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Attenuation of Slugging in Unstable Oil Wells by Nonlinear Control

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Abstract: This paper illustrates the potential of nonlinear model-based control applied for stabilization of unstable flow in oil wells. A simple empirical model is developed that describes the qualitative behavior of the downhole pressure during severe riser slugging. A nonlinear controller is designed by an integrator backstepping approach, and stabilization for open-loop unstable pressure setpoints is demonstrated. The proposed backstepping controller is shown in simulations to perform better than PI and PD controllers for low pressure setpoints, and is in addition easier to tune. Operation at a low pressure setpoint is desirable since it corresponds to a high production flow rate.

Keywords: Nonlinear control, slugging, backstepping, stabilization.

1. INTRODUCTION

Multiphase flow instabilities present in all phases of the lifetime of a field, however, the likelihood for multiphase flow instabilities increases when entering tail production. In tail production, *i.e.* oil production from mature fields where the reservoir is about to be drained, unstable multiphase flow from wells or severe slugging is an increasing problem. In particular, unstable flow causes reduced production and oil recovery as the well must be choked down for the downstream processing equipment on the platforms to be able to handle the resulting variations in liquid and gas flow rates.

Research on handling severe slugging in unstable wells has received much attention in the literature and in the industry, such as Pickering et al. [2001], Storkaas [2005]. The schematic of the severe slugging cyclic behavior is shown in Figure 1. The active control of the production choke at the well head is used to stabilize or reduce these instabilities. The motivation for using active feedback control is that one can operate the pipeline/well in an unstable operating region, where the system is open-loop unstable. Several publications use the active feedback control to stabilize the flow, see for examples, [Henriot et al., 1999, Drengstig and Magndal, 2002, Molyneux et al., 2000, Dalsmo et al., 2002, Kinvig and Molyneux, 2001, Godhavn et al., 2005, Storkaas, 2005, Siahaan et al., 2005, Storkaas and Skogestad, 2007]. Some works used a detailed model and only proved stability linearly, whereas Siahaan et al. [2005] proved nonlinear stability with a simplified model.

This paper illustrates the potential of nonlinear model-based control applied to stabilize unstable flow in wells.

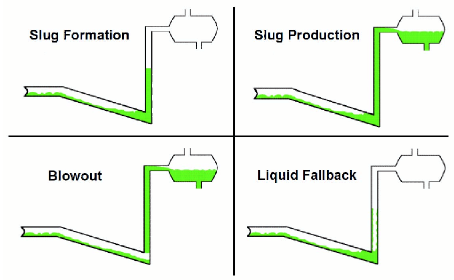

Fig. 1. Schematics of the severe slug cycle in flowline riser systems Pickering et al. [2001]

A simple empirical model is developed that describes the qualitative behavior of the downhole pressure in case of severe slugging in unstable wells. The model is used to develop a model-based control law which more intelligently counteracts the destabilizing mechanisms in unstable flow, *i.e.*, balances the pressure oscillations in the well. Two nonlinear control schemes are designed by an integrator backstepping approach, and stabilization in the unstable region is demonstrated. The first scheme is an exact cancelling design because we simple cancel existing dynamics including some stabilizing nonlinearities, which may waste control effort and make the control law complicated. To avoid cancellation of useful nonlinearities, a better controller is developed by taking input saturation into account. It is shown that the second control scheme can guarantee the asymptotically stable of the closed-loop system with saturated control. It is shown that the proposed backstepping controller can stabilize for smaller pressure than the PI controller and PD controller.

2. MODELLING

The oscillating behavior of the downhole pressure of a slugging well can be characterized as a stable limit cycle. Severe slugging exhibits qualitatively the same behavior as the slightly modified van der Pol equation

$$\dot{p} = w, \tag{1}$$
$$\dot{w} = a_1(\beta - p) + a_2(\zeta - w^2)w, \tag{2}$$

where the states p and w are the down hole pressure in the riser and its time derivative, respectively. The coefficients in (1)–(2) can be explained as follows.

- β: steady state pressure.
- a_1: frequency or stiffness of the system.
- a_2, ζ: local "degree of the stability/instability" and amplitude of the oscillation.

