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On the Settling Time in Repetitive Control Systems

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Abstract: Repetitive control seeks to converge to zero tracking error when a feedback control system has a periodic command or a periodic disturbance. It is of interest to examine how long one must wait until convergence is reached. Without the repetitive control loop, any feedback system has a settling time often defined as four times the longest time constant in the characteristic polynomial. When a repetitive control loop is put around such a system, there are p additional roots to the characteristic polynomial, where p is the period in time steps, which can be very large. Again one can define the settling time, which might best be measured in units of periods, representing the number of periods needed to essentially complete the convergence process. This paper studies the convergence rate for several general classes of repetitive controller design methods.

1. INTRODUCTION

Repetitive control (RC) is a relatively new field (Inoue *et al.* 1981, Omata *et al.* 1984, Hara *et al.* 1985) that seeks to converge to zero tracking error when following a periodic command, or following a constant command in the presence of a periodic disturbance of known period. This is usually done by adjusting the command given to a feedback control system every time step, based on the error observed in the previous period. RC differs from iterative learning control (ILC) in that ILC looks at the previous run instead of the previous period, and the system is reset to the same initial condition before each run.

Stability and stability robustness are serious issues in RC, and as a result most of the attention in creating RC designs is directed to these issues. But, as in more general control design problems, one is also interested in the settling time needed before one has essentially reached convergence to zero tracking error. It is the purpose of this paper to supply some evaluation of this issue. There are several general approaches to RC design that we consider: (1) the simplest form of RC that is a single gain times the error at an appropriate time step one period back, (2) using the inverse of the system transfer function as a compensator, (3) using the inverse modified to produce zero phase for any zeros that are not stably invertible, (4) using an FIR fit to the inverse of the steady state frequency response, and (5) combining the FIR design with inversion of poles and "stable" zeros. The properties of the settling time for these approaches are investigated here.

2. REPETITIVE CONTROL FORMULATION

Figure 1 gives the basic structure used for the repetitive control system. The $G(z)$ normally represents the closed loop transfer function of a feedback control system, and is assumed to be asymptotically stable. The RC wraps a feedback loop around this with RC controller

$$R(z) = \phi F(z)/(z^p - 1) \tag{1}$$

The ϕ is the repetitive controller gain or learning gain, $F(z)$ is the RC compensator, p is the period of the periodic command or periodic disturbance given in time steps of sample time interval T, $Y_D(z)$ is the desired periodic or constant output, and $V(z)$ is a periodic output disturbance. The periodic disturbance might enter the feedback control system in various places around its loop, but wherever it enters, there is an equivalent periodic output disturbance which is used here. Using block diagram algebra, the difference equation giving the error as a function of time step k, and the characteristic polynomial can be written respectively as

$$\{z^p - [1-\phi F(z)G(z)]\}E(z) = (z^p - 1)[Y_D(z) - V(z)] \tag{2}$$

$$\phi F(z)G(z)/(z^p - 1) = -1 \tag{3}$$

For simplicity, and to aid in obtaining a general understanding, we consider that $G(z)$ comes from feeding a continuous time system $G(s)$ through a zero order hold. Based on the results in (Åström *et al.* 1980), this allows us to know where the zeros are that are introduced by the discretization, at least asymptotically as T tends to zero, based on the pole excess of the original continuous time transfer function.

According to (Longman 2000) a sufficient condition for asymptotic stability and convergence to zero tracking error from all possible initial error histories, is

$$\left|1-\phi F(z)G(z)\right| < 1 \quad \forall \quad z = \exp(i\omega T), \quad 0 \le \omega T \le \pi \tag{4}$$

And (Songschon *et al.* 2003) shows that this condition is very close to being both sufficient and necessary, except for very small values of p, that are usually not reasonable in applications.

3. COMPUTING THE SETTLING TIME

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In classical control theory, the transients decay as exponentials (possibly multiplied by sines or cosines), and in theory take an infinite amount of time to converge to zero. In order to have a standard concept of how long one needs to wait until the transients are negligible, it is common practice to define a settling time that is equal to four time constants of the slowest decaying solution to the homogeneous equation. This corresponds to waiting for the exponential factors of the solution to decay to 0.01832 times its original value, i.e. about 2% of the original value remains for the slowest decaying solution. Of course, this is a somewhat arbitrary definition, but nevertheless it is a useful one. It puts the main emphasis on the slowest decaying solution, and it can happen that this part does not dominate, at least at the start of the transient decay. We will use this definition here on RC systems.

