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Power-Grid-Partitioning Model and its Tabu-Search-Embedded Algorithm for Zonal Pricing

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Abstract: Power grid partition is a major problem in power markets based on zonal marginal price (ZMP). As the preparation of modeling, the zone of a node set is defined as the minimum-girthed convex polygon containing these nodes, and a theorem judging common nodes between two zones of node sets is proposed. Then, the power-grid-partitioning problem is described as a combinatorial optimization model, and solved by a heuristic algorithm embedded by tabu search (TS). Finally, two actual examples comprising 27 and 323 nodes from northeastern power grid of China prove the validity of the proposed algorithm. In contrast to other partitioning methods, the proposed one in this paper is electrical-information considered and automatically finished, and therefore applicable to actual power system.

Keywords: power system; zonal marginal price (ZMP); grid partition; combinatorial optimization problem; tabu search (TS)

1. INTRODUCTION

Uniform marginal price (UMP) is the original pricing mode in power markets since the introduction of the microeconomics theory (Ding and Fuller, 2005). Locational marginal price (LMP) is then proposed in order to present the market participants with economic signals (Litvinov et al., 2004). In theory, the LMP mode is perfect, as it correctly accounts for transmission constraints (and losses in some certain versions). But, the large number of prices is confusing, and the economic signals are very unstable. Therefore, as the trade-off of UMP and LMP, zonal marginal price (ZMP) mode is applied to many true power markets, such as new electricity trading arrangement (NETA) in England and Wales (Clarke, 2002; Hesmondhalgh, 2003), Australia national electricity market, the markets in Norway and Texas, and so on.

The partition of power grid is a major problem in ZMP-based power market (Wu, 2005; Zhang, 2006). However, the operators in actual power market operation usually establish zones manually based on their experiences without mathematical analysis (Yang et al., 2005). It is reported in (Alomoush and Shahidehpour, 1999) that marginal price of a node should be close to those of the other nodes in the same zone. Yang et al. (2005) have proposed a scale hierarchical clustering method based on the radial basis function network (RBFN) for grid partitioning, but the congestion information is needed. However, in NETA, the transmission lines are supposed to have infinite capacities. As a result, no congestion information can be obtained. Therefore, the zones in this mode should be established according to nodal electrical information. Kang et al. (2004) partition a power market from China completely based on its administrative regions with electrical information unconsidered. National Grid (2007) presents only some elementary principles but no detailed operable partitioning method.

In this paper, an automatically power-grid-partitioning method for zonal pricing is presented. Generally speaking, the nodes within zones should be adjacent to each other in geographical location. This constraint is usually realized by limiting the maximum distances within zones (Yang et al., 2005), which theoretically results in round contours of zones. Nevertheless, in fact, the contour lines of zones may be round, square, or triangular, and so on. As a result, the concept of the zone of a node set is proposed and defined as the minimum-girthed convex polygon containing these nodes. If there is no common node between any different two zones of node sets, the contours of zones can be in any shape and all nodes within zones are adjacent to each other. Besides this constraint, of course, the key values of nodes, e.g., the nodal prices, within zones should be approximate. A combinatorial optimization model of the power-grid-partitioning problem is built. A heuristic algorithm embedded by tabu search (TS) is developed to solve the model. And finally, the model and the algorithm are tested by some actual examples and proved to be efficient and applicable to actual power market.

2. PROBLEM FORMULATION

2.1 Power-Grid-Partitioning Problem

All the nodes in a power grid form the universal node set I_N. For each node $n \in I_N$, the nodal price is v_n, which is the most important electrical information of the node. Each node $n \in I_N$ has a geographical location (x_n, y_n) in Cartesian coordinates, where x_n and y_n are the horizontal and vertical coordinates, respectively. The set I_N is to be hard divided into some non-empty subsets. For each subset S_s, the nodes belonging to it should be adjacent to each other in geographical location, and the prices of the nodes belonging to it should be as proximate as possible. Additionally, the number of the subsets should be as small as possible.

The division of generation zones in NETA is very similar to the above partitioning problem. The nodal price v_n here can be defined as the cost of supplying an incremental load (demand) at the corresponding location in NETA (Zhang, 2005; National Grid, 2007). Moreover, the problem described above can be easily applied to other ZMP-based power markets if the nodal price is defined as the historical peak LMP or the average LMP at that location.