2.1 The equilibrium downhole pressure β

The equilibrium point (p^*, w^*) of the system (1)–(2) becomes

$$\begin{bmatrix} p^* \\ w^* \end{bmatrix} = \begin{bmatrix} \beta \\ 0 \end{bmatrix},$$

which means that the parameter β is simply the equilibrium downhole pressure p^*. The equilibrium downhole pressure $p^* = \beta$ is given by

$$\beta = \bar{\rho} gH + \Delta p_f + \Delta p_c + p_0. \tag{3}$$

where $\bar{\rho} gH$ is the static head with $\bar{\rho}$ being the average density in the riser, Δp_f is the frictional pressure drop, Δp_c is the pressure drop over the production choke, and p_0 is the pressure downstream the choke. For a given reservoir influx w_{res}, the differential pressure over the production choke is given by its flow characteristic according to

$$\Delta p_c\left(w_{res}\right) = \frac{w_{res}^2}{\left(K_c u_c\right)^2 \rho_c}, \tag{4}$$

where ρ_c is the density upstream the choke, u_c the choke opening, and K_c the flow constant of the choke. The frictional pressure drop $\Delta p_f\left(w_{res}\right)$ is a increasing function of w_{res} according to

$$\Delta p_f = K_f w_{res}^2.$$

In the simplest case, we may assume constant influx w_{res} such that β can be given in the lumped form

$$\beta\left(q\right) = b_0 + b_1 q, \tag{5}$$

where b_0 and b_1 are positive constants, and q is proportional to the differential pressure Δp_c at steady-state flow w_{res}. In Figure 2, β is plotted as a function of the choke opening.

2.2 Local Degree of Stability/Instability a_2,ζ

The parameters a_2 and ζ are related to the amplitude of oscillation and stability properties of the fixed point. This can be seen by linearizing system (1)–(2) to get

$$\dot{\Delta p} = \Delta\omega, \tag{6}$$
$$\dot{\Delta\omega} = -a_1\Delta p + a_2\zeta\Delta\omega. \tag{7}$$

The eigenvalues of the system are $\lambda = \frac{a_2\zeta \pm \sqrt{a_2^2\zeta^2 - 4a_1}}{2}$, which means that (assuming $a_1 > 0$ and $a_2 > 0$)

- $\zeta = 0$, bifurcation point.
- $\zeta < 0$, system is stable.
- $\zeta > 0$, system is unstable.

In the simplest case, we may assume constant flow rates of liquid and gas from the reservoir. Then

$$\zeta\left(q\right) = c_0 - c_1 q, \tag{8}$$

where c_0/c_1 denotes the bifurcation point and c_0, c_1 are positive constants.

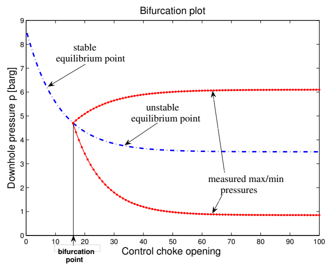

Fig. 2. Bifurcation plot

2.3 Transportation Delay

The variable q is related to the effect of the differential pressure over the production choke. Due to transport delay in the well, a time-lag is expected between application of the control signal to the choke and seeing the effect in (1)–(2). This time-lag is modelled as follows

$$\dot{q} = -\frac{1}{\tau}q + \frac{1}{\tau}\delta, \tag{9}$$

where δ represents the control input and is a strictly decreasing function of the production choke opening $u \in [0, 1]$. Thus, when δ is computed, the actual control signal to apply to the choke is found by inverting $\delta(u)$. It is assumed that $\delta \to \infty$ as $u \to 0$, and that $\delta \geq \delta_{min} \geq 0$. Without loss of generality, we let $\delta_{min} = 0$.