For the purpose of this section only, let z_i be the ith root of the characteristic polynomial (3). Corresponding to this root is a solution to the difference equation given by $(z_i)^k$. An exponential $\exp(-\alpha t)$ with time constant $1/\alpha$, is given by $(e^{-\alpha T})^k$ at the sample times $t = kT$. Let $\rho = \max_i |z_i|$. Then we define the settling time given in units of time (seconds) t_s, given in units of time steps t_{ss}, and given in units of periods of the command or disturbance t_{sp}, as follows

$$t_s = -4T / \ln\rho \quad ; \quad t_{ss} = -4 / \ln\rho \quad ; \quad t_{sp} = -4 / (p \ln\rho) \qquad (5)$$

4. THE SIMPLEST FORM OF RC

The simplest form of RC can be described for a robot link as follows: if the link angle was 2 degrees to small at this phase of the previous period, add 2 degrees to the command, or add gain ϕ times 2 degrees to the command. Consider a general asymptotically stable first order system $G(s) = ab/(s+a)$. When fed by a zero order hold, the time delay through $G(z) = b(1-\exp(-aT))/(z-\exp(-aT))$ is one time step. To account for this delay, one looks one step ahead in the previous period, making the RC compensator equal $F(z) = z$. The characteristic polynomial (3) becomes

$$\phi' z / [(z^p - 1)(z - e^{-aT})] = -1 \qquad (6)$$

where $\phi' = \phi b[1-\exp(-aT)]$. This has one zero at the origin, one pole given by the system, and p poles evenly spaced around the unit circle. Setting p equal to 8, Fig. 2 shows the resulting root locus when the DC gain $b = 1$, and $a = 22.31$ producing a discrete time pole at $z = 0.8$. The p roots on the unit circle initially move inward making the RC system stable, and then curve and start moving outward, heading for infinity. There is an optimum choice of the gain ϕ' roughly equal to 1.6 as seen in Fig. 3 that plots the settling time t_{ss} as a function of the RC gain. We note that when using this learning law, the best possible settling time is a function of the transfer function of the system, and of the number of time steps in a period, and one can only influence the settling time by optimizing the single repetitive control gain. Although this law is always stable for small enough gain for first order systems, we will see in the next section that there is no point in using this learning law for first order systems. The law might be stable for higher order systems provided that the pole excess of $G(s)$ is one. This law is most likely unstable when applied to other higher order systems. It is necessary that the number of poles and the number of zeros inside the unit circle be equal for the product $F(z)G(z)$ in order to satisfy stability condition (4), and this requires a pole excess of one in $G(s)$ when $F(z) = z$. With a higher pole excess, one could consider using $F(z) = z^2$ or z^3 etc. in order to get an equal number of poles and zeros inside the unit circle, but there is no guarantee that one can stabilize the system, and each system will have its own root locus properties. Design methods given in Sections 5 and 6 are generally needed for higher order systems.

5. USING THE SYSTEM INVERSE AS A COMPENSATOR

In certain respects the ideal compensator is the inverse of the system, $F(z) = G^{-1}(z)$. Whereas the above compensator design is guaranteed to work only on first order systems, this method is guaranteed to work on systems $G(s)$ with pole excess equal to one with all zeros being minimum phase. Furthermore, the approach works on second order systems with no zeros, and is likely to work on higher order systems that are minimum phase with pole excesses of two. As shown by (Åström *et al.* 1980), asymptotically as the sample time interval T gets small, the process of converting $G(s)$ fed by a zero order hold into discrete time form $G(z)$, will generically introduce enough zeros to produce a pole excess of one in $G(z)$. For all even order pole excesses there is a zero that approaches -1. For stable second order systems one can show that this approaches -1 from inside the unit circle, and hence makes a stably invertible system. For a pole excess of 3 there is a zero introduced at -3.7321, for a pole excess of 4 there is a zero introduced at -9.8990, for pole excess of 5 there are two zeros outside the unit circle at -23.2039 and -2.3225, etc. Hence, at least for sufficiently fast sample rates one cannot stably invert the discrete time transfer function when the pole excess is 3 or more. A slow enough sample rate will make any minimum phase asymptotically stable transfer function stably invertible, but most often the sample rate required is far too slow in applications. Here we simply consider RC design using a system inverse compensator for first order and second order systems, but first we consider what we call the base case of a zeroth order system for which $G(z) = 1$ for which we pick $F(z) = 1$.

5.1 The Base Case: Zeroth Order System

The characteristic equation for the case of $G(z) = F(z) = 1$ from (3) is $z^p - (1-\phi) = 0$ producing p roots evenly spaced around a circle, whose radius is given by the pth root of $(1-\phi)$. We consider gains $0 < \phi \le 1$ since larger gains simply cause over correction. The root locus plot is shown in Fig. 4 where all poles on the unit circle move radially inward at the same rate, and reach the origin when $\phi = 1$. This corresponds to deadbeat control, producing zero error in finite time, after one period. One may chose to use this gain, but often one picks a smaller gain to make less aggressive changes each time step. The final value of the error when there is plant and measurement noise can be improved by using a smaller gain. When the gain is less than unity, the settling time is non-zero and is given by $t_{ss} = -4p/\ln(1-\phi)$ expressed in units of time steps. This makes a settling time that is linear in p. And when the settling time is given in periods, it becomes a constant, independent of the period, $t_{sp} = -4/\ln(1-\phi)$. As the gain approaches unity, these settling times approach zero, meaning that the error is zero starting after the initial conditions of one period of operation, or as soon as the RC can be turned on.