2.2 Preparative Definition and Theorem

Definition. The zone covered by a node set (for short, the zone of a node set) is the minimum-girthed convex polygon containing these nodes. The vertices of a node set are the vertices of the polygon. Joining all the vertices of the zone of a node set in turn, we can get a closed folded line. The closed folded line is defined as the outline of the set.

Theorem. Suppose the zones of node sets S_1 and S_2 are Z_1 and Z_2, and the contours are F_1 and F_2, respectively. There is no common node between Z_1 and Z_2, if and only if:

1) there is no common node between F_1 and F_2; and

2) any node in set S_1 is not in zone Z_2; and

3) any node in set S_2 is not in zone Z_1.

To judge common node between F_1 and F_2 is to judge common node between several pairs of line segments. To judge whether a node is in a zone is to judge whether a node belongs to a half-plane for several times. The detailed judging procedure of the theorem is omitted here for the limited space.

If there is no common node between any different two zones of node sets, then the nodes within any node sets are adjacent to each other and the contours of zones can be in any shape. This result is much closer to actual system than the single type of round contours resulted from the usual method of limiting distances. Furthermore, the above convexity is reasonable because there are only finite discrete nodes to be grouped. For example, though the state boundaries of a country in map may be of any shape, the contours of some big cities within states are convex in most circumstances.

3. MATHEMATICAL MODEL

The power-grid-partitioning problem can be modelled as follows:

$$\min |I_S| = \left|\left\{S_{s_1}, S_{s_2}, \cdots\right\}\right| , \tag{1}$$

$$\text{s.t. } \max_{n\in S_s} v_n - \min_{n\in S_s} v_n \le e, \ \ \forall S_s \in I_S , \tag{2}$$

$$Z_{s_1} \cap Z_{s_2} = \Phi, \ \forall s_1 \ne s_2 , \tag{3}$$

$$S_{s_1} \cap S_{s_2} = \Phi, \ \forall s_1 \ne s_2 , \tag{4}$$

$$\bigcup_{\forall S_s \in I_S} S_s = I_N , \tag{5}$$

$$S_s \ne \Phi, \ \ \forall S_s \in I_S . \tag{6}$$

Where, s, s_1, and s_2 are all the index of node sets; S_s, S_{s_1}, and S_{s_2} are the nodes in node sets s, s_1, and s_2, respectively; Z_{s_1} and Z_{s_2} are the zones of node sets s_1 and s_2, respectively; $I_S = \left\{S_{s_1}, S_{s_2}, \cdots\right\}$ consists of all the node sets; e is the maximum spread permitted of nodal prices within node sets; Φ stands for empty set; and $|\bullet|$ stands for the size of the particular set.

In model (1)-(6), the objective (1) minimizes the size of I_S, i.e. the number of the subsets; constraint (2) means the maximum spread of nodal prices within subsets; constraint (3) guarantees that there is no common node between any different two zones of subsets; equation (4) means that there is no common node between any different two subsets; (5) guarantees the union set of all subsets is I_N; and (6) tells us none of the subsets can be empty. Constraints (4)-(6) tells us the solution of the model is a hard division of I_N. In other words, each node should belong to and only belong to one subset.

The above model can exactly explain the division of generation zones in NETA; see (National Grid, 2007).

4. TABU-SEARCH-EMBEDDED ALGORITHM

4.1 Imitating Out-Point Method

In order to solve the grid-partitioning model, a heuristic algorithm is developed. Because the algorithm is similar to the traditional out-point method on the whole, it is named as *imitating out-point method*.

The following is the main idea of the algorithm. At first, let the number of subsets N_S equal to a relatively smaller value. Find the feasible solutions of the model with the given value of N_S. Increase the value of N_S little by little and then find feasible solutions once more with the new value of N_S if no feasible solution is found. Continue such search procedure until a feasible solution is found. The first found feasible solution is also an optimal one.

The step-by-step procedure of the algorithm:

Step 1: Find the lower bound of N_S.

Sub-Step 1-1: Initialize the sub-algorithm. $U = I_N$; $v_{\max} = \max_{n \in U} v_n$; and $N_S = 1$. Where, U is the set consisting of the ungrouped nodes; and $v_{\max}$ is the maximum nodal price in U.

