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Modeling and adaptive attitude control of observation spacecrafts in view of flexible structure

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Abstract: The paper discusses a computerized method for deriving of angular motion equations of observation spacecrafts, whose structural flexibility is determined by large-size solar battery panels. Reorientation dynamics problems that arise because of structural flexibility and inaccurate settings of parameters of the object model are considered. An adaptive system for control of such space objects is suggested. The system provides a robust control and stability with respect to vibrations. Robustness and stability are achieved by using Kalman filtration of flexible modes and by providing optimal phase oscillation conditions for switching of control actions.

1. INTRODUCTION

Since mid-60s, leading world experts have been paying a close attention to the complex problems of deriving mathematical models of flexible spacecrafts (FS), developing a justified simplification of these models and providing a robust control with respect to a poorly defined parameter vector of a flexible object (Junkins and Kim, 1993), (Kirk (ed.), 1990, 1993, 1996, 1999). The essential control problem with flexible spacecrafts, that is, the interaction of the object's attitude control system with the flexible oscillations of the object's structure still needs to be addressed.

In fact, a motion control of FS often implies a conflict between the main goal of the control of the flexible object as a rigid body and the necessity to restrict the magnitude of structural vibrations that are caused by control actions of the main regulator. There exists a strong tendency to excite vibrations in the control process of the main ("rigid") motion of FS. A level of oscillations that exceeds a critical value leads to a system instability, where the flexible oscillations capture the regulator. The crux of the problem is the lack of information with regards to the state of the flexible body as structural vibration detectors are not available and the mathematical model of the object is often defined poorly.

Observation spacecrafts often perform reorientation maneuvers, which are accompanied by long lasting vibrations that affect the carrying body and blur the pictures generated by satellite surveillance technique. The paper addresses this problem by developing the following tools: (a) computerized method of obtaining 3D-orientation motion equations of observation spacecrafts with respect to a flexibility of solar battery panels; (b) simplifying transformation of the obtained model to a modal-physical form with respect to three almost independent planes; (c) flexible object orientation adaptive control system, which

- provides a robustness with respect to inaccurate settings of FS parameter vector,
- week excitation of flexible structural vibrations,
- minimal vibrations dissipation time after the reorientation is over.

2. COMPUTER AIDED DERIVATION OF MOTION EQUATIONS FOR OBSERVATION SPACECRAFTS IN VIEW OF FLEXIBLE STRUCTURE.

Let us consider the problem of computer aided derivation and transformations of motion equations for observation spacecrafts in view of flexible structures.

We take the kinematical structure of a mechanical system as a main rigid carrying body with s rigid carried bodies that are attached to the main one. At the attaching points there are the springs that imitate the flexibility of the carried bodies. In such a statement the problem is solved in (Zemlyakov *et al.*, 2007). Namely, the mathematical model (MM) of the mechanical system is derived as the equation

$$A(q)\ddot{q}+\sum_{s=1}^{N}\left[\dot{q}^{T}D_{s}(q)\dot{q}\right]e_{s}+Cq=S(q)M \qquad (1)$$

where $q^T=(q_1, q_2,\dots,q_N)$ is the vector of generalized coordinate; $M\in R^3$ is the vector of control moments acting with respect to the main body axes. In (Zemlyakov *et al.*, 2007)concrete mathematical formulas for matrices $A(q)$, $D_s(q)$ $(s=\overline{1,n}), C, S(q)$ are presented for computer calculating.

In this work we will turn our attention on the problem how from MM (1) to go to more simple MM that would be more convenient for a system control synthesis in the case of observation spacecrafts. For this goals we need to find mathematical formulas for: - linearization of the MM (1) taking into account small deviations from a desired motion; - transfer of linearized MM (1) to the normal coordinates; - decomposition of the linearized MM to a series partial MM; - transformation of a partial MM to a modal-physical model (MPM) (Glumov *et al.,* 1998) that is the most convenient for the synthesis and analysis of a control system.

2.1. Linearization of the mathematical model.

The observation spacecraft to be regarded is moving along an orbit. A control system has to guarantee the prescribed dynamic of an orientation. It could be supposed beforehand that deviations of controlled coordinates are small. Then it is possible to linearize the MM (1) relatively desired motions, for example $q = q^0$, $\dot{q} = 0$, where all components of the vector q^0 are zeros except $q_1 = q_1^0 = \mathrm{const} \neq 0$. In this case the MM (1) could be rewritten in the form

$$A\ddot{q} + Cq = SM \tag{2}$$

where $A = A(q)$, $S = S(q)$ for $q = q^0$.

