahi et al. [2005], we compute the stability *regions* (gray shaded) of supply network dynamics on the first quadrant of h versus τ plane, Figure 6, for the choice of $\alpha_S = 0.2$ and $\alpha_{SL} = 0.2$.

[4] Recall that we adapt Sterman's notation to maintain easier comparison among models.

In Figure 6, the shaded region is the stability region, while the remaining regions indicate instability. Furthermore, the curves separating stable and unstable behavior correspond to the locations in τ and h for which the dynamics becomes a perfect oscillator, see Gu [2005], Sipahi et al. [2005] for further discussions on these curves. In this figure, along the h-axis, i.e. when $\tau = 0$, one recovers the study in Riddalls et al. [2002a], while for $\tau \neq 0$ the effects of transportation delays can be seen. We wish to point out that the stability region has an intricate geometry which may serve counter-intuitive. For instance, when $h = 18$ and $\tau = 20$, the supply chain dynamics is stable, while *decreasing* τ down to $\tau = 10$ will create *instability*. Furthermore, it is

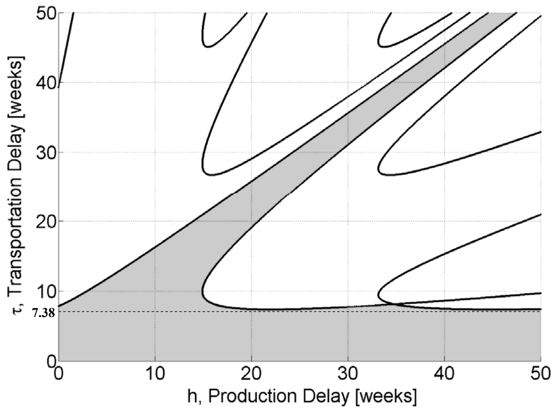

Fig. 6. Stability of the supply network dynamics in the parameter space of the delays.

interesting to observe that for $\tau < 7.38$, the dynamics is stable independent of the choice of h, while on the other hand for any $\tau > 7.38$, the supply chain manager should be careful, since stable and unstable behavior is possible depending on the choice of h and τ.

Borrowing from Riddalls et al. [2002a], the inventories $i(t)$ are expressed as $\frac{di(t)}{dt} = O(t-h) - d(t)$, where $d(t)$ is the demand (part of the non-homogeneous terms). Deploying non-homogenous part of the dynamics from the cited work, we present in Figure 7 how inventories (where $i(0) = 200$) behave in response to a step change in *demand* (20 units increase) for the two choices **(a)** $h = 6$, $\tau = 0$ (the case analyzed by Riddalls) and **(b)** $h = 6$, $\tau = 2$ (the new case studied here). The remaining parameters, which have no effect on the stability, remain the same as in Riddalls et al. [2002a].

The simulations in Figure 7 indicate that taking into account the effects of transportation delays may make the inventories more prone to oscillations and even towards their depletion.

5. CONCLUSION

Continuous time deterministic mathematical models of supply network dynamics are surveyed along with models of delays arising in these networks due to production lead times, material deliveries, information and decision lags and transportation times. These models are in the

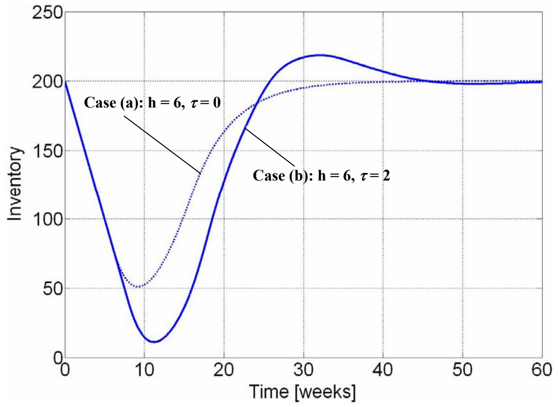

Fig. 7. Time domain simulation of inventories with production delay $h = 6$ weeks, while transportation delay is either $\tau = 0$ (dotted curve) or $\tau = 2$ (solid curve) weeks.

form of ordinary differential equations representing the behavior of inventory levels, production and demand rates and the delays. The connection of the problem with system dynamics and especially with the field of time delay systems is demonstrated and an illustrative case study in this context is presented. This work is intended to establish a step towards our comprehension of supply network dynamics via such connections. It is foreseen that this effort will motivate multi-disciplinary research and put light on some parts of the field unrevealed so far.

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