5.2 First Order Systems

Now consider the use of $F(z) = G^{-1}(z)$ as an RC compensator for the first order system above equation (6). Substituting into the characteristic polynomial (3) demonstrates that the root locus is identical to the base case above, except that there is a pole zero cancellation at the system pole $z = \exp(-aT)$. The polynomial can be written as

$$(z - e^{-aT})[z^p - (1-\phi)] = 0 \tag{7}$$

The conclusion is that if the gain ϕ is above some threshold, the p roots from the square bracket term are closer to the origin that the system pole, and hence the system pole dominates with its fixed time constant independent of the gain. And this situation applies when $\phi = 1$, so that the RC system is no longer deadbeat, but instead has the settling time of the original feedback control system $G(z)$. If the gain is below this threshold, the p roots dominate and the settling time coincides with that of the base case:

$$t_{ss} = \max\left[\frac{4}{aT},\ \frac{-4p}{\ln(1-\phi)}\right];\ t_{sp} = \max\left[\frac{4}{a(pT)},\ \frac{-4}{\ln(1-\phi)}\right] \tag{8}$$

Graphs of these are formed from a zero slope section and a nonzero slope section as shown in Figs. 5-7. In these plots the value of a is set to 74.25 and $p = 8$ when ϕ is varied, and $\phi = 0.8$ when period p is varied. No plot of t_{sp} versus ϕ is given since it is just a scaled version of Fig. 5. For a fixed p, the gain at which the slope changes is given by $\phi_c = 1 - \exp(-apT)$ where pT is the period of the command or disturbance. If $\phi \in [\phi_c,\ 1)$, the t_{ss} is the settling time of $G(s)$, and otherwise it is the settling time of the base case. For fixed gain ϕ, the period p at which the plots have a discontinuous slope is given by $p_c = -\ln(1-\phi)/(aT)$. If the period p is shorter than p_c, the settling time is determined by the system poles, and otherwise it is determined by the base case. Note that when using $F(z) = z$ there is a pole at +1 and a pole on the real axis from the system, and hence the root locus will exist on the line between these with a breakaway somewhere in the middle. This indicates that the inverse design here can always be made to have a better settling time than the simplest form of RC.

5.3 Second Order Systems

Now consider $G(s) = b\omega_n^2/(s^2 + 2\zeta\omega_n + \omega_n^2)$, a general second order system with no zeros. When fed by a zero order hold and discretized one obtains the following form

$$G(z) = b'(z - z_1)/[(z - p_1)(z - p_2)] \tag{9}$$

where the p_i are images of the continuous time poles, and z_1 is a zero introduced by the discretization. This zero is normally inside the unit circle, but as the sample time T gets small, it approaches the point -1 on the unit circle. Hence, one can still entertain using the system inverse as a compensator. Substituting (9) and this compensator into the characteristic equation (3) produces

$$(z - p_1)(z - p_2)(z - z_1)[z^p - (1-\phi)] = 0 \tag{10}$$

This follows the same pattern as observed above for the first order system, having the system poles introduced along with the base case roots, except that this time the zero of the discrete time system is also a root of the RC system characteristic equation. As an example, consider that $b = 1$, $\omega_n = 37$ rad/sec (5.9 Hz), and $\zeta = 0.5$. Then the zero location as a function of the sample rate is given in Fig. 8 showing how it approaches -1. Figure 9 plots the value in time units of four time constants of the associated pole used to cancel the zero, i.e. the settling time for a root at the zero location. It is clear that this root can easily dominate, and determine the settling time of the system. Hence, it is clear that the settling time can be long when using this type of compensator on a second order system. This statement generalizes to any system with a pole excess of two. A method of addressing this is given in a later section.

6. COMPENSATOR DESIGN METHODS

Several general methods have been developed to address RC design problems for which the above methods do not apply, which are summarized here.

