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A Modified Design for the VS-MRAC Based on the Indirect Approach: Stability Analysis

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Abstract: Recently, an alternative to achieve a robust controller that provides a straightforward and intuitive design and tuning of its parameters named Indirect Variable Structure Model Reference Adaptive Controller (IVS-MRAC) was presented for relative degree one LTI plants, suggesting to be globally asymptotically stable with superior transient behavior and disturbance rejection properties. Its novelty is in the procedure to obtain the bounds for the relay's amplitudes, used in the switching laws. These bounds are now associated with the plant parameters, instead of the controller parameters. In this paper, a modification is made on the plant high frequency gain switching law, in order to develop a first formal stability analysis, considering the presence of input disturbances and unmodeled dynamics. It is shown that the overall system error is stable with respect to some small residual set.

1. INTRODUCTION

The acronym VS-MRAC designates the class of variable structure (VS) controllers (Utkin, 1992) which require only input/output measurements to be implemented, first proposed in (Hsu and Costa, 1989), for relative degree one plants, then extended in (Hsu, 1990), for the general case. The main interest of the VS-MRAC relies on its remarkable stability and performance robustness properties (Costa and Hsu, 1992; Hsu et al., 1994). Since (Hsu and Costa, 1989), several developments have been made and the VS-MRAC already was applied to SISO linear and nonlinear systems (Min and Hsu, 2000) and MIMO linear and nonlinear systems (Cunha et al., 2003). Practical aspects, as chattering elimination (Peixoto et al., 2001) and simplified algorithms (Hsu et al., 1994) has been studied. All these works are based on the direct adaptive control approach, where the switching laws are designed for the controller parameters. In the controller design for higher order plants, the bounds for the controller parameters may become harder to find, mainly due to the increasingly complexity of the matching equations, which, by the way, depend on the nominal plant parameters and their respective uncertainties.

In view of this fact, a natural solution would be design switching laws for the plant parameters instead of the controller parameters, as in the direct case. Thus, it was presented in Oliveira and Araujo (2004) a redesign for the VS-MRAC using the indirect adaptive control approach. It was named Indirect VS-MRAC (IVS-MRAC). The IVS-MRAC leads to a straightforward design for the relays amplitudes, since they are related with the plant parameters, which present uncertainties that can be known easier than in the direct approach, considering that they are related with physical parameters, as inertia moments, friction coefficients, resistances, capacitances and so on. Likewise the direct VS-MRAC, simulations have suggested fast transient and external disturbance rejection. Moreover, experimental results (Oliveira and Araujo, 2004), besides its feasibility, have suggested the robustness of the controller in the presence of unmodeled dynamics. The robustness of the direct VS-MRAC to unmodeled dynamics and external disturbances was considered in (Costa and Hsu, 1992), based on the singular perturbation approach (Kokotovic and Khalil, 1986).

It is noteworthy that indirect variable structure controllers proposed by Stotsky (1994) differs from the VS-MRAC essentially in the structure, dynamical behaviour and stability properties. It uses a least-squares-like adaptation mechanism on the plant parameters for ideal parameter matching and introduces a discontinuous term in the control law. Controllers that present integral adaptation are often associated with the necessity of some richness condition to achieve the control goal. This persistent excitation condition may be undesirable or even impossible to obtain in certain applications.

The procedure of deriving the IVS-MRAC switching laws from the conventional integral laws for the plant parameters estimates used in the indirect MRAC (Ioannou and Sun, 1996) generates a compound discontinuous function at the plant high frequency gain (k_p). This type of function is also called "nested" discontinuity and is outside the scope of Filippov's theory. Some works on combined sliding modes observer-controller introduce some low-pass filtering to handle it (Weiwen and Gao, 2003). In this paper, the direct handling with this function is avoided, by a suitable modification on the k_p VS law. It is substituted by an integral law and, then, not only the nested discontinuity but also an inherent algebraic loop is overcome. In doing this, the IVS-MRAC becomes a combined algorithm, as in Stotsky (1994), but in a different point of view. The combined IVS-MRAC has parametric adaptation restricted to only one system parameter, which guarantees and preserves the non oscillatory and fast transient provided by VS based algorithms, as well as their robustness properties. Simulations are presented to reinforce and clarify the results.