2.4 Simplified Model of Riser Slugging

Based on (5) and (8), the system dynamics (1)–(2) and (9) can be assembled into

$$\dot{p} = w \tag{10}$$
$$\dot{w} = -a_1 p + h\left(w\right) + g\left(w\right)q + a_1 b_0 \tag{11}$$
$$\dot{q} = -\frac{1}{\tau}q + \frac{1}{\tau}\delta, \tag{12}$$

where the functions h and g are defined as

$$h\left(w\right) = a_2 c_0 w - a_2 w^3 = h_0 w - h_1 w^3 \tag{13}$$
$$g\left(w\right) = a_1 b_1 - a_2 c_1 w = g_0 - g_1 w. \tag{14}$$

The positive constants a_i, b_i and c_i $(i = 1, 2)$ are empirical parameters that are adjusted to produce the right behavior of the downhole pressure p.
The system (10)–(12) can capture some of the qualitative properties in the downhole pressure during riser slugging.

- Decreasing control gain: A characteristic property of riser slugging is that the static gain decreases with choke opening.
- Bifurcation: The model exhibits the characteristic bifurcation that occurs at a certain choke opening c_0/c_1, *i.e.*, the steady-state response of the downhole pressure exhibits changes from a stable point when choke opening is smaller than c_0/c_1 to a stable limit cycle when choke opening is larger that c_0/c_1 (see Figure 2).
- Time lag: The transportation delay between a change in choke opening to the resulting change in downhole pressure p is modeled by simple 1st-order lag.

Our objective is to design a control law for the control input δ which stabilizes p at the desired set-point p_{ref}.

3. CONTROLLER DESIGN

In this section we design stabilizing controllers using backstepping. Thus, we iteratively look for a change of coordinates in the form

$$z_1 = p - p_{ref}, \tag{15}$$

$$z_2 = w - \alpha_w, \tag{16}$$

$$z_3 = q - \alpha_q, \tag{17}$$

and an accompanying Lyapunov function. The functions α_w and α_q are virtual controls to be determined.

3.1 Control Scheme I

Step 1 — virtual control law α_w
From (10), (15) and (16), we obtain that

$$\dot{z}_1 = \alpha_w + z_2.$$

Then we design a virtual control law α_w

$$\alpha_w = -C_1 z_1. \tag{18}$$

The time-derivative of $U_1 = \frac{1}{2}z_1^2$ becomes

$$\dot{U}_1 = -C_1 z_1^2 + z_1 z_2. \tag{19}$$

Step 2 — virtual control law α_q
We start by computing the time-derivative of z_2 using (11) and (15)–(17), obtaining

$$\begin{aligned}\dot{z}_2 = &-a_1(z_1 + p_{ref} - b_0) + h(w)\\ &+ g(w)\alpha_q + g(w) z_3 - \dot{\alpha}_w.\end{aligned} \tag{20}$$

If we for now ignore (14) and instead assume that $g(w) \geq g_0 > 0$, we may choose the virtual control α_q as

$$\begin{aligned}\alpha_q = &\frac{1}{g(w)}(-C_2 z_2 - z_1 + a_1(z_1 + p_{ref} - b_0)\\ &-h(w) - a_1 b_0 + \dot{\alpha}_w).\end{aligned} \tag{21}$$

Consider the CLF $U_2 = U_1 + \frac{1}{2}z_2^2$. The time derivative of U_2 is

$$\dot{U}_2 = -C_1 z_1^2 - C_2 z_2^2 + g(w) z_2 z_3. \tag{22}$$

Step 3 — Final control law δ
The dynamics of z_3 is obtained as

$$\dot{z}_3 = \dot{q} - \dot{\alpha}_q = -\frac{1}{\tau}q + \frac{1}{\tau}\delta - \dot{\alpha}_q. \tag{23}$$

Selecting

$$\delta = -\tau C_3 z_3 - \tau g(w) z_2 + \alpha_q + \tau\dot{\alpha}_q, \tag{24}$$

the derivative of the control Lyapunov function $U_3 = U_2 + \frac{1}{2}z_3^2$ becomes

$$\begin{aligned}\dot{U}_3 &= -C_1 z_1^2 + g(w) z_2 z_3 + z_3\left(-\frac{1}{\tau}q + \frac{1}{\tau}\delta - \dot{\alpha}_q\right)\\ &\leq -C_1 z_1^2 - C_2 z_2^2 - C_3 z_3^2,\end{aligned} \tag{25}$$