6.1 Compensator Design for Phase Cancellation Only

One method develop by (Tomizuka et al. 1989) applies techniques from filtering theory to handle the nonminimum phase zeros introduced on the negative real axis by the discretization process. For simplicity of understanding, consider a third order system composed of $a/(s+a)$ times the second order $G(s)$ above. When fed by a zero order hold and converted to discrete time, one has

$$G(z) = \bar{b}(z - z_1)(z - z_2)/[(z - p_1)(z - p_2)(z - p_3)] \tag{11}$$

Asymptotically as T gets small, the zero approaches $z_1 = -3.7321$, and z_2 approaches the reciprocal location. The compensator design uses the reciprocal of all terms in (11) that are stably invertible, and to address the zero outside the unit circle, a zero is introduced inside the unit circle at the reciprocal location, and a pole is placed at the origin. One can introduce a gain $\bar{b}'$ that adjusts the DC gain of the repetitive control process. Then

$$F(z) = \left[\frac{(z - p_1)(z - p_1)(z - p_1)}{\bar{b}(z - z_2)}\right]\left[\frac{\bar{b}'(z - z_1^{-1})}{z}\right] \tag{12}$$

This cancels the influence on phase of the zero outside the unit circle, but does not cancel its frequency dependent influence on the amplitude. To see this, one can write the terms related to the zero outside, and substitute $z = \exp(i\omega T)$ to obtain the frequency response

$$\frac{(z-z_1)(z-z_1^{-1})}{z} = \left(z+\frac{1}{z}\right) - \left(z_1+\frac{1}{z_1}\right) = \cos(\omega T) + R \quad (13)$$

where asymptotically $R = 3.7321 + 1/3.7321$. Since ωT goes from 0 to π as the frequency goes from zero to Nyquist, (13) is a real number and is always positive. This implies that for appropriately chosen gain ϕ, one can always satisfy the stability condition (4). The approach generalizes in an obvious way to handle larger pole excesses, and hence forms a general approach to designing RC compensators.

6.2 FIR Frequency Response Inversion Compensator Design

(Panomruttanarug et al. 2004) presents a method of designing an FIR filter

$$F(z) = a_1 z^{m-1} + a_2 z^{m-2} + \cdots + a_n z^{-(n-m)} \quad (14)$$

as a compensator that approximates the frequency response of the system inverse, by picking the gains to minimize

$$J = \sum_{j=1}^{N} [1 - G(e^{i\omega_j T}) F G(e^{i\omega_j T})] W_j [1 - G(e^{i\omega_j T}) F G(e^{i\omega_j T})]^* \quad (15)$$

where the sum is taken over a suitably dense sampling of frequencies from zero to Nyquist, the W_j is a weight factor if desired, and asterisk indicates the complex conjugate. Experience suggests that the value of *m* should be chosen to equal $1 + n/2$ when *n* is even, and $1 + (n+1)/2$ when *n* is odd. This is a very simple compensator to implement, requiring only that one compute a linear combination of *n* errors from the previous period.

Note that the FIR filter supplies $n - m$ poles at the origin, and it can pick $n-1$ zeros to minimize (15). When given the appropriate number of gains, the zeros form special patterns. First they cancel the poles of the system. For any zero inside or outside the unit circle, additional zeros are introduced, symmetrically placed around a circle with radius equal to the zero location.

6.3 Combination of FIR and System Inverse

The FIR compensator is only able to place poles at the origin, so it cannot place poles to cancel any zero inside the unit circle. Of course, for such invertible zeros, we can perform the cancellation, and then use the optimization (15) to design a compensator for the system with the zeros already cancelled. Thus, one can invert all terms of the transfer function that are invertible, and then for whatever zeros of $G(z)$ exist outside the unit circle, we design and FIR filter using (15) to approximate the frequency response of these zeros. We will study the comparison between these two approaches. The resulting design then cancels everything inside the unit circle, and places zeros evenly spaced around a circle of radius given by the system zero. Implementing this compensator requires running the error signal through the somewhat non-standard filter formed by the inversion of everything inside the unit circle, and then applying the FIR filter as a linear combination of the resulting values.

7. RC FOR 3rd ORDER SYSTEMS

Here we consider RC on third order systems with no continuous time zero. One zero outside the unit circle is introduced, which approaches -3.7321 as the sample time goes to zero, and there is another zero introduced inside the unit circle by the discretization, which approaches the reciprocal location. The zero outside cannot be stably inverted, and its influence is addressed by one of the above design methods. The results here are indicative of what happens for higher order systems with odd pole excesses. Even pole excesses of 4 or more have a zero or zeros outside the unit circle as in this section, and also have a zero that approaches -1 on the unit circle as in the second order problem treated above (also considered in the next section). Hence, considerable understanding of higher order systems is provided.

Many of the results are generated for the asymptotic locations of the zeros as indicated above. Since these zeros do not move dramatically with the sample time interval *T*, by looking at the asymptotic locations one obtains an understanding of what will happen for a large class of third order systems. Some results are generated for a specific third order system $G(s)$ formed by multiplying the second order system used above by $a/(s+a)$ where $a = 8.8$, and using the sample time interval $T = 1/100$. For this example, the zeros introduced by discretization are at -3.3104, and -0.2402.