II PROBLEM STATEMENT

2.1 Plant Parameterization and Assumptions

This paper considers the control of the linear, single input/single output, relative degree one plant $(n^* = 1)$ with a singularly perturbed state space representation given in “actuator form” (Kokotovic and Khalil, 1986) as

$$\begin{aligned} \dot{x} &= A_1 x + A_{12} z + b_1 (u + d) \\ \mu \dot{z} &= A_2 z + b_2 (u + d) \\ y &= h^T x; \quad h^T = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \end{aligned} \tag{1}$$

where $x \in R^n$ is the state vector, $z \in R^k$ is the vector associated with the parasitics, u is the input, y is the output, d is some input disturbance and μ is a small positive constant. Any system in the general form

$$\begin{aligned} \dot{x} &= A_{11} x + A_{12} \zeta + b_1' (u + d), & x \in R^n \\ \mu \dot{\zeta} &= A_{21} x + A_{22} \zeta + b_2' (u + d), & \zeta \in R^m \\ y &= h^T x \end{aligned} \tag{2}$$

with μ sufficiently small and A_{22} nonsingular, can be transformed to the actuator form (1) by the transformation $z = Lx + \zeta$, where L is one solution of the algebraic Riccati equation $0 = A_{21} + \mu L A_{11} - A_{22} L - \mu L A_{12} L$ and $A_1 = A_{11} - A_{12} L$, $A_2 = A_{22} + \mu L A_{12}$, $b_1 = b_1'$ and $b_2 = b_2' + \mu L b_1'$. This parameterization is quite suitable for analysing systems with simultaneous fast and slow dynamics (Costa and Hsu, 1992).

The nominal model of the plant used in the controller design is given by a reduced order approximation of the plant (1) obtained by formally making $\mu = 0$, that is

$$\begin{aligned} \dot{x} &= A_r x + b_r u \qquad (d = 0) \\ y &= h_r^T x \end{aligned}$$

where $A_r = A_1, b_r = b_1 - A_{12} A_2^{-1} b_2$ and $h_r^T = h^T$. The corresponding transfer function is denoted by

$$W_r(s) = h_r^T (sI - A_r)^{-1} b_r = k_p \frac{n_r(s)}{d_r(s)} \tag{3}$$

where $k_p = h_r^T b_r = h^T \left(b_1 - A_{12} A_2^{-1} b_2\right)$ is the high frequency gain of the plant. The matrix A_r and vectors b_r, h_r are uncertain, but belong to known sets. From (3), n_r and d_r are monic polynomials written as

$$n_r(s) = s^{n-1} + \sum_{i=1}^{n-1} \beta_i s^{n-1-i} \tag{4}$$

$$d_r(s) = s^n + \alpha_1 s^{n-1} + \sum_{i=1}^{n-1} \alpha_{i+1} s^{n-1-i} \tag{5}$$

The reference model is defined by

$$y_m(s) = M(s) r(s), \quad M(s) = k_m \frac{n_m(s)}{d_m(s)} \tag{6}$$

where y_m is the output. The reference signal r is assumed piecewise continuous and uniformly bounded. As in (4)-(5), n_m and d_m are given by

$$n_m(s) = s^{n-1} + \sum_{i=1}^{n-1} \beta_{m,i} s^{n-1-i} \tag{7}$$

$$d_m(s) = s^n + \alpha_{m,1} s^{n-1} + \sum_{i=1}^{n-1} \alpha_{m,i+1} s^{n-1-i} \tag{8}$$

From (3)-(5), the vector of nominal plant parameters is defined as

$$\theta_p = \begin{bmatrix} k_p & \beta^T & \alpha_1 & \alpha^T \end{bmatrix}^T \tag{9}$$

where $\beta \in R^{n-1}$ contains the elements $\beta_i (i = n-1, \ldots, 1)$ of (4), $\alpha_1 \in R$ is the element α_1 of (5), $\alpha \in R^{n-1}$ contains the elements $\alpha_{i+1} (i = n-1, \ldots, 1)$ of (5) and, similarly, it is defined $\beta_m, \alpha_{m,1}, \alpha_m$, with respect to (6)-(8). The following assumptions regarding the plant and the reference model are made:

(A1) the reduced model is observable and controllable with $\text{degree}(d_r) = n$ and $\text{degree}(n_r) = n-1$, n known; **(A2)** $\text{sign}(k_p) = \text{sign}(k_m)$ (positive, for simplicity); **(A3)** $n_r(s)$ is Hurwitz, i.e., $W_r(s)$ is minimum phase; **(A4)** M(s) has the same relative degree of $W_r(s)$ and is chosen to be strictly positive real (SPR); **(A5)** both $r(t)$ and $d(t)$ are assumed piecewise continuous and uniformly bounded, i.e., $\sup|r(t)| \le \bar{r}$ and $\sup|d(t)| \le \bar{d}$ for some constants $\bar{r}$ and $\bar{d}$; **(A6)** upper bounds for the nominal plant parameters are known; **(A7)** the neglected dynamics is stable, i.e., $\text{Re}(\text{eig}(A_2)) < 0$.