2.2. Transformation of the linearized mathematical model to normal coordinates.

As a rule for the desired observation spacecraft orientation only coordinates of the vector $q_0^T = (q_1, q_2, ..., q_6)$ and their velocities are measured. Then the MM (2) more comfortable to present in the form

$$A\ddot{q} + Cq = SM, \quad q_0 = (E_6 \quad O_{6\times n})q, \tag{3}$$

where $O_{6\times n}$ is $6\times n$ zero matrix, $n = N - 6$.

In (2) and (3) matrices *A* and *C* are positive and negative definite, respectively. Then there exists (Lurye, 1961) nonsingular transformation $q = \Phi s$, that reduces MM (3) to the normal coordinates $s^T = (s_1, s_2, ..., s_N)$. To find the matrix Φ, we represent MM (3) to the form

$$\begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}\begin{pmatrix} \ddot{q}_0 \\ \ddot{q}_c \end{pmatrix} + \begin{pmatrix} O_6 & O_{6\times n} \\ O_{n\times 6} & C_{22} \end{pmatrix}\begin{pmatrix} q_0 \\ q_c \end{pmatrix} = \begin{pmatrix} S_{11}(q) \\ O_{n\times 3} \end{pmatrix} M,$$
$$q_0 = \begin{pmatrix} E_6 & O_{6\times(n-6)} \end{pmatrix}\begin{pmatrix} q_0 \\ q_c \end{pmatrix} \tag{4}$$

where $q^T = (q_0^T, q_c^T)$.

Proposition 1, (Zemlyakov *et al*., 2007). *To reduce MM* (4) *to normal coordinates* s*, one should apply nonsingular transformation* $q = \Phi s$*, where the matrix* Φ *is given by the equality*

$$\Phi = \begin{pmatrix} A_{11}^{-1/2} & -A_{11}^{-1}A_{12}Q \\ O_{(n-6)\times 6} & Q \end{pmatrix},$$
$$Q = a^{-1/2}T, \quad a = A_{22} - A_{12}^T A_{11}^{-1} A_{12}, \tag{5}$$

where the orthogonal matrix T *is obtained from the relation* $T^T a^{-1/2} C_{22} a^{-1/2} T = \Omega$.

In the vector s we note subvectors $s_0^T = (s_1, s_2, ..., s_6)$ and $s_c^T = (s_7, s_8, ..., s_N)$. Then the MM (3) relatively normal coordinates takes the form

$$\begin{pmatrix} \ddot{s}_0 \\ \ddot{s}_c \end{pmatrix} + \begin{pmatrix} O_6 & O_{6\times n} \\ O_{n\times 6} & \Omega \end{pmatrix}\begin{pmatrix} s_0 \\ s_c \end{pmatrix} = \begin{pmatrix} RS_{11}(q) \\ HS_{11}(q) \end{pmatrix} M,$$
$$q_0 = (R \quad H^T)\begin{pmatrix} s_0 \\ s_c \end{pmatrix}, \tag{6}$$

where $R = A_{11}^{-1/2}$, $H = -T^T a^{-1/2} A_{12}^T A_{11}^{-1}$.

Now we introduce for the MM (6) the transformation $\begin{pmatrix} s_0 \\ s_c \end{pmatrix} = \begin{pmatrix} R^{-1} & O_{6\times n} \\ O_{n\times 6} & E_n \end{pmatrix}\begin{pmatrix} \bar{x} \\ s_c \end{pmatrix}$ and as a result we receive a MM that contains a coordinate of stiff motion $\bar{x}$

$$\begin{pmatrix} \ddot{\bar{x}} \\ \ddot{s}_c \end{pmatrix} + \begin{pmatrix} O_6 & O_{6\times n} \\ O_{n\times 6} & \Omega \end{pmatrix}\begin{pmatrix} \bar{x} \\ s_c \end{pmatrix} = \begin{pmatrix} A_{11}^{-1}S_{11}(q) \\ HS_{11}(q) \end{pmatrix} M,$$
$$q_0 = (E_6 \quad H^T)\begin{pmatrix} \bar{x} \\ s_c \end{pmatrix}. \tag{7}$$