7.1 Settling Time for Phase Cancellation

Denote the zeros by z_1, z_2 with z_1 outside the unit circle, and the poles by p_1, p_2, p_3. Then the characteristic polynomial (3) becomes

$$(z-p_1)(z-p_2)(z-p_3)(z-z_2)[(z^p - 1) + \phi \bar{b}'(z-z_1)(z-z_1^{-1})] = 0 \quad (16)$$

As before, the system poles or the system zero inside the unit circle can dominate, and otherwise the settling time is determined by the square bracket term. But this term no longer corresponds to the base case. Figure 10 shows the root locus plot for this term using the asymptotic location for the zero. It can be proven that the poles on the unit circle depart radially inward, as in the base case. But they do not continue radially inward to the origin, and instead curve and go unstable. Figures 11-13 use the given third order system and present the settling time versus period and versus gain, showing discontinuities when the domination switches from poles or zeros to the bracket term. As before in Fig. 7, the settling time in periods vs. the length of the period approaches a constant value as the period gets long.

7.2 Settling Time for FIR

When one uses a 10 gain compensator (14) on the given third order system, the resulting product $F(z)G(z)$ for frequencies from zero to Nyquist is plotted in the complex plane in Fig. 14. If the plot were just a point at +1, the FIR compensator would be a perfect inverse of the system frequency response. Here it stays very close to +1 for all frequencies up to Nyquist, making a stable and easily calculated inverse of the frequency response. Examining the design, one sees that there are four compensator poles at the origin, and nine zeros introduced as follows: three zeros that cancel the system poles, three zeros around a circle of the radius of the zero outside, and similarly for the zero inside. Using this information, the characteristic equation can be written as

$$0 = (z-p_1)(z-p_2)(z-p_3)(z-z_2)[(z^p-1)z^4 + \overline{\phi}(z-z_1)(z+z_1)(z-z_1 i)(z+z_1 i)(z-z_2)(z+z_2)(z-z_2 i)(z+z_2 i)] \tag{17}$$

Figure 15 is a root locus plot of the roots for the square bracket term, when the period is given as $p = 8$. This makes a very symmetric situation. Note that the optimization (15) automatically creates a negative value for $\overline{\phi}$, so that the root locus plot is shown for negative gains. Figure 16 illustrates how the plot changes when $p = 7$, and similar modifications apply when not enough gains are supplied to complete a pattern. Figures 17-18 present the settling time vs. period for a 10 gain RC designed for the given third order system using (15). Comparing Figs. 17 and 18 to Figs. 11 and 12, it is clear that the FIR frequency response inverse compensator design is dramatically superior to the phase-cancellation-only design above, for this third order system.

To create a more general understanding, we examine the settling time associated with the bracket term as a function of learning gain, using the asymptotic locations of the zeros, Fig. 19. Again, the performance is dramatically faster than in Fig. 13. When making an RC design one can obtain roughly the performance indicated here (based on asymptotic locations), until the settling time indicated gets faster than the slowest of the zeros and poles outside the square bracket in (17). For third order systems (and higher order systems with odd poles excess) it is not likely that the zero inside the unit circle is the dominant root, in which case the RC system can reach the settling time associated with the given feedback control system $G(s)$.

7.3 Settling Time for FIR Combined with System Inverse

The FIR compensator is only able to introduce poles inside the unit circle that are located at the origin. Therefore, it cannot create a pole located to cancel the zero inside the unit circle, the way inverse designs would do. Of course we can always introduce this pole to cancel the zero, and then present what is left to optimization criterion (15) to design an FIR compensator for this part of the system. The overall result will then be that the compensator cancels all poles and all zeros inside the unit circle, leaving only the one zero outside the unit circle on the negative real axis. The characteristic polynomial is then

$$(z-p_1)(z-p_2)(z-p_3)(z-z_2)\bullet [(z^p-1)z^4 + \overline{\phi}(z-z_1)(z+z_1)(z-z_1 i)(z+z_1 i)] = 0 \tag{18}$$

Figure 20 presents the root locus plot for the square bracket term using the asymptotic zero location. Note that the roots move further toward the origin before changing direction, so there is a potential for faster settling times by comparison to Fig. 15 that has zeros inside the unit circle. The number of gains for (14) for this design is 4 to introduce three zeros while Fig. 15 uses 7 to introduce 6 zeros. Figure 19 also presents the settling time in time steps for this square bracket term, as a function of the gain ϕ. For higher gains this combined compensator design can have a substantially faster settling time indicated in the figure. This improvement is realizable in practice provided the roots in the square bracket term still dominate for the gain of interest. When the gain ϕ is unity, the settling time reaches $t_{ss} = 3.6796$ corresponding to a root location with $|z_i| = 0.3372$.