The control aim is to achieve asymptotic convergence of the output or tracking error

$$e_o(t) = y(t) - y_m(t) \tag{10}$$

to zero or, despite the presence of input disturbance (bounded but not necessarily small) and unmodeled dynamics, guarantee that every signal in the resulting closed-loop system remains uniformly bounded and the output error e_o becomes ultimately small in some sense.

2.2 Error Equation

From standard MRAC, the plant input and output filters are given by

$$\dot{v}_1 = \Lambda v_1 + gu, \quad \dot{v}_2 = \Lambda v_2 + gy \tag{11}$$

where v_1, v_2, $g = [0 \ \ldots \ 0 \ \gamma]^T \in R^{n-1}$, $\gamma > 0$ and Λ is chosen such that $\det(sI - \Lambda) = n_m(s)$. The regressor vector is $\omega = [v_1^T \ y \ v_2^T \ r]^T$. When the plant is perfectly known, a control law which achieves matching between the system closed-loop transfer function and $M(s)$ is given by (Sastry and Bodson, 1989)

$$u^* = \theta_{v_1}^{*T} v_1 + \theta_n^* y + \theta_{v_2}^{*T} v_2 + \theta_{2n}^* r = \theta^{*T}\omega \tag{12}$$

where the nominal controller parameters vector is

$$\theta = [\theta_{v_1}^T \ \ \theta_n \ \ \theta_{v_2}^T \ \ \theta_{2n}]^T \tag{13}$$

From this control parameterization, it is convenient that (12) satisfies the inequality $\sup_t |u(t)| \le K_\omega \sup_t \|\omega(t)\| + K_\delta; \ \ \forall t$, where $K_\omega, K_\delta > 0$. This prevents the finite time escape.

Defining $F := A_2 z + b_2 u$ and $X^T = [x^T \ \ v_1^T \ \ v_2^T]$, the plant (1) and the filters (11) can be represented as

$$\begin{aligned} \dot{X} &= \bar{A}X + \bar{b}u + \bar{A}_{12}F + \bar{b}_1 d \\ \mu\dot{z} &= F + b_2 d \end{aligned} \tag{14}$$

where

$$\bar{A} = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & \Lambda & 0 \\ gh^T & 0 & \Lambda \end{bmatrix}, \quad \bar{b} = \begin{bmatrix} b_r \\ g \\ 0 \end{bmatrix}, \quad \bar{A}_{12} = \begin{bmatrix} A_{12}A_2^{-1} \\ 0 \\ 0 \end{bmatrix}, \quad \bar{b}_1 = \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix}.$$

Now, adding and subtracting $\bar{b}\theta^{*T}\omega$ in (14) and using the relation $\omega_r = WX$, where W is a constant matrix with elements 0 or 1, one has

$$\dot{X} = A_c X + b_c r + \bar{b}(u - \theta^{*T}\omega) + \bar{A}_{12}F + \bar{b}_1 d \tag{15}$$

where $A_c = \bar{A}X + \bar{b}\theta_r^{*T} W$ and $b_c = \bar{b}\theta_{2n}^*$. The above equation is valid only for the direct case (Costa and Hsu, 1992). Since IVS-MRAC derives from indirect MRAC (Ioannou and Sun, 1996), (15) must be rewritten to explicitly include the plant parameters. Let $\hat{k}_p, \hat{\beta}, \hat{\alpha}_1, \hat{\alpha}$ be the estimated values for $k_p, \beta, \alpha_1, \alpha$. Defining the error on the plant parameters vector as

$$\tilde{\theta}_p = [\tilde{k}_p \ \ \tilde{\beta}^T \ \ \tilde{\alpha}_1 \ \ \tilde{\alpha}^T]^T \tag{16}$$

$$\tilde{k}_p = \hat{k}_p - k_p^*; \quad \tilde{\beta} = \hat{\beta} - \beta^* \tag{17}$$