2.3. Construction of mathematical models of partial motion with respect to each of measurable coordinates.

With the MM (7) it appears the possibility to separate the partial MM that represents the spatial motion of our mechanical system with respect to only one of the coordinates of the vector q_0, for example, q_i $(i = \overline{1, 6})$

$$\begin{pmatrix} \ddot{\bar{x}}_i \\ \ddot{s}_c \end{pmatrix} + \begin{pmatrix} 0 & O_{1\times n} \\ O_{n\times 1} & \Omega \end{pmatrix}\begin{pmatrix} \bar{x}_i \\ s_c \end{pmatrix} = \begin{pmatrix} a_i \\ HS_{11}(q) \end{pmatrix} M,$$
$$q_i = (1 \quad h^{iT})\begin{pmatrix} \bar{x}_i \\ s_c \end{pmatrix} \tag{8}$$

where $a_i = (A_{11}^{-1})_i S_{11}(q)$, $(A_{11}^{-1})_i$ is the *i*-th row of the matrix A_{11}^{-1}, h^i is the *i*-th column of the matrix H.

2.4. Constructing of modally physical models of partial motions [4].

We consider the diagonal $n\times n$- matrix H^i with diagonal consisting of the components of the column vector h^i, i.e. $H^i = \mathrm{diag}(h_{1i}, h_{2i}, ..., h_{ni})$. Now it is possible to formulate

Proposition 2, (Zemlyakov *et al*., 2007). *The nonsingular transformation* $\begin{pmatrix} \bar{x}_i \\ s_c^i \end{pmatrix} = \begin{pmatrix} 1 & O_n \\ 0 & (H^i)^{-1} \end{pmatrix}\begin{pmatrix} \bar{x}_i \\ \tilde{x}^i \end{pmatrix}$ *allows to receive the modally physical models of partial motions [4] in the form*

$$\begin{pmatrix} \ddot{\bar{x}}_i \\ \ddot{\tilde{x}}^i \end{pmatrix} + \begin{pmatrix} 0 & O_{1\times n} \\ O_{n\times 1} & \Omega \end{pmatrix}\begin{pmatrix} \bar{x}_i \\ \tilde{x}^i \end{pmatrix} = \begin{pmatrix} a_i \\ K^i \end{pmatrix} M,$$
$$q_i = \bar{x}_i + \tilde{x}_i, \quad \tilde{x}_i = \sum_{j=1}^{n} \tilde{x}_j^i, \tag{9}$$

where $K^i = L^{iT} H^i H S_{11}(q)$.

The modal-physical model (MPM) (9) takes into account disturbances to coordinate $q_i \ (i=\overline{1,\ 6})$ from the spatial interconnected motions of all other coordinates $q_j \ (j=\overline{1,\ N},\ \ j\neq i)$. Sometimes such an interconnection so small that it is possible to neglect it. If so then the MPM (9) becomes more simply

$$\ddot{\bar{x}}_i = a_i^i M_i,$$

$$\ddot{\tilde{x}}_j + \omega_j^2 \tilde{x}_j = k_j^i M_i\ , q_i = \bar{x}_i + \tilde{x}_i,\ \ \tilde{x}_i = \sum_{j=1}^{n} \tilde{x}_j^i, (j=\overline{1,\ n}).$$

where a_i^i and k_j^i are components of vector a_i and matrix K^i respectively.

In (Glumov *et al.,* 1998) MPM for one angle coordinate without interconnected disturbances takes the more concrete form

$$\begin{aligned} &(a) \quad \ddot{\bar{x}} = m(u),\ m(u) = M(u) I^{-1}, \\ &(b_i) \quad \ddot{\tilde{x}}_i + \tilde{\omega}_i^2 \tilde{x}_i = \tilde{k}_i m(u),\ i = \overline{1,n}\ , \\ &(c) \quad x = \bar{x} + \tilde{x},\ \tilde{x} = \sum_{i=1}^{n} \tilde{x}_i \end{aligned} \tag{10}$$

where $x = q_i \ \left(i=\overline{1,\ 3}\right)$ is a measured and controlled angular coordinate; $\bar{x}$ is a coordinate of "rigid" motion; $\tilde{x}$ is an additional angle motion of the main body due to the elastic oscillations of connected bodies; I is a central spacecraft moment of inertia; $\tilde{\omega}_i,\ i=\overline{1,n}$ are the fundamental frequencies of elastic oscillations; $\tilde{k}_i$ are coefficients excitability for elastic modes; $M(u)$ is a control action; $u = u(x,\dot{x},t)$ is a control law.