8. IMPROVING THE SETTLING TIME IN SECOND ORDER SYSTEMS

Note that the difficulty in using the inverse compensator on second order systems came from the fact that the zero location approaches the unit circle as the sample time gets short, making a settling time for the pole that cancels this zero approach t_s equal to roughly 0.324 sec. This is slower by 50% than the settling time of the second order system model, which is $t_s = 0.2162$. One can avoid having this limitation to the settling time of the RC system by introducing an extra pole to the analogue system, for example by introducing a first order anti-aliasing filter. Then one obtains a third order system, with a zero outside the unit circle, and the zero inside is moved much closer to the origin, to a location where it would not normally dominate. Then one can design the compensator for the resulting third order system, using the techniques of the previous section. Therefore, it is suggested that one should not use the inverse as a compensator for second order systems, but instead introduce the extra root to the continuous time system and design the RC for the resulting third order system. The same method should work to improve performance of RC of higher order systems $G(s)$ with even pole excesses.

9. CONCLUSIONS

(1) The simplest form of RC works for first order systems and can work in some other situations, but other methods can always give better settling times.
(2) RC using a compensator that is the inverse of the system works on first order and second order systems, and on higher order systems with continuous time pole excess of one provided it is minimum phase, and it is likely to work for pole excesses of two as well. The settling time can be made to equal the larger of the settling time of the system and the zero location nearest the unit circle. The zero location can make the settling time rather long, and other methods can improve on this.
(3) RC compensators that cancel the stably invertible part of the system and cancel the phase influence of the rest, work on general systems. Settling times are related to the nature of the root locus plot for the zero(s) outside the unit circle, and the cancelled system zeros or poles, and can be long.

(4) RC using an FIR inverse of the system frequency response can give substantially faster settling times.
(5) The combination of inversion of all parts of the system that are stably invertible and the FIR design, have the potential to have still faster settling times.

(6) A method is presented to improve the settling times of systems with even pole excess, by introducing an analogue filter to increase the system pole excess to an number.

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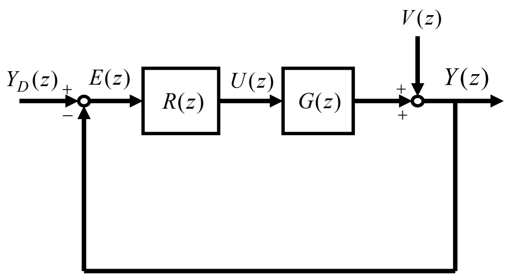

Fig. 1. Repetitive control system block diagram.

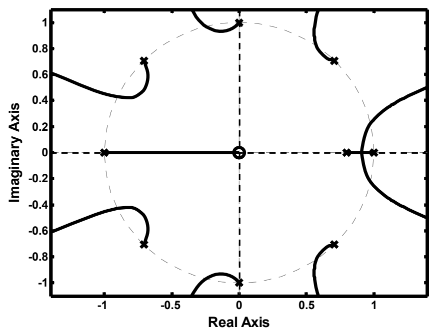

Fig. 2. Root locus for 1st order system with $F(z) = z$.

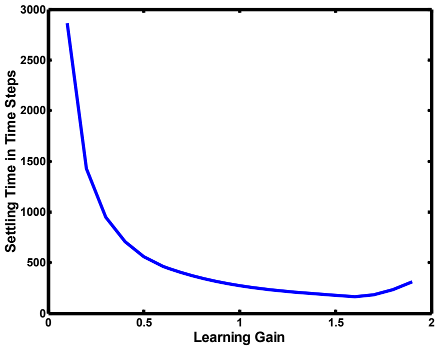

Fig. 3. Settling time in time steps for the 1st order system with $F(z) = z,\ \ p = 8$.

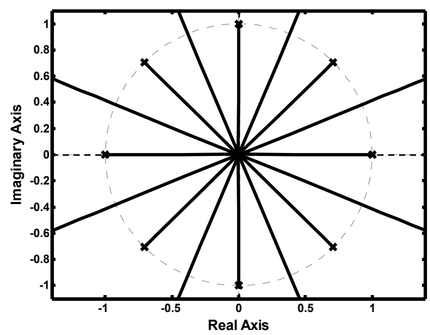

Fig. 4. Root locus for base case, 0th order system.

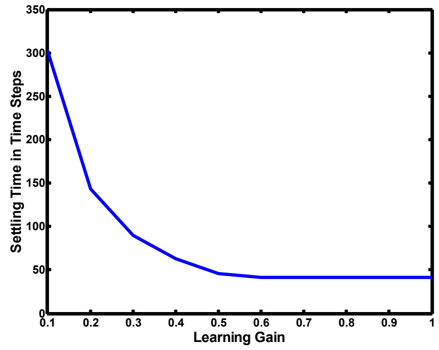

Fig. 5. Settling time in time steps vs. learning gain ϕ for the 1st order system with inverse compensator.