$$\tilde{\alpha}_1 = \hat{\alpha}_1 - \alpha_1^*; \quad \tilde{\alpha} = \hat{\alpha} - \alpha^* \tag{18}$$

and adding and subtracting $\hat{k}_p u$ in the third term of (15), using (12) and their respective matching equations (Ioannou and Sun, 1996) given by

$$\theta_{v_1}^* = \frac{\beta_m - \beta^*}{\gamma}; \ \theta_n^* = \frac{\alpha_1^* - \alpha_{m,1}}{k_p^*} \tag{19}$$

$$\theta_{v_2}^* = \frac{\alpha^* - \alpha_m + (\alpha_{m,1} - \alpha_1^*)\beta_m}{k_p^*\gamma}; \ \theta_{2n}^* = \frac{k_m}{k_p^*} \tag{20}$$

the following error system description arises

$$\begin{aligned} \dot{e} &= A_c e + \frac{b_c}{k_m}(\tilde{\theta}_p^T\zeta) + + \bar{A}_{12}F + \bar{b}_1 d \\ \mu\dot{z} &= F + b_2 d; \quad e_o = h_c^T e \end{aligned} \tag{21}$$

where ζ is an auxiliary signal vector given by $\zeta = [\zeta_p \ \ k_p^*\zeta_\beta^T \ \ \zeta_1 \ \ \zeta_\alpha^T]^T$, which has the elements

$$\zeta_p = \frac{\beta_m^T v_1}{\gamma_p} - u - \frac{\hat{\beta}^T v_1}{\gamma_p} \tag{22}$$

$$\zeta_\beta^T = [\zeta_{\beta_1} \ \ \ldots \ \ \zeta_{\beta_{n-1}}], \quad \zeta_{\beta_i} = \frac{-v_{1,i}}{\gamma} \tag{23}$$

$$\zeta_1 = y - \frac{\beta_m^T v_2}{\gamma_1} \tag{24}$$

$$\zeta_\alpha^T = [\zeta_{\alpha_2} \ \ \ldots \ \ \zeta_{\alpha_n}], \quad \zeta_{\alpha_i} = \frac{v_{2,i-1}}{\gamma} \tag{25}$$

with $\zeta_p, \zeta_1 \in R$, $\zeta_\alpha, \zeta_\beta \in R^{n-1}$ and $\gamma, \gamma_1, \gamma_p > 0$. Therefore, $\tilde{\theta}_p^T\zeta = \tilde{k}_p\zeta_p + k_p^*\tilde{\beta}^T\zeta_\beta + \tilde{\alpha}_1\zeta_1 + \tilde{\alpha}^T\zeta_\alpha$.

III DESIGN OF THE IVS-MRAC

Recently (Oliveira and Araujo, 2004), it was proposed a controller similar to direct VS-MRAC (Hsu and Costa, 1989), but with the VS laws designed for the plant parameters (PP), whereas in direct case they are on the controllers parameters (CP). The idea is to simplify the design, mainly in the stage of tuning the relays amplitudes, since that now the uncertainties on the PP can be known easier, making their adjustment more intuitive. Instead of parametric adaptation (Andrievsky et al., 1996), the IVS-MRAC relies on signal synthesis, being the control signal generated from a switching mechanism (Utkin, 1992). It was proposed the following VS laws for (9):

$$\hat{k}_p = k_p^{nom} - \bar{k}_p \operatorname{sgn}(e_o\zeta_p) \tag{26}$$

$$\hat{\beta}_i = -\bar{\beta}_i \operatorname{sgn}(e_o\zeta_{\beta_i}), i = 1, \ldots, n-1 \tag{27}$$

$$\hat{\alpha}_1 = -\bar{\alpha}_1 \operatorname{sgn}(e_o\zeta_1) \tag{28}$$

$$\hat{\alpha}_i = -\overline{\alpha}_i \operatorname{sgn}(e_o \zeta_{\alpha_i}), i = 2, \ldots, n \tag{29}$$

where the auxiliary signals in ζ are given as in (22)-(25) and k_p^{nom} was introduced to guarantee assumption (A2), due to its switching behavior. A first attempt to formalize the stability properties of the IVS-MRAC is presented in this paper. Initially, one sees that, expanding (26) and using (22) and (27), one has

$$\hat{k}_p = k_p^{nom} - \overline{k}_p \operatorname{sgn}\left\{ e_o \left\{ \left[\beta_m + \overline{\beta} \operatorname{sgn}(e_o \zeta_\beta) \right]^T \frac{v_1}{\gamma} - u \right\} \right\} \tag{30}$$