3. SOME PECULIARITIES OF AN INTERCONNECTION OF THE FS ORIENTATION SYSTEM WITH THE STRUCTURE VIBRATIONS

One of the principal problems of the FS orientation is the essential interconnection of the control system with the structure vibrations. This interconnection becomes especially strongly at applying relay or discrete control and at a boundedness of the information about the flexible object's state vector.

It is well-known (Rutkovsky and Sukhanov, 1996) that when the regulator with discrete variable of the control action level is used the vibrations are excited and amplitudes of the elastic modes $\rho_i(t) = |\tilde{x}_i(t)|$ are changed at each instant of the control action switching. In this case, as a rule, a dominant mode $\tilde{x}_d(t),\ d \in i$, is appeared. Its amplitude $\rho_d(t)$ increases more quickly in compare with the others and in some time the inequality $\rho_d(t) \gg \rho_{i\neq d}(t)$ will be correct. So the summary intensity of the vibrations $\rho(t) = \sum |\tilde{x}_i(t)|$ can be estimated as $\rho(t) \approx \rho_d(t)$. In (Rutkovsky and Sukhanov, 1996) it was shown that at $\overset{*}{\rho}_d \gg \mu^m$, $\mu^m \approx |m_{\bar{u}}| \tilde{k}_d \tilde{\omega}_d^{-2}$ ($m_{\bar{u}} \doteq m(\bar{u})$ is the base control that is synthesized if the spacecraft would be rigid) the process of the amplitude $\rho(t)$ changing can be described as follows

$$\rho(t_k) \doteq \rho_k \approx \overset{*}{\rho}_d + \left|\mu^m\right| \cdot \sum_{j=1}^{k} \delta_j \cos\beta_j [\operatorname{sign} \dot{m}_j(u)] \tag{11}$$

where δ_j is the coefficient of the control action level changing at the *j*-th switching, β_j is the corresponding phase of the dominant mode, $\dot{m}_j(u) = \dot{m}(t_j) = dm(u)/dt\Big|_{t=t_j}$.

Consequently, the character of the amplitude ρ_d changing is defined by the totality of the system's state at the switching instants, that is the amplitude ρ_k can increase or decrease in accordance with the $\operatorname{sign} \dot{m}_j(u)$, and by the value of the phase $\beta_j = \beta(t_j)$. As an optimal condition with respect to the phase β_j is the case when the amplitude ρ_k after the switching will be the smallest from all possible ones at prescribed direction of the control action switching $\operatorname{sign} \dot{m}_j(u)$. Optimal conditions are defined by the correlation

$$\beta_j = \begin{cases} 2\pi n & \forall\ \operatorname{sign} \dot{m}_j(u) = +1, \\ \pi(2n+1) & \forall\ \operatorname{sign} \dot{m}_j(u) = -1,\ \ n = 0,1,2,\ldots\ . \end{cases} \tag{12}$$

The changing of the $\operatorname{sign} \dot{m}_j(u)$ to the opposite in (12) leads to the worst condition of switching and leads to the maximal increase of the amplitude ρ_k. All intermediate values of the phase $\beta_j = 2\pi n \vee \pi(2n+1),\ n = 0,1,2,\ldots$ define either favorable (decrease of the amplitude) or unfavorable (increase of the amplitude) conditions of the switching.

Thus, the phases of the vibration amplitudes at the switching instants define the character of the oscillating processes at the FS' control. Let us note that exceeding of the intensity $\rho(t) = |\tilde{x}(t)|$ of a critical level ρ_{cr} (mainly at the expense of the dominant mode $\tilde{x}_d$ growth) leads to the system instability (to the "capture" of the regulator by vibrations).

4. SYNTHESIS OF ADAPTIVE ALGORITHM OF THE FS REORIENTATION

Manoeuvres of the initial orientation and further reorientations at the change of the watched object are the most important regimes of the observation spacecrafts operation. Very often, these manoeuvres are realized with the help of control moments created by flywheels.