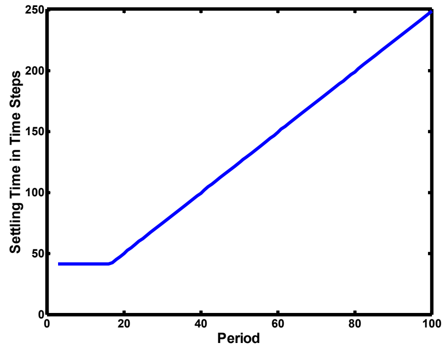

Fig. 6. Settling time in time steps vs. period for the 1st order system.

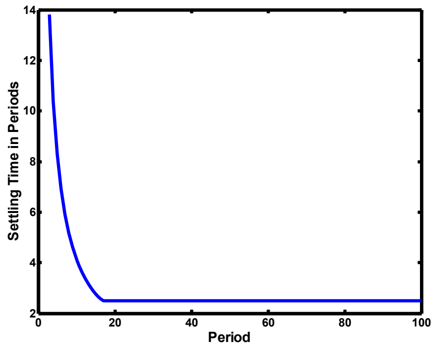

Fig. 7. Settling time in periods vs. period for the 1st order system.

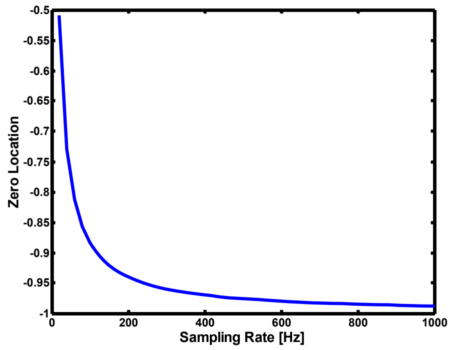

Fig. 8. Zero location vs. sample rate for the 2nd order system.

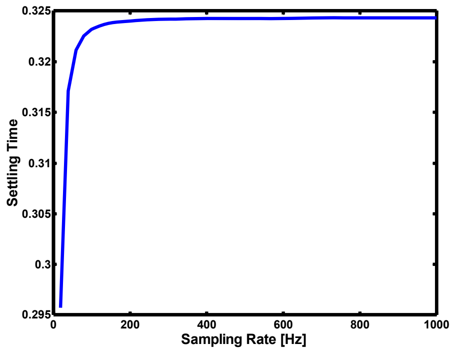

Fig. 9. Settling time in seconds vs. sample rate for a pole at the zero location, 2nd order system.

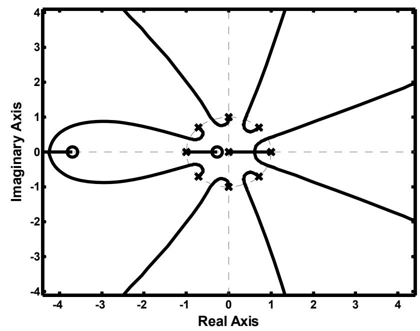

Fig. 10. Root locus plot for phase cancellation compensator design.

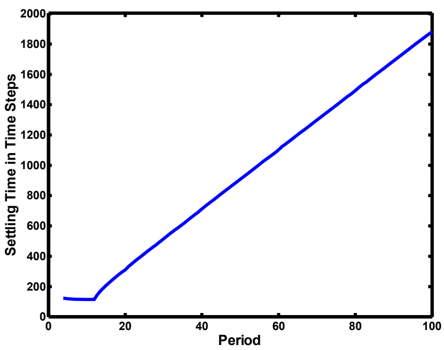

Fig. 11. Settling time in time steps vs. period, phase cancellation compensator design, $\phi = 0.8$.

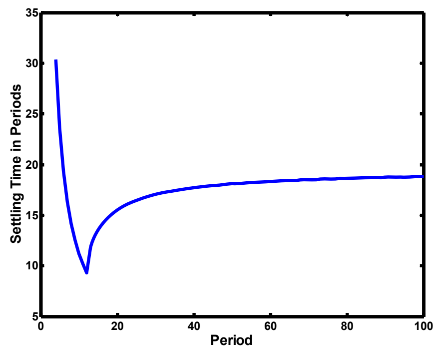

Fig. 12. Settling time in periods vs. period, phase cancellation compensator design, $\phi = 0.8$.