The $\hat{k}_p$ VS law presents a "nested" discontinuity of the form

$$\hat{k}_p = k_p^{nom} - \overline{k}_p \operatorname{sgn}(k_1 + k_2 \operatorname{sgn}(\cdot) + k_3) \tag{31}$$

with k_1, k_2 and k_3 defined from (30). This type of compound function is outside the scope of the Filippov's theory and is common in sliding modes observer-controller schemes (Wang and Gao, 2003), where they overcome it by an appropriate filtering. Others avoid it, using continuous weighted VS laws (Nunes et al., 2004). In this paper, the direct handling with it is avoided through a suitable substitution, called k_p – modification. Equation (26) will be replaced by a simple integral adaptation law (Ioannou and Sun, 1996) as

$$\dot{\hat{k}}_p = \begin{cases} -\gamma_p e_o \zeta_p & \text{if } |\hat{k}_p| > k_0 \text{ or} \\ & \text{if } |\hat{k}_p| = k_0 \text{ and } e_o \zeta_p \le 0 \\ 0 & \text{otherwise} \end{cases} \tag{32}$$

where $\hat{k}_p(0) \ge k_0 > 0$ and k_0 is a known lower bound for $|k_p^*|$. This combined IVS-MRAC differs from traditional combined algorithms (Andrievsky and Fradkov, 2003), since the adaptation is restricted to only one parameter. The further analysis and simulations will show that the fast transient of the VS algorithms is preserved.

IV STABILITY ANALYSIS

In the following, c_i denote positive constants.

THEOREM 1 Consider system (1), the overall system error (21), the VS laws (27)-(29) and the integral law (32). If all assumptions (A1)-(A7) are satisfied, and, in (27)-(29), one has $\overline{\alpha}_1 > |\alpha_1^*|$, $\overline{\beta}_i > |\beta_i^*|, i = 1, \ldots, n-1$ and $\overline{\alpha}_i > |\alpha_i^*|, i = 2, \ldots, n$, then every trajectory of the system enters an invariant compact residual set

$$D_R = \left\{ (e, z, \tilde{k}_p) : V(e, z, \tilde{k}_p) \le \left(c_1 \sqrt{\mu} + c_2 \overline{d} + \delta \right)^2 + c_3 \tilde{k}_p^2 \right\} \tag{33}$$

in some finite time, where $\delta > 0$ is arbitrarily small.

REMARK 1 The following auxiliary Lemma is necessary.

LEMMA 1 (Jiang, 1988) – The matrix $\begin{bmatrix} P & R^T \\ R & S \end{bmatrix}$ with $P = P^T > 0$ and $S = S^T$ is positive definite if and only if

$$S - RP^{-1}R^T > 0 \tag{34}$$

PROOF – THEOREM 1 This proof follows closely Costa and Hsu (1992). It is proposed the following candidate Lyapunov function

$$2V(e, z, \tilde{k}_p) = \begin{bmatrix} e^T & \mu z^T & \tilde{k}_p \end{bmatrix} \begin{bmatrix} P & R^T & 0 \\ R & S & 0 \\ 0 & 0 & (\gamma_p k_m)^{-1} \end{bmatrix} \begin{bmatrix} e \\ \mu z \\ \tilde{k}_p \end{bmatrix} \tag{35}$$

From (37), two cases must be considered.

Case 1: $\dot{\hat{k}}_p = -\gamma_p e_o \zeta_p$, $\gamma_p > 0$

The computation of $\dot{V}(e, z, \tilde{k}_p)$ along the system trajectories (21) gives

$$\begin{aligned} \dot{V}(e, z, \tilde{k}_p) &= -e^T Q e + \frac{e_o}{k_m} \left(k_p^* \tilde{\beta}^T \zeta_\beta + \tilde{\alpha}_1 \zeta_1 + \tilde{\alpha}^T \zeta_\alpha \right) \\ &+ \mu z^T \left[\frac{1}{2} \left(A_2^T + S A_2 \right) + R \overline{A}_{12} A_2 \right] z + \mu z^T R A_c e \\ &+ \frac{\mu z^T R b_c}{k_m} \left(\tilde{k}_p \zeta_p + k_p^* \tilde{\beta}^T \zeta_\beta + \tilde{\alpha}_1 \zeta_1 + \tilde{\alpha}^T \zeta_\alpha \right) \\ &+ \mu z^T \left(R \overline{A}_{12} + S \right) b_2 u + d \left(\overline{b}_1^T P + b_2^T R \right) e + + \mu z^T \left(R \overline{b}_1 + S b_2 \right) d \end{aligned} \tag{36}$$