which proves that the equilibrium $(z_1, z_2, z_3) = 0$ is globally exponentially stable, and in particular p is regulated to the setpoint p_{ref}. The rate of convergence is adjustable through the constants C_1, C_2, and C_3, and we may in principle have any desirable performance of the system. The resulting control law is

$$\begin{aligned}&\delta(p, w, q)\\ &= -\tau C_3 q - \tau g(w)(w + C_1(p - p_{ref}))\\ &+ \frac{1}{g^2(w)}\Big[\tau\big((C_3+1)g(w) - g'(w)\big(h(w) +\\ &-a_1(p - b_0) + g(w)q\big)\big)\big(-(C_1+C_2)w - h(w)\\ &-(C_1C_2 + 1 - a_1)(p - p_{ref}) + a_1(p_{ref} - b_0)\big)\Big]\\ &- \frac{1}{g(w)}\Big[\tau(C_1 + C_2 + h'(w))\big(-a_1(p - b_0)\\ &+h(w) + g(w)q\big) + \tau(C_1C_2 + 1 - a_1)w\Big]\end{aligned} \tag{26}$$

Remark 1. We refer to this choice of α_q as an exact *cancelling design* because we simply cancel existing dynamics and replace it with some desirable linear feedback terms: $-C_1 z_1$ and $-C_2 z_2$. Note that this design is not necessarily the best choice of control law because stabilizing nonlinearities may be cancelled, potentially wasting control effort, losing robustness to modelling errors, and making the control law overly complicated. As can be seen in (26), the controller becomes quite complicated as a result of the virtual controls and their time derivatives occuring in it. It is desirable to obtain a simpler control law, which is possible if simple virtual controls can be found by avoiding cancellation of useful nonlinearities.

3.2 Control Scheme II

The design of the previous section is a straight forward application of the backstepping technique. However, it ignores (14) as well as the fact that the control input δ saturates at 0. In this section, a better control law will be obtained by exploiting the structure of the system in terms of the specific choices for $h(w)$ and $g(w)$ in (13)–(14), and the flexibility of the backstepping procedure in selecting virtual control laws.
By inspection of the second step of backstepping in the previous section, we recognize that the terms $-h_1 w^3$ and $-g_1 wq$ are expected to be stabilizing, since physically $q \geq 0$. Hence, cancelling these terms is not necessary at this point in the design. Substituting (13) and (14) into (20), and selecting $\alpha_w = 0$ and

$$\alpha_q = -\frac{C_2 + h_0}{g_0} z_2 + \frac{a_1}{g_0}(p_{ref} - b_0), \qquad (27)$$

$$U_2 = \frac{a_1}{2} z_2^2 + \frac{1}{2} z_2^2, \qquad (28)$$

gives

$$\dot{U}_2 = -(C_2 + g_1 q) z_2^2 - h_1 z_2^4 + g_0 z_2 z_3. \qquad (29)$$

Here, we notice that the $z_1 z_2$-cross-term was cancelled, due to the particular choice of U_2 and α_w. The stabilizing terms $-h_1 z_2^3$ and $-g_1 \alpha_q z_2$ increase negativity of $\dot{U}_2$, and need not be compensated at this point. Consider now the CLF

$$U_3 = U_2 + \frac{1}{2} z_3^2. \qquad (30)$$

It's time derivative is

$$\begin{aligned}\dot{U}_3 = &-(C_2 + g_1 q) z_2^2 - h_1 z_2^4 \\ &+ z_3 \left(g_0 z_2 - \frac{1}{\tau} q + \frac{1}{\tau}\delta - \dot{\alpha}_q \right), \end{aligned} \qquad (31)$$

and we may select

$$\delta = -\tau C_3 z_3 - \tau g_0 z_2 + q + \tau \dot{\alpha}_q, \qquad (32)$$

to obtain

$$\dot{U}_3 = -(C_2 + g_1 q) z_2^2 - h_1 z_2^4 - C_3 z_3^2. \qquad (33)$$

LaSalle's invariance principle now implies that the origin is asymptotically stable. The following result formalizes this, and in addition takes saturation of δ into account.