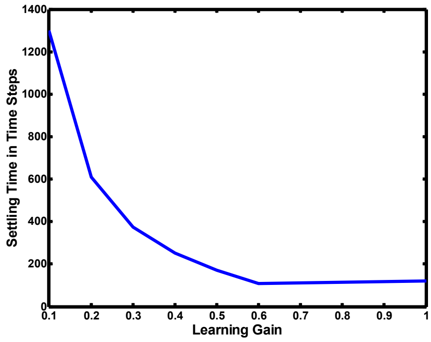

Fig. 13. Settling time in time steps vs. learning gain ϕ , phase cancellation compensator design, $p = 8$.

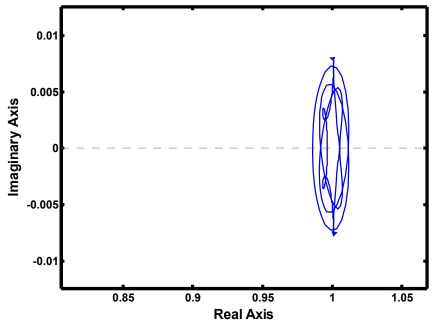

Fig. 14. Polar plot of $F(e^{i\omega T}) \cdot G(e^{i\omega T})$, 10 gain compensator, the 3rd order system..

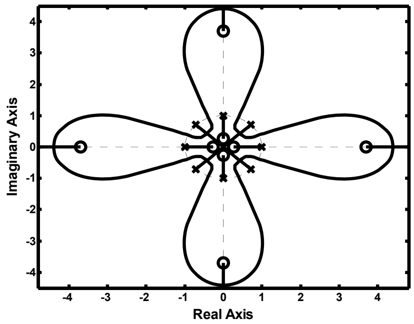

Fig. 15 Root locus plot for asymptotic zero locations for FIR compensator design, $p = 8$.

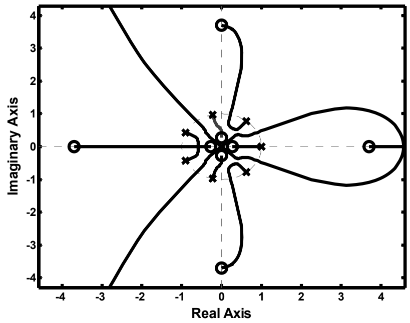

Fig. 16. Root locus plot for asymptotic zero locations for FIR compensator design, $p = 7$.

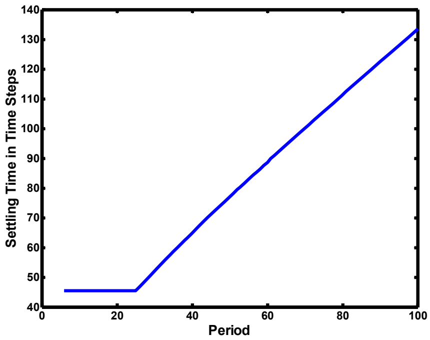

Fig. 17. Settling time in time steps vs. period for the 3rd order system using 10 gain FIR compensator, $\phi = 1.0$.

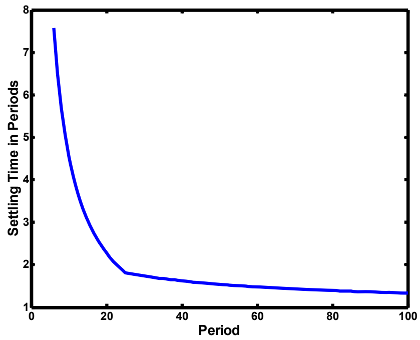

Fig. 18. Settling time in periods vs. period for the 3rd order system using 10 gain FIR compensator, $\phi = 1.0$.

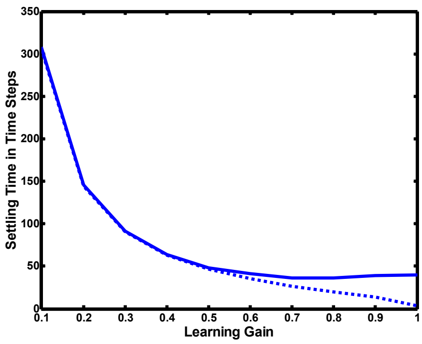

Fig. 19. Settling time in time steps vs. learning gain ϕ using 9 gain for zeros inside and outside (solid line), 4 gain for zero outside only (dashed) FIR compensator using asymptotic zero locations, $p = 8$.

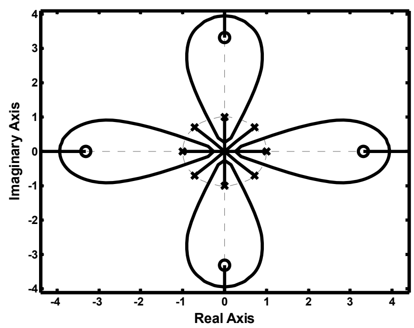

Fig. 20. Root locus plot for 4 gain FIR compensator with asymptotic zero location outside unit circle, $p = 8$.