Since A_2 is Hurwitz then there exist matrices $P_1 = P_1^T$ and $Q_1 = Q_1^T > 0$ such that

$$A_2^T P_1 + P_1 A_2 = -2Q_1 \tag{37}$$

By choosing $S = \alpha P_1$ with $\alpha > 0$ sufficiently large one can satisfy (34) and simultaneously assure that the quadratic term $\mu z^T \left[\frac{1}{2} \left(A_2^T + S A_2 \right) + R \overline{A}_{12} A_2 \right] z$ is negative definite and bounded above by $-\mu z^T Q_2 z < 0$, $Q_2 > 0$. From (36), using the relation $u = \theta^T \omega = \theta_r^T W(e + X_m) + \theta_{2n} r$ and knowing that the second term of (36), $\frac{e_o}{k_m} \left(k_p^* \tilde{\beta}^T \zeta_\beta + \tilde{\alpha}_1 \zeta_1 + \tilde{\alpha}^T \zeta_\alpha \right) = \frac{1}{k_m} \left(\tilde{\theta}_{p_r}^T \zeta_r \right) e_o$ is bounded above by

$$-\frac{k_{\theta_p}}{k_m} \|\zeta_r\| |e_o|, \ k_{\theta_p} = \min_i \left\{ \tilde{\theta}_{p_r, i} - \left| \theta_{p_r, i}^* \right| \right\} > 0, \ i = 1, \ldots, 2n-1.$$

where the vectors $\tilde{\theta}_{p_r}$ and ζ_r are constructed from (16), (23)-(25), one has

$$\begin{aligned} \dot{V}(e, z, \tilde{k}_p) \le \mu z^T &\left(Q_3 e + Q_4 r + Q_5 X_m + Q_6 d + Q_7 \zeta_p \right) + \\ &+ e^T Q_8 d - e^T Q e - \mu z^T Q_2 z \end{aligned} \tag{38}$$

where the bound for the second term of (36) was neglected. The auxiliary matrices in (38) are given by

$$
\begin{aligned}
Q_3 &= \frac{Rb_c}{k_m} Rb_c\left(\hat{\theta}_{pr} - \theta^*_{pr}\right)^T H + RA_c \\
&+ \left(R\overline{A}_{12} + S\right)b_2\theta_r^T W \\
Q_5 &= Q_3 - RA_c; \quad Q_6 = R\overline{b}_1 + Sb_2 \\
Q_7 &= \frac{Rb_c}{k_m}\left(\hat{k}_p - k_p^*\right), \quad Q_8 = R^T b_2 + P\overline{b}_1
\end{aligned} \tag{39}
$$

where $H = \begin{bmatrix} 0 & \frac{-k_p^*}{\gamma} & 0 \\ 0 & 0 & \frac{1}{\gamma} \\ h^T & 0 & \frac{-\beta_m^T}{\gamma} \end{bmatrix}$. Defining $\tilde{z} := \sqrt{\mu} z$ and using suitable upper bounds of each term of (39) gives

$$
\begin{aligned}
\dot{V}(e,z,\tilde{k}_p) \le &\left(-k_1\|e\|^2 - k_2\|\tilde{z}\|^2 + k_3\sqrt{\mu}\|e\|\|\tilde{z}\|\right) + k_4\sqrt{\mu}\|\tilde{z}\| + \\
&+ k_5\sqrt{\mu}\|\tilde{z}\|\overline{d} + k_6\sqrt{\mu}\|\tilde{z}\|\left|\zeta_p\right| + k_7\|e\|\overline{d}.
\end{aligned} \tag{40}
$$

The signal $\left|\zeta_p\right|$ in (40), since it depends on the control signal, is easily proven to be bounded and is also associated with μ and $\overline{d}$. It is clear that there exists $\overline{\mu} > 0$ sufficiently small such that $\left(-k_1\|e\|^2 - k_2\|\tilde{z}\|^2 + k_3\sqrt{\mu}\|e\|\|\tilde{z}\|\right) \le k_8\|\tilde{\varepsilon}\| < 0$, for all $\mu \in (0, \overline{\mu}]$, $\overline{\mu} > 0$, where $\tilde{\varepsilon} := \left[e^T \quad \tilde{z}^T\right]$. Then

$$
\dot{V}\left(\tilde{\varepsilon}, \tilde{k}_p\right) \le -k_8\|\tilde{\varepsilon}\|^2 + \left(k_4\sqrt{\mu} + k_5\sqrt{\mu}\overline{d} + k_6\sqrt{\mu}\left|\zeta_p\right| + k_7\overline{d}\right)\|\tilde{\varepsilon}\| \tag{41}
$$