Theorem 1. Let $p_{ref} > b_0$, $C_2 > 0$ and $C_3 > 0$. Then the equilibrium $x_{ref} = (p_{ref}, 0, a_1(p_{ref} - b_0)/g_0)$ of system (10)–(12) in closed loop with the saturated control

$$\delta = \max\{0, \delta_a\}$$

where

$$\begin{aligned}\delta_a = &\frac{(C_2 + h_0)}{g_0}\Big[\tau a_1 p(t) - \tau(C_3 + h_0) w(t) + \tau h_1 w^3(t) \\ &+ \tau g_1 w(t) q(t) - \tau a_1 b_0\Big] + \frac{a_1 \tau C_3}{g_0}(p_{ref} - b_0) \\ &- \tau g_0 w(t) + (1 - \tau(h_0 + C_2 + C_3)) q(t) \end{aligned} \qquad (34)$$

is asymptotically stable. If

$$C_2 \leq \frac{1}{2\tau} - h_0, \qquad (35)$$

then the set

$$\mathcal{A} = \left\{ (p, w, q) \,\middle|\, p \geq \underline{p}_0, \underline{w}_0 \leq w \leq \bar{w}_0, q \geq 0 \right\} \qquad (36)$$

where

$$\underline{p}_0 = \frac{1}{4}(3 p_{ref} + b_0) \qquad (37)$$

$$\underline{w}_0 = -\min \left\{ \frac{g_0}{2\tau g_1 (C_2 + h_0)}, \sqrt[3]{\frac{a_1(p_{ref} - b_0)}{4 h_1}} \right\} \qquad (38)$$

$$\bar{w}_0 = \frac{a_1 (C_2 + h_0)(p_{ref} - b_0)}{4(g_0^2 + C_2 h_0 + h_0^2)} \qquad (39)$$

is contained in the region of attraction of x_{ref}.

Proof: The condition $p_{ref} > b_0$ ensures that $\delta_a > 0$ at the equilibrium $z = (z_1, z_2, z_3) = 0$. Thus, in view of (30) and (33), there exists a constant $c > 0$ such that $D = \{z \,| U_3(z) < c\}$ is positively invariant and $\delta_a > 0$ and $q(t) > 0$ for all $z \in D$. Thus, from (33) we have

$$\dot{U}_3 \leq -C_2 z_2^2 - C_3 z_3^2 \qquad (40)$$

in D. Furthermore, only $z(t) \equiv 0$ stays forever in $S = \left\{ z \in D \,\middle|\, \dot{U}_3 = 0 \right\}$ since $\dot{z}_2 = -a_1 z_1$ for $z \in S$. Therefore, by Corollary 4.1 of Khalil [2002] $z = 0$ is asymptotically stable.

The estimate of the region of attraction is obtained by analyzing $\dot{U}_3$ when δ is saturated as follows. From the condition $q(0) \geq 0$, equation (12), and the fact that $\delta(t) \geq 0$ for all $t > 0$, we have that $q(t) \geq 0$ for all $t > 0$. So, from (31) we have

$$\dot{U}_3 \leq -C_2 z_2^2 + z_3 \left(g_0 z_2 - \frac{1}{\tau} q + \frac{1}{\tau} \delta - \dot{\alpha}_q \right). \qquad (41)$$

Now, let $\delta_a < 0$. Then, $\delta = 0$,

$$\dot{z}_3 = -\frac{1}{\tau} q - \dot{\alpha}_q, \qquad (42)$$

and the derivative of U_3 satisfies

$$\dot{U}_3 \leq -C_2 z_2^2 + z_3 \left(g_0 z_2 - \frac{1}{\tau} q - \dot{\alpha}_q \right). \qquad (43)$$

We will now consider two cases: a) $z_3 \leq 0$ and b) $z_3 > 0$.

a) $z_3 \leq 0$. Since $\delta_a < 0$, we have from (32), which is equivalent to (34) but written in the z coordinates, that

$$-C_3 z_3 < g_0 z_2 - \frac{1}{\tau} q - \dot{\alpha}_q, \qquad (44)$$

so

$$z_3 \left(g_0 z_2 - \frac{1}{\tau} q - \dot{\alpha}_q \right) < -C_3 z_3^2. \qquad (45)$$