Sub-Step 1-2: For node $n \in U$, if $v_n \in [v_{\max} - e,\ v_{\max}]$, then it is grouped into Subset N_S; and hence it never belongs to U.

Sub-Step 1-3: If U is not empty, then $v_{\max} = \max_{n \in U} v_n$; $N_S = N_S + 1$; and go Sub-Step 1-2.

Sub-Step 1-4: Otherwise, $N_{SL} = N_S$, where N_{SL} is the found lower bound of N_S. End the sub-algorithm.

Step 2: Initialize the solution with the given group number N_{SL}. The network is equally divided into the given number of small square grids. The solution derived from this action can meet constraint (3) surely, which can save the times of iterations to a certain degree.

Step 3: Find solutions meeting constraint (2) with the initial solution and the given fixed group number.

Step 4: If no solution meeting (2) is found, then divide the subset with the maximum spread of nodal prices into two ones; and the initial solution for the next iteration is the obtained new solution with the increased group number. $N_S = N_S + 1$. Go Step 3.

Step 5: Otherwise, the solution meeting (2) found in Step 3 is feasible and hence optimal. Stop.

The detailed description of Step 3 is in Subsection 4.2.

4.2 Embedded TS Algorithm

If we exchange (1) and (2), and reformulate them by little, we can get a sub-model. The new objective is

$$\min\ E = \max_{S_s \in I_S} (\max_{n \in S_s} v_n - \min_{n \in S_s} v_n) . \tag{7}$$

And the new constraint is

$$\text{s.t. } |I_S| = \left| \{ S_{s_1}, S_{s_2}, \cdots \} \right| = C . \tag{8}$$

Where, E is the maximum spread of nodal prices within subsets and C represents a number that never changes. The sub-model with objective (7) and constraints (3)–(6), (8) minimizes the maximum spread of nodal prices within subsets with the number of subsets fixed at its current value. Step 3 of the imitating out-point method is to solve this sub-model by TS.

TS, as a kind of meta-heuristic approach after the emergence of genetic algorithm (GA) and simulated annealing (SA), was first proposed in (Glover, 1989; Glover, 1990). In TS, to prevent the search from cycling, the attributes of recently visited solutions are memorized in a tabu list as a forbidden area for a number of iterations. The corresponding solutions to the attributes in the tabu list are forbidden, unless they meet a so-called aspiration criterion. TS has been successfully applied in many fields, mainly including a widespread variety of combinatorial optimization problems (Hammami and Ghedira, 2003; Hajji et al., 2005; Kannan et al., 2005; Sait et al., 2006; Wang et al., 2007).

The embedded TS algorithm of Step 3 is designed as follows:

1) Coding: The algorithm is coded with natural numbers. Each solution consists of $|I_N|$ elements with the values ranging from 1 to $|I_S|$ representing the subsets that they belong to. For example, if 10 nodes are to be divided into 3 groups, the following is a valid solution:

$$[2,\ 2,\ 3,\ 3,\ 1,\ 3,\ 1,\ 1,\ 2,\ 3].$$

In the above solution, the fist element "2" denotes that the first node belongs to the 2nd subset (group). Furthermore, in the 2nd subset, there are in total 3 nodes: the first, the second, and the ninth nodes.

2) Neighborhood structure: If any element of the current solution is changed into any other value of $\{1, 2, \cdots, |I_S|\}$, then we get a neighbour solution.

3) Tabu list: The pair of a node and its group number is a cell of the tabu list. For example, the element (2, 3) means that the 2nd node belongs to the 3rd subset. The maximum length of the tabu list is very important for the performance of the algorithm. It can be decided according to the number of nodes and be modified flexibly along with the iteration.

4) Aspiration criterion: If a neighbour solution is better than the optimum one in history, it will be selected as the current solution regardless its tabu status.

5) Stop rule: If the maximum number of iterations is reached, the iteration will stop.

5. SIMULATION RESULTS AND DISCUSSIONS

A lot of grid-partitioning problems have been effectively solved by using the model and algorithm developed in this paper. The programmes are written in matlab 6.5.1 and run in Dell personal computer with 512MB memories and Intel 2.80GHz CPU.