Discrete analog of linear PD-algorithm (at each interval of the digitization $T_0 = \text{const}$ corresponding constant moment is applied to the satellite) is used usually in this case. After completion of the initial orientation or reorientation the control system realizes the process of stabilization. One of the wellknown algorithms of relay-logistic type (Raushenbakh and Tokar, 1974) is a base algorithm for this regime. At using the algorithms of such type in the system stable limit cycle takes place that guarantees required accuracy and economics control. In any case discontinuous character of the control actions is the cause of the excitation of the solar panels vibrations.

For the vibrations damping it is possible to use suggested in (Rutkovsky and Sukhanov, 1974) the principle of the phase control of the single-frequency FS (FS with only one elastic mode). This principle is based on the use of the algorithm with precise information about elastic mode phase. Briefly, the essence of this algorithm can be explained like that.

Let $\bar{u} = f_0(x,t)$, $x \doteq \bar{x} + \tilde{x}$, is the base algorithm which is synthesized provided that the spacecraft is rigid. Then the algorithm $u = f_1(\bar{u}, \beta)$ with additional signal about the phase β will be called as an extended one. The switching instant t_j of the control action will be called as phase-controlled if it depends on not only the coordinate $x(t)$ but the phase β also. At this the direction of the control action ($\operatorname{sign} \dot{m}_j(u)$) switching is defined uniquely by the base algorithm but the switching is delayed until the phase $\beta(t)$ will be equal to its optimal value or its favorable one.

Using correlation (11) it is possible to obtain the increment of the elastic mode in the i-th period of the limit cycle containing R switchings of the control action:

$$\Delta\rho_i \doteq \rho_i - \rho_{i-1} \approx \left|\mu^m\right| \cdot \sum_{r=1}^{R} \delta_r \cos\beta_r [\operatorname{sign} \dot{m}_r(u)] \quad (13)$$

It is obvious that for stability of the control system motion it is necessary to have optimal or favorable values of the phases β_r at least at $R/2$ points of the switchings. In this case at $\delta_r = \text{const}\ \forall r \in [1,R]$ the increment $\Delta\rho_j \le 0$ that guarantees damping of the vibrations.

For this algorithm using to orientation of multi-frequency FS with poorly defined parameters and at absence of the elastic modes sensors it is necessary to solve two tasks: 1) to get and realize the algorithm of the dominant mode number identification, 2) synthesize the subsystem of the elastic modes $\tilde{x}_i(t)$ and the object's parameters estimation, that makes it possible to calculate the current value of the phase $\beta(t)$.

4.1 *The algorithm of the dominant mode number identification*

Let $Z = \{z\}$ is the block of the measurements at the discrete instants $t_z \in T_z$. The process of the dominant mode changing can be defined by the function's analysis that is given by blocks Z_m, T_m. These blocks are obtained on the basis of processing of the rectified output signal x^{Σ} of the FS orientation sensors. At the presence of dominant mode the majority of the differences $t_m[l] = \{t_m[l] - t_m[l-1]\}$, that are the adjacent elements of the block $T_m = \{t_m[l]\}$, coincide with the semiperiod of the mode $\tilde{x}_d(t)$, that is $\Delta t_m[l] \approx 0.5\tilde{T}_{dl}$. Using the average $\tilde{T}_d = \frac{2}{L-1}\sum_{l=1}^{L-1} \Delta t_m[l]$, ($L = \dim T_m$) the dominant mode frequency $\omega_d = 2\pi\tilde{T}_d^{-1}$ is defined. For identification of the dominant mode number the differences $\Delta\omega = \left|\omega_i - \omega_d\right|_{(i=\overline{1,n})}$ are investigated and it is assumed $d = i_d$ according to the minimal difference $\Delta\omega_i = \left|\omega_i - \omega_d\right| = \min\limits_i$. Here ω_i, $i = \overline{1,n}$, are known frequencies of the elastic modes that are taken into account. This result is considered as the correct, if the correlation $\Delta\omega_i / \omega_d \le \delta \approx 0{,}01$ is fulfilled.

Availability of this algorithm is illustrated by the example of simulation (Fig. 1) of multi-frequency FS (n=6) motion.