From (41) and using Barbalat's lemma to extend (41) for asymptotic convergence, one concludes that $\dot{V}\left(\tilde{\varepsilon}, \tilde{k}_p\right) < 0$ outside the ball $D = \left\{\tilde{\varepsilon} : \|\tilde{\varepsilon}\| \le M\right\}$, where

$$
M = \left(k_9\sqrt{\mu} + k_{10}\sqrt{\mu}\overline{d} + k_7\overline{d}\right)/k_8
$$

Returning back to the variables e and z one has that inside D, $\|e\| \le M$ and $\|z\| \le \frac{M}{\sqrt{\mu}}$. In order to show that any trajectory of the system enters a residual domain of the form $D_R = \left\{\left(e, z, \tilde{k}_p\right) : V(e, z, \tilde{k}_p) \le k_R\right\}$ in some finite time, one should estimates the constant $k_R > 0$ such that in the set $W_R = \left\{\left(e, z, \tilde{k}_p\right) : V(e, z, \tilde{k}_p) \ge k_R\right\}$ one has $\dot{V} \le -\delta_R$ for some constant $\delta_R > 0$. Through Rayleigh's inequality, finding an upper bound for the Lyapunov function (35) and using the above upper bounds for $\|e\|$ and $\|z\|$, a possible choice of k_R is $k_R = \left(c_1\sqrt{\mu} + c_2\sqrt{\mu}\overline{\zeta}_p + c_3\overline{d} + \delta\right)^2 + c_4\tilde{k}_p^2$, $\delta > 0$ arbitrarily small, i.e., $D_R = k_R$, which assures that $D \subset \overline{D} \subset D_R$ and $\dot{V}$ is strictly negative in W_R.

COROLLARY 1 – If the trajectory of the system is inside D_R then $\|e\| \le c_4\sqrt{\mu} + c_5\overline{d} + c_6\tilde{k}_p$ and $|u|$ is ultimately of order $\|z\| \le c_7\sqrt{\mu} + c_8\overline{d} + c_9\tilde{k}_p + c_{10}\sup|r| + c_{11}\sup\|X_m\|$.

PROOF – Using a lower bound for (35) and the definition of D_R it follows that

$$
\frac{\lambda_{\min}\left(\overline{P}\right)\|e\|^2}{2} \le \left(c_1\sqrt{\mu} + c_2\overline{d} + \delta\right)^2 + c_3\tilde{k}_p^2
$$

with $\overline{P}$ being the matrix in (35) and $\lambda_{\min}$ its minimum eigenvalue. Then, inside D_R

$$
\|e\| \le c_4\sqrt{\mu} + c_5\overline{d} + c_6\tilde{k}_p \tag{42}
$$

Since $e_o = h_c^T e$, it follows that $\left|e_o\right|$ is $O(\|e\|)$. Substituting each term of the relation $u = \theta^T\omega = \theta_r^T W(e + X_m) + \theta_{2n} r$ by some upper bound and using (42) results that

$$
|u| \le c_8\sqrt{\mu} + c_9\overline{d} + c_{10}\tilde{k}_p + c_{11}\sup|r| + c_{12}\sup\|X_m\|
$$

Since $\mu\dot{z} = A_2 z + b_2 u$ and A_2 is Hurwitz by assumption (A7), then $\|z\|$ is ultimately of the same order as $|u|$.

Case 2: $\dot{\hat{k}}_p = 0$

The proof follows exactly as the case 1, except for the adding of the term $\frac{e_o\tilde{k}_p\zeta_p}{k_m}$, which must be strictly negative. Since, according to (32), this case occurs when $e_o\zeta_p \operatorname{sgn}\left(\hat{k}_p\right) > 0$ and $\left|\hat{k}_p\right| = k_0$, one has

$$
e_o\zeta_p > 0 \Rightarrow \operatorname{sgn}\left(\hat{k}_p\right) > 0 \Rightarrow \hat{k}_p = k_0 \Rightarrow \tilde{k}_p < 0 \Rightarrow \frac{\tilde{k}_p e_o \zeta_p}{k_m} < 0 \tag{43}
$$

Thus, the Theorem 1 and the Corollary 1 are also valid.