Thus, we obtain

$$\dot{U}_3 \leq -C_2 z_2^2 - C_3 z_3^2. \qquad (46)$$

b) $z_3 > 0$. In this case, we have from (43), by inserting for $\dot{\alpha}_q$ and rearranging terms, that

$$\begin{aligned}\dot{U}_3 \leq &-C_2 z_2^2 - \frac{C_2 + h_0}{4 g_0} a_1 (p_{ref} - b_0) z_3 \\ &- q z_3 \left(\frac{1}{2\tau} - (C_2 + h_0) \right) \\ &- q z_3 \Big(\frac{1}{2\tau} + \frac{C_2 + h_0}{g_0} g_1 z_2 \Big) \\ &- \frac{C_2 + h_0}{4 g_0} a_1 z_3 \Big(4 z_1 + (p_{ref} - b_0) \Big) \\ &- \frac{C_2 + h_0}{4 g_0} z_3 \Big(4 h_1 z_2^3 + a_1 (p_{ref} - b_0) \Big) \\ &- \frac{z_3}{4 g_0} \Big((C_2 + h_0) a_1 (p_{ref} - b_0) \\ &- 4(g_0^2 + C_2 h_0 + h_0^2) z_2 \Big). \end{aligned} \qquad (47)$$

Using (35), and imposing the conditions

$$z_1 \geq -\frac{1}{4}(p_{ref} - b_0) \qquad (48)$$

$$z_2 \leq \frac{a_1 (C_2 + h_0)(p_{ref} - b_0)}{4(g_0^2 + C_2 h_0 + h_0^2)} \qquad (49)$$

$$z_2 \geq -\min\left\{\frac{g_0}{2\tau g_1(C_2+h_0)}, \sqrt[3]{\frac{a_1(p_{ref}-b_0)}{4h_1}}\right\} \quad (50)$$

we obtain

$$\dot{U}_3 \leq -C_2 z_2^2 - \frac{C_2+h_0}{4g_0} a_1(p_{ref}-b_0)\,|z_3|\,. \quad (51)$$

In view of (40),(46) and (51), LaSalle's invariance principle can be invoked as in the first part of this proof to establish asymptotic stability of $z=0$ and that initial conditions satisfying (48)–(50) are contained in the region of attraction of $z=0$. Finally, we note that the conditions (48)–(50), written in terms of (p,w,q), exactly characterize the set $\mathcal{A}$, as given by (36)–(39).

4. SIMULATION RESULTS

In this section we test our proposed backstepping controller on model (1)–(2). For simulation studies, the following values are selected as "true" parameters for the system: $h_0=1$, $h_1=50$, $g_0=0.125$, $g_1=5$, $a_1=0.025$, $b_0=3.5$, and $\tau=0.1$. The design objective is to stabilize p at the desired set point $p_{ref}=3.51$. With the proposed backstepping control scheme II, we take the following set of design parameters: $C_2=0.2$ and $C_3=5$. The initials are set as $p(0)=3.51, w(0)=q(0)=0$ and $u_0=[0.10, 0.90]$, respectively. Figure 3 illustrates the backstepping controller applied for stabilization in the unstable region at reference pressure $p_{ref}=3.51$. Figure 4 shows that the system looses controllability at the pressure $p_{ref}=3.49$, which is below the point $p=b_0=3.5$. The simulation results verify our theoretical findings.

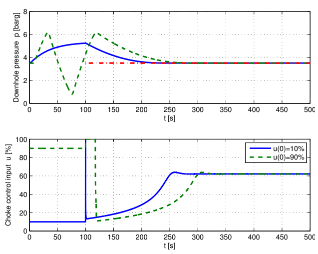

Fig. 3. Simulations of stabilization in the unstable region at $p_{ref}=3.51$ using scheme II.

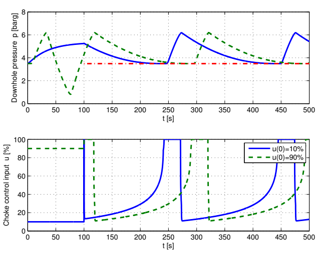

Fig. 4. Simulations of an attempt to stabilize at $p_{ref}=3.49$, which is below what is physically feasible.