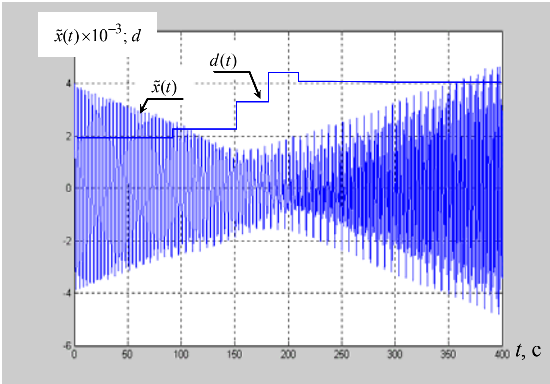

Fig. 1. The example of the dominant mode number identification.

During the observed interval, the second dominant mode $\tilde{x}_{d=2}$ was damped and the fourth one $\tilde{x}_{d=4}$ occurs. This fact is fixed very clearly by step-like changing of the output signal $d(t)$ of the identification subsystem.

4.2 *Estimation of the FS elastic modes by Kalman filter*

In synthesis of the base algorithm for spacecrafts orientation, as a rule, the signals of the attitude sensors and rate sensors are used. The absence of the reliable devices for coordinates $\tilde{x}_i$ measurement is the cause that bounds the use of the suggested approach in synthesis of the FS high-effective control.

This problem and the problem of inaccurate setting parameters of the object can be solved with the help of the method of joint estimation of the parameters and modal-physical coordinates of the FS motion (Sukhanov *et al*., 2003). This method is based on the combination of the Kalman discrete filtration and the theory of the statistical hypothesizes testing.

Taking into account the transducers noises the MPM (10) of the FS in discrete form can be written as follows:

$$\begin{aligned} X_{k+1} &= \Phi_{k+1}(Y_{k+1})X_k + \partial_{k+1}(Y_{k+1})u_k + \\ &+ \Psi_{k+1}(Y_{k+1})W_k, \quad Y_{k+1} = M_{k+1}Y_k, \end{aligned} \quad (14)$$

$$Z_{k+1} = H(Y_{k+1})X_{k+1} + V_{k+1}, \quad (15)$$

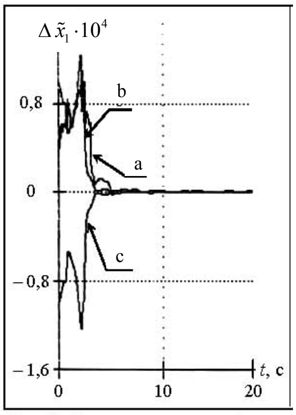

Fig.2,a. The error of estimation of the dominant mode coordinate $\tilde{x}_1$: a) $\Delta\tilde{x}_1$, b) $+2\sigma_{\tilde{x}_1}$, c) $-2\sigma_{\tilde{x}_1}$,

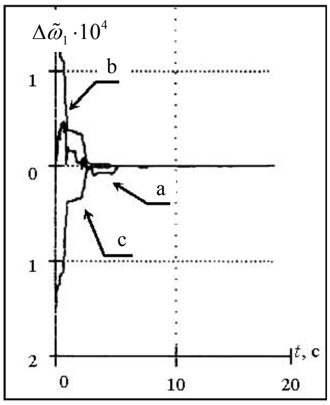

Fig. 2,b. The error of estimation of the dominant mode frequency $\tilde{\omega}_1$: a) $\Delta\tilde{\omega}_1$, b) $+2\sigma_{\omega 1}$, c) $-2\sigma_{\omega 1}$,

where $X_k = (\bar{x}_k, \dot{\bar{x}}_k, \tilde{x}_{1k}, \dot{\tilde{x}}_{1k}, \ldots, \tilde{x}_{nk}, \dot{\tilde{x}}_{nk})^T$ is the coordinate vector, $Y_k = (\omega_{1k}, \tilde{k}_{1k}, \ldots, \omega_{nk}, \tilde{k}_{nk})^T$ is the object parameter vector; $Z_k = (x_k^*, \dot{x}_k^*)^T$ is the measurement vector, W_k and V_k are the noises of the object and measurements. For fixed vector $Y_k = Y$,equations (14) and (15) are linear with respect to the vector X_k. In this case for the joint estimation of the vectors X_k and Y it is possible to use aforementioned method. Taking as the hypothesizes D_j, $j = \overline{1,l}$, the assembly of the parameters Y concrete values, that is $D_j = (y_1^j, \ldots, y_s^j, \ldots, y_{2n}^j)^T$, the optimal estimations of the vectors X_N and Y_N can be obtained from the system of equations for estimations, covariance matrices and functionals (Sukhanov *et al.*, 2003)