V SIMULATIONS RESULTS

Using the notation introduced in (3)-(8), the following example is considered:

$$
W_r(s) = k_p \frac{s + \beta_1}{s^2 + \alpha_1 s + \alpha_2} = \frac{s+3}{s^2 + 3s - 10}
$$

$$
M(s) = \frac{k_m}{s + \alpha_{m,1}} = \frac{1}{s+1} \ ; v = \ \dot{v}_1 = -v_1 + u, \quad \dot{v}_2 = -v_2 + y
$$

$$
\Delta(s) = \frac{-\mu s + 1}{\mu s + 1} \ ; \ W(s) = \Delta(s) W_r(s)
$$

The relays amplitudes for the VS laws (27)-(29) are easily adjusted from W_r, obeying the sufficient conditions of the Theorem 1. The used parameters are $\overline{\beta}_1 = 3.5, \overline{\alpha}_1 = 3.5$ and $\overline{\alpha}_2 = 11$. The design parameters for the adaptation law are

$\gamma_p = \gamma = 1$ and $\hat{k}_p(0) = k_0 = 0.8$. Initial condition is $y(0) = 1$. The integration step is $h = 10^{-3}$ and the derivatives are approximated by Euler method. The multiplicative parasitics $\Delta(s)$ makes the system non minimum phase.

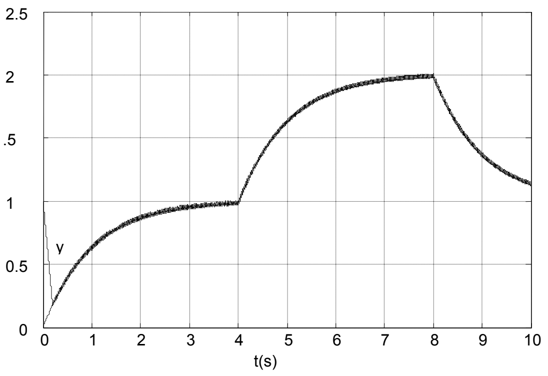

Fig 1.Reference tracking in the ideal case ($\mu = 0; d = 0$)

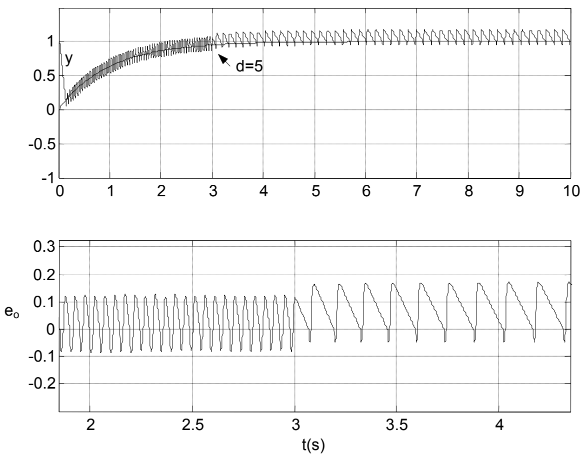

Fig 2. Simultaneous effect of parasitics with $\mu = 0{,}005$ and step disturbance (d=5) introduced at t=3s

Fig. 1 shows the output of the system free of input disturbances and parasitics in the tracking mode. In Fig. 2, the simultaneous presence of unmodeled dynamics and disturbance is shown. In all figures, one sees the absence of oscillations and overshoot during the transient, being preserved its fast behaviour. This fact is according to Theorem 1, since $\tilde{k}_p$ in the steady-state is small. When the sufficient conditions for the relays amplitudes are obeyed (always considered a fact in practice), the integral adaptation has little influence in the performance.

VI CONCLUSIONS

The Combined IVS-MRAC is shown to be remarkably robust with respect to disturbances and unmodeled dynamics, even when an integral adaptation law acts on the high frequency gain of the plant. This adaptation does not affect the performance if the sufficient conditions for the relays amplitudes are obeyed. This new combined algorithm preserves the fast transient response, inherent in VS schemes. The stability analysis has shown that the system is globally asymptotic stable with respect to a small residual set. Further works will deal with the nested discontinuity, in order to design an IVS-MRAC totally based on signal synthesis and with an equivalent structure to the direct case.

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