4.1 PI control

The conventional way to stabilize riser slugging is by applying a simple control law u_{PI} of the form

$$u_{PI} = u_I - K_p\,(p-p_{ref})\,, \quad (52)$$

where u_I is the bias for a given pressure set-point p_{ref}, generated by slow integral action according to

$$\dot{u}_I = -\frac{K_i}{T_i}\,(p-p_{ref})\,. \quad (53)$$

By linearizing the closed loop dynamics, the characteristic equation is

$$\lambda^3 + \left(\frac{1}{\tau} - h_0 + g_1 q_{ref}\right)\lambda^2 + \frac{1}{\tau}\left(\tau a_1 - h_0 + g_1 q_{ref}\right)\lambda + \frac{g_0}{\tau}\delta'\left(u_I\right)K_p + \frac{a_1}{\tau} = 0. \quad (54)$$

According to the Hurwitz criterion, it turns out that local exponential stability can be achieved by PI control if

$$p_{ref} > b_0 + \frac{h_0 g_0}{a_1 g_1} - \min\left\{\frac{g_0\tau}{g_1}, \frac{g_0}{g_1\tau a_1}\right\} \quad (55)$$

and K_p satisfies

$$\underline{K}_p < K_p < \bar{K}_p \quad (56)$$

where

$$\underline{K}_p = \frac{\left(\frac{1}{\tau} - h_0 + g_1 q_{ref}\right)\left(\tau a_1 - h_0 + g_1 q_{ref}\right) - a_1}{\delta'\left(u_I\right)g_0}, \quad (57)$$

$$\bar{K}_p = \frac{-a_1}{g_0\delta'\left(u_I\right)}, \quad (58)$$

$$q_{ref} = \frac{a_1}{g_0}\left(p_{ref}-b_0\right). \quad (59)$$

Here, we have treated u_I as constant, corresponding to the choke opening at the equilibrium $(p_{ref}, 0, a_1(p_{ref}-b_0)/g_0)$. The bifurcation point corresponds to

$$p_{ref} = b_0 + \frac{h_0 g_0}{a_1 g_1}. \quad (60)$$

Figure 5 illustrates PI controller applied for stabilization in the unstable region at reference pressure $p_{ref}=4.498$. Figure 6 shows that the system looses stability at the pressure $p_{ref}=4.45$, which is below the required reference $p_{ref} > 4.4975$. The bifurcation point corresponds to $p_{bifur}=4.5$. The design parameters are chosen as $K_p=0.1$, $K_i=0.1$ and $T_i=25$.

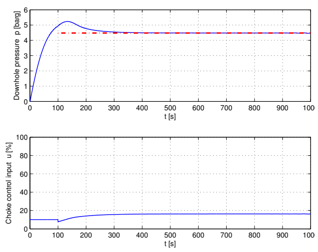

Fig. 5. Simulations of PI stabilization at a pressure in the unstable region $p_{ref}=4.498$ using PI controller.

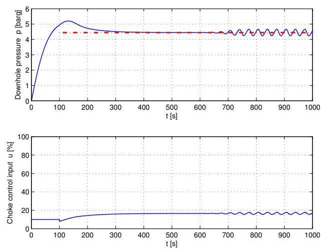

Fig. 6. Simulations of an attempt to stabilize at a pressure in the unstable region $p_{ref}=4.45$ using PI controller.

4.2 PD control

The another way to stabilize riser slugging is by applying a simple control law u_{PD} of the form

$$u_{PD} = u_I + u_D - K_p\,(p-p_{ref})\,, \quad (61)$$

where u_I is the bias for a given pressure set-point p_{ref}, and u_D is the derivative action according to

$$u_D = -K_d\frac{d(p-p_{ref})}{dt} = -K_d w. \quad (62)$$

By linearizing the closed loop dynamics, the characteristic equation is

$$\lambda^3 + \left(\frac{1}{\tau} - h_0 + g_1 q_{ref}\right)\lambda^2 + \left(\frac{g_0}{\tau}\delta'(u_I)K_p + \frac{a_1}{\tau}\right)$$
$$+\frac{1}{\tau}\left(\tau a_1 - h_0 + g_1 q_{ref} + K_d g_0 \delta'(u_I)\right)\lambda = 0. \tag{63}$$