$$\hat{X}_{jk+1} = \Phi_{jk+1}\hat{X}_{jk} + \partial_{jk+1}u_k + K_{jk+1}\Delta_{jk+1},$$
$$\Delta_{jk+1} = Z_{k+1} - H(\Phi_{jk+1}\hat{X}_{jk} + \partial_{jk+1}u_k),$$
$$K_{jk+1} = \bar{P}_{jk+1}H^T(H\bar{P}_{jk+1}H^T + R)^{-1};$$
$$\bar{P}_{jk+1} = \Phi_{jk+1}P_{jk}\Phi^T_{jk+1} + \Psi_{jk+1}Q\Psi^T_{jk+1},$$
$$P_{jk+1} = \bar{P}_{jk+1} - \bar{P}_{jk+1}H^T(H\bar{P}_{jk+1}H^T + R)^{-1}H\bar{P}_{jk+1};$$
$$I_{jk+1} = I_{jk} + \Delta^T_{jk+1}\sum\nolimits^{-1}_{jk+1}\Delta_{jk+1} + \varepsilon_{jk+1},\ \varepsilon_{jk+1} =$$
$$= \ln\left(|\bar{P}_{jk+1}|\,|H\bar{P}_{jk+1}H^T + R|/|P_{jk+1}|\right),$$
$$\sum\nolimits^{-1}_{jk+1} = \left[E - (H\bar{P}_{jk+1}H^T + R)^{-1}H\bar{P}_{jk+1}H^T\right]\times$$
$$\times(H\bar{P}_{jk+1}H^T + R)^{-1}. \qquad (16)$$

In Fig. 2 the example of the simulation of the estimation processes of the object's parameters and the dominant mode is shown. It was assumed that for each number of the hypothesizes $j = \overline{1,l}$, the vector Y components are constant, that is $Y_j = const\ \forall j \in [1,l]$. The initial values of the vector Y_j components y_{ji} were chosen between their possible maximal and minimal values for $j = \overline{1,l}$.

Further they are changed discretely with corresponding digitization steps. Using equations (16) the final values of the functionals I_{jN} were calculated. The hypothesis with the number ν was chosen as the most probable where ν corresponds to the minimum of the functional $I_{\nu N} = \min_j I_{jN}$.

The estimation $\hat{X}_{\nu N}$ is the optimal one of the vector X_N and the components of the vector Y_ν are the estimations of the vector Y components.

Fig. 2 is shown that at initial displacements $\hat{\tilde{\omega}}_i(0) \le 1{,}4\tilde{\omega}_i(0)$ it is guaranteed not only admissible rate of the estimations convergence but their high accuracy. This information is sufficient for the dominant mode current phase $\beta(t)$ calculation at any instant.

4.3 Adaptive system of the FS orientation

On the basis of the dominant node number identification and calculation current value of the phase $\beta_d(t)$ the task of the FS orientation system design is solved.

Block-scheme of the adaptive orientation system, that is realized phase control of the multi-frequency FS, is shown in Fig. 3.

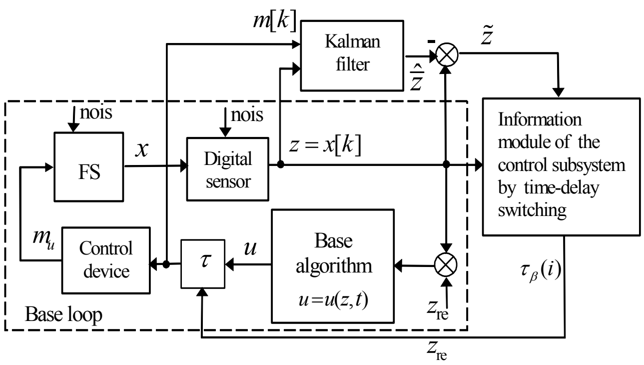

Fig. 3. Block-scheme of adaptive control system for FS.