5.1 A 27-Node Example

The 500-kilo-volt power system in northeastern grid of China is taken as an example. There are 27 nodes in the system. For each node $n \in I_N = \{1, \cdots, 27\}$, the geographical location (x_n, y_n) measured from a map in centimetre (cm) and nodal price v_n derived from a transport model (National Grid, 2007) in kilometre (km) are shown in Table 1. The nodal prices here in unit of km are nodal marginal kilometres in (National Grid, 2007), which will become actual price if multiplied by a constant. The units of x_n, y_n, and v_n do not affect the result in this example.

The given maximum spread of nodal prices within subsets is 300 km. For simplicity, the size of the tabu list is fixed at 25. The difficulty of solving the sub-model by TS depends on the current subset number. Therefore, the maximum number of iterations of the TS algorithm varies along with the increasing subset number.

Table 1. Given data of the 27-node example

n	x_n (cm)	y_n (cm)	v_n (km)
1	4.4	53.1	1037
2	22.0	52.3	659
3	25.8	49.8	553
4	34.5	43.8	344
5	37.3	46.5	301
6	44.0	48.5	120
7	46.7	53.4	0
8	50.0	51.2	281
9	31.8	33.9	496
10	36.3	34.7	576
11	30.0	29.3	564
12	26.9	29.4	637
13	35.7	27.9	398
14	28.5	21.0	241
15	26.8	20.0	262
16	28.5	23.7	349
17	29.7	18.8	219
18	27.0	17.1	151
19	26.8	14.8	171
20	25.1	4.0	426
21	25.1	3.0	436
22	21.0	15.3	325
23	10.7	18.8	539
24	18.7	13.3	401
25	16.5	9.3	513
26	16.7	8.1	527
27	13.5	7.1	679

The lower bound of the subset number N_{SL} derived from Step 1 of the algorithm is 5. The grid-partitioning result is shown in Figure 1 with the subset number N_S being 6.

The simulation result indicates the validity of the model and algorithm. On one hand, the geographical locations of nodes within subsets (groups) are adjacent to each other. On the other hand, nodal prices within subsets are approximate. For example, we can find that the nodal price of Node 1 (the left-upper node in Fig. 1) is much higher than any other nodal prices. As a result, there is only one node in its group.

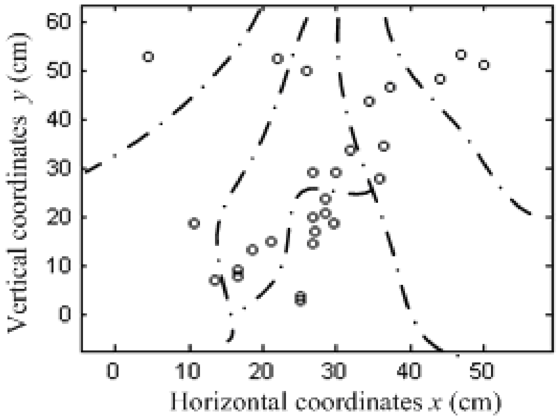

Fig. 1. Grid-partitioning result of the 27-bus example

Furthermore, it can be found that the lower bound of the subset number derived from Step 1 is very close to the final minimum subset number. It is much easier to start iterating from the derived lower bound of subset number than from the value of one. Therefore, the introduction of Step 1 of the algorithm contributes much to the algorithm.

5.2 A 323-Node Example

There are some 220 kilo-volt nodes and lines besides the 500 kilo-volt nodes and lines in the northeastern electrical transmission system. There are 323 nodes in the northeastern power system. According to the operation data in the year 2005, the nodal prices are calculated. Then, the whole grid is divided into 9 zones by using the grid-partitioning algorithm (Figure 2). All the detailed original data and the calculation parameters are omitted because of the limited space.

In Figure 2, each zone is outlined with a closed folded line and numbered at its centre. The geographical location data of the two examples are from two different maps. Therefore, the nodal coordinates are quite different. But this difference does not affect the results in our examples.

The 323-node example can prove the reasonableness of convexity of zones better than the 27-node example. Though the counters of node sets are defined to be convex and it is constrained that there is no common node between any two zones, reasonable practicable results can be successfully derived from the proposed algorithm. In Figure 2, the shapes of zones are very near to the actual regional boundaries in map.