According to the Hurwitz criterion, it turns out that local exponential stability can be achieved by PD control if

$$p_{ref} > b_0 + \frac{h_0 g_0}{a_1 g_1} - \min\left\{\frac{g_0\tau}{g_1} + \frac{g_0^2 K_d}{g_1 a_1}\delta'(u_I), \frac{g_0}{g_1\tau a_1}\right\}, \tag{64}$$

K_p satisfies

$$\underline{K}_p < K_p < \bar{K}_p, \tag{65}$$

and K_d satisfies

$$K_d < \frac{g_0\delta'(u_I)K_p + a_1}{\left(\frac{1}{\tau} - h_0 + g_1 q_{ref}\right)g_0\delta'(u_I)} - \frac{\tau a_1 - h_0 + g_1 q_{ref}}{\delta'(u_I)g_0}, \tag{66}$$

where

$$\underline{K}_p = \frac{\left(\frac{1}{\tau} - h_0 + g_1 q_{ref}\right)\left(\tau a_1 - h_0 + g_1 q_{ref}\right) - a_1}{\delta'(u_I)g_0}$$
$$+K_d\left(\frac{1}{\tau} - h_0 + g_1 q_{ref}\right), \tag{67}$$

$$\bar{K}_p = \frac{-a_1}{g_0\delta'(u_I)}, \tag{68}$$

$$q_{ref} = \frac{a_1}{g_0}\left(p_{ref} - b_0\right). \tag{69}$$

Figure 7 illustrates PD controller applied for stabilization at reference pressure $p_{ref} = 4.6$. The design parameters are chosen as $K_p = 2$ and $K_d = 2$, which satisfy the stability conditions. Figure 8 shows that the system looses stability at the pressure $p_{ref} = 3.51$. The design parameters are chosen as $K_p = 0.02$ and $K_d = -1$, which satisfy the stability conditions. When the pressure is small, feasible K_p and K_d according to the Hurwitz criterium, give an aggressive actuation that the choke saturates repeatedly and stabilization is not achieved.

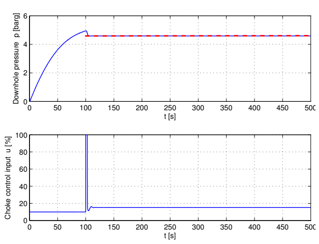

Fig. 7. Simulations of PD stabilization at a pressure $p_{ref} = 4.60$ using PD controller.

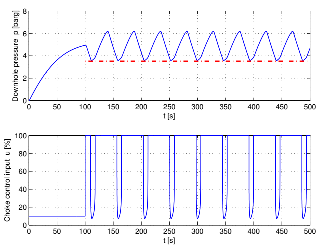

Fig. 8. Simulations of an attempt to stabilize at $p_{ref} = 3.51$ using PD controller.

5. CONCLUSION

This paper illustrates the potential of nonlinear model-based control applied for stabilization of unstable flow in oil wells. A simple empirical model is developed that describes the qualitative behavior of the downhole pressure in case of severe riser slugging. Two control schemes are developed using the integrator backstepping approach. The first scheme is an exact cancelling design because we simply cancel existing dynamics including some stabilizing nonlinearities, which may waste control effort and make the control law complicated. To avoid cancellation of useful nonlinearities, a better controller is developed, which in addition takes input saturation into account. It is shown that the proposed backstepping control scheme can guarantee asymptotic stability of the closed-loop system with saturated control. The proposed backstepping controller can stabilize at lower pressure setpoints, corresponding to higher flow rates, than PI and PD controllers. When the pressure setpoint is low, parameters of the PD controller that are feasible according to the Hurwitz criterium, give a very aggressive actuation causing the choke to saturate repeatedly and stabilization is not achieved. For the same pressure setpoint, the proposed backstepping controller is easy to tune. Simulation results are presented to illustrate the performance of the proposed control scheme.

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