This system can guarantee high accuracy of the reorientation, stabilization with respect to a new direction and damping of the constriction vibrations. In Fig. 3 z_{re} is the signal of reorientation. The control loop of the main ("rigid") FS motion is depicted by a dot line. It includes an additional link that realizes the time-delay of the control action switching until the phase β_d will be as optimal. The value of the phase β_d is calculated in the informational module of the time-delay switching subsystem. In this module, the time-delay $\tau = t^*_{sw}(\beta_d) - t_{sw}$ is calculated also. Here t^*_{sw} is the instant, when the phase β_d is equal to its optimal value, t_{sw} is the instant of the control action switching according to the base algorithm $u = u(z,t)$. So $u(t) = \bar{u}(t - \tau)$.

After reorientation of the satellite, the system passes to the regime of the controlled coordinate x stabilization with respect to the new position of an observation axis. At thus corresponding stable limit cycle is formed. In half of its switching points the optimal phases β realized as it was suggested early.
In fig. 4 the example of the adaptive system operation is shown. The processes of stabilization ($0 < t < 300\,s$ and $400\,s \le t < 800\,s$) and reorientation ($300\,s \le t < 400\,s$) simulated.

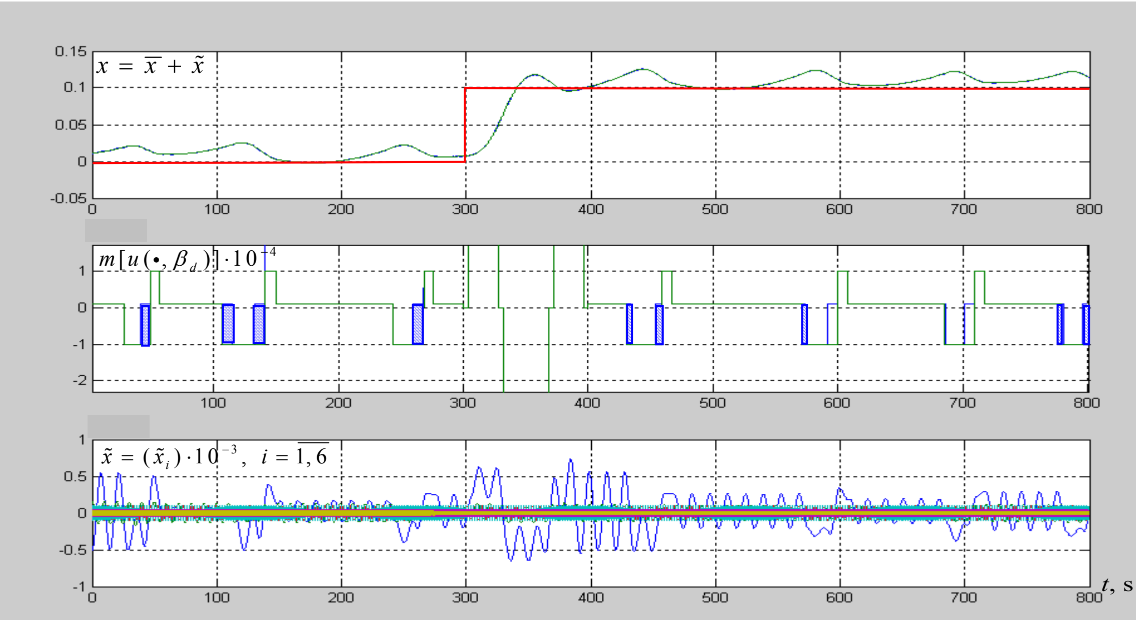

Fig. 4. Transient processes in regimes of the FS stabilization and reorientation at use of adaptive phase control.

The intervals of time-delay of control actions switching are shaded. From graphs it is clear that realization of the optimal conditions switching with respect to the phase of dominant mode guaranties the vibrations damping before and after reorientation without additional consumption of energy.

5. CONCLUSION

Suggested approach in the designing of the FS control system realizes the adaptive tuning of the base algorithm with a view to get optimal phase of the dominant mode in the instants of the control action switching. This guarantees the damping of the construction vibrations in the case of poorly defined object mathematical model without additional consumption of energy for control.

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