Of course, the proposed algorithm containing the judgment of convexity is a little time-consuming. However, in practice, the grid-partitioning problem is solved at a very low frequency, just once or twice a year. As a result, it is unnecessary for us to solve the problem in real-time. Therefore, the running time of the program is not very important and hence unconsidered.

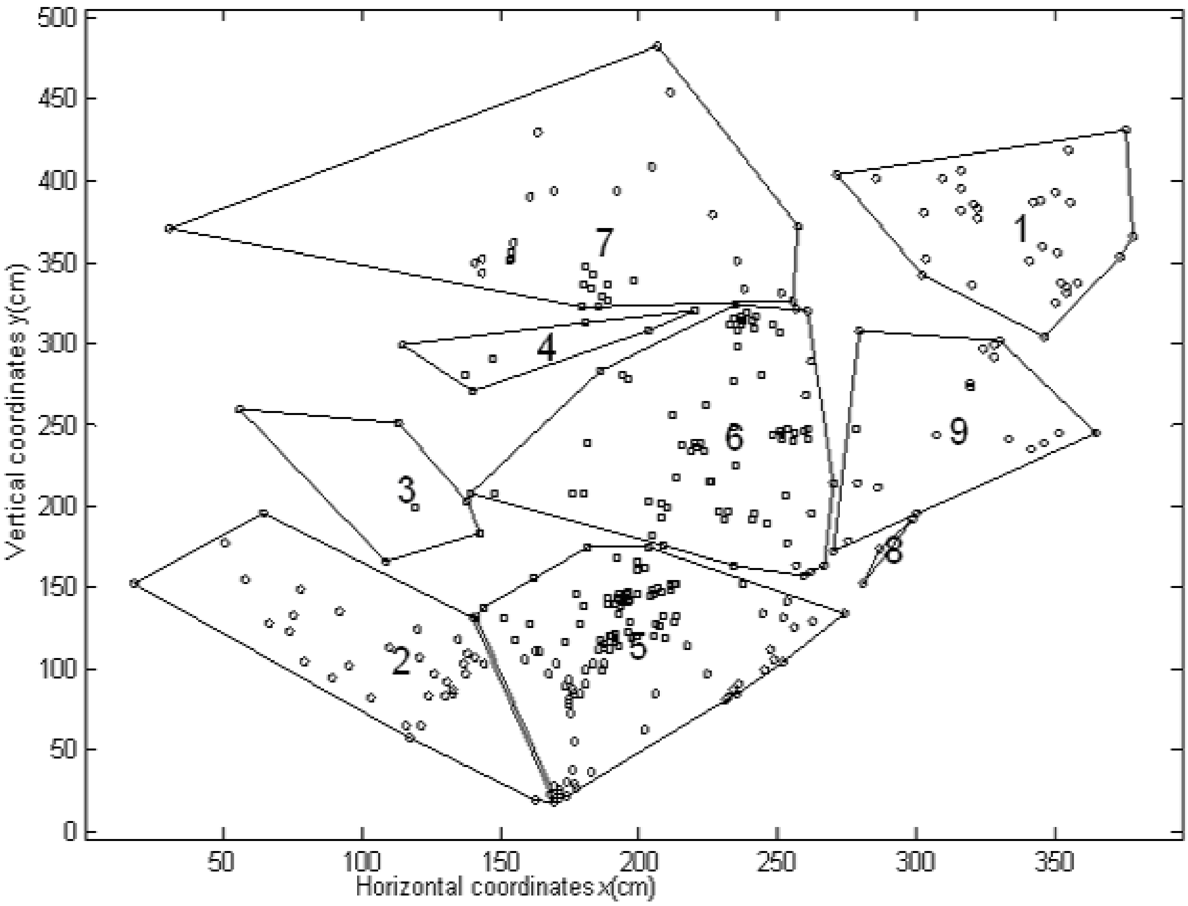

Fig. 2. Grid-partitioning result of the 323-node example

6. CONCLUSIONS

A combinatorial optimization model is built and a tabu-search-embedded heuristic algorithm is developed. The actual examples from northeastern power grid of China have proved the reasonableness and validity of the algorithm. In contrast to other grid-partitioning method, the proposed method is based on electric information and can be operated automatically. Therefore, it can be applied to actual zonal-pricing-based power system.

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