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Modification of Model Predictive Control to Reduce Cross-Coupling

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Abstract: We consider the problem of reducing interaction in the closed-loop response of model predictive control (MPC). Interaction in MPC may be caused by diagonal weighting of inputs in the MPC cost function that are not diagonally related to the outputs. If instead of weighting the plant inputs a suitable decoupled input signal is used in the MPC cost function, then a significant reduction in cross coupling can occur. In the case where the plant has a static interaction matrix, complete decoupling occurs. Simulation examples show that the procedure can be implemented via a simple modification to standard MPC algorithms, and is applicable to ill-conditioned and non-minimum phase plants.

1. INTRODUCTION

Decoupling (by which we mean diagonalisation), or approximate decoupling, has a long history in multivariable control systems design, going back at least to the 1960s. It is particularly valuable as a tool to permit extension of classical SISO control techniques to multivariable systems (see for example more 'recent' texts such as Maciejowski [1989], Morari and Zafiriou [1989]). More recently, optimisation techniques have been applied to multivariable system to deal with the complexity and interactions that may be present, and these offer a powerful way of achieving robust, stable, high performance control.

One such optimisation technique is Model Predictive Control (MPC). MPC is a process control strategy which has been studied extensively for dealing with complex, interacting, multivariable systems (see for example the survey article Rawlings [2000], and texts such as Comacho and Bordons [1995]). It has also seen widespread adoption by the process industry with many commercially available implementations. However, MPC, as with many other optimisation-based control approaches, does not directly consider 'interaction' as part of the cost to be optimised.

Decoupling of the closed-loop response of a multivariable system may be desirable even when using optimisation-based techniques, such as MPC, to design robust feedback controllers. In particular, since it is almost always the case that there is a hierarchy of control actions, decoupling at intermediate or lower levels, such as those utilising MPC, will simplify the job of higher level (e.g. supervisory) controls. Furthermore, from a process operator's viewpoint, decoupled (or approximately decoupled) responses greatly simplifies their tasks, and may improve their ability to diagnose and respond appropriately to plant disturbances. Reducing interactions may also enable processes to operate closer to their economic limits. In this paper, we therefore seek to design and simulate modifications to MPC that improve the level of decoupling achieved.

A number of other researchers have studied problems of decoupling in model predictive control, or it's close allies. Demircioglu and Gawthrop [1992] study a continuous time version of predictive control and show that in the limit, as the control weighting (λ in (Demircioglu and Gawthrop [1992])) vanishes, decoupled control is achieved. In Chai *et al* [1994], a technique for introducing a decoupling compensator to the reference signals is described. Other authors (for example Niemi *et al* [1997], Lu and Tsai [2001]), for specific applications, give decoupling schemes suitable for their particular application.

Our aim here is to propose a scheme which

- is generically applicable;
- significantly reduces the cross coupling (or ideally achieves perfect decoupling);
- requires only simple modification to the regular MPC cost function (and is therefore compatible with constrained control); and
- applies to systems without zero input weighting (and therefore can reasonably be applied to non-minimum phase systems).

The outline of this paper is as follows. Section 2 introduces some notation, presents the general MPC problem under consideration. Section 3 describes a well-known decoupling method based on state variable feedback (SVF), and some recent alternatives. Section 4 describes the re-formulation of the SVF decoupling method (from Section 3) as an MPC problem, and also describes some approximations and extensions. Section 5 presents some simulations of the new method, and Section 6 concludes the paper.

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2. MODEL AND PROBLEM FORMULATION

We begin by considering a stabilisable and detectable, square, multivariable discrete-time state space plant model, with state $x_k^p \in \mathbb{R}^{n_p}$, input $u_k \in \mathbb{R}^m$ and output $y_k \in \mathbb{R}^m$:

$$\begin{aligned} x_{k+1}^p &= A_p x_k^p + B_p u_k \\ y_k &= C_p x_k^p. \end{aligned} \tag{1}$$

Frequently, this model is augmented with additional state variables to incorporate integral action, with the new 'control' variables as being effectively the control 'increments' or control 'moves':

$$(\delta u)_k := u_k - u_{k-1}. \tag{2}$$

Combining (2) with (1) we obtain the extended state space model:

$$\begin{aligned} \begin{bmatrix} x_{k+1}^p \\ u_k \end{bmatrix} &= \begin{bmatrix} A_p & B_p \\ 0 & I \end{bmatrix} \begin{bmatrix} x_k^p \\ u_{k-1} \end{bmatrix} + \begin{bmatrix} B_p \\ I \end{bmatrix} (\delta u)_k \\ y_k &= [\, C_p \;\; 0 \,] \begin{bmatrix} x_k^p \\ u_{k-1} \end{bmatrix}. \end{aligned} \tag{3}$$

For simplicity, (3) can be written in the standard state space form

$$\begin{aligned} x_{k+1} &= A x_k + B(\delta u)_k \\ y_k &= C x_k \end{aligned} \tag{4}$$

with augmented state $x_k^T = \left[\, (x_k^p)^T \;\; u_{k-1}^T \,\right]$. Provided the original plant model, (1) has no transmission zeros at $z = 1$, and has at least as many outputs as inputs, then the augmented model (3) retains the stabilisability and detectability properties of the original model.

2.1 MPC Formulation

For simplicity, consider an unconstrained model predictive control problem, where at a particular time step, the inputs to be computed are denoted by:

$$U = \begin{bmatrix} (\delta u)_k \\ (\delta u)_{k+1} \\ \vdots \\ (\delta u)_{k+N_u-1}^T \end{bmatrix} \in \mathbb{R}^{mN_u}. \tag{5}$$

Similarly we define

$$Y = \begin{bmatrix} y_{k+1} \\ y_{k+2} \\ \vdots \\ y_{k+N_y} \end{bmatrix}, R = \begin{bmatrix} r_{k+1} \\ r_{k+2} \\ \vdots \\ r_{k+N_y} \end{bmatrix} \in \mathbb{R}^{mN_y} \tag{6}$$

for the output vector and reference vector respectively. We also define the error vector as $E = R - Y$. The output vector, Y can computed from the plant equation:

$$Y = \mathcal{P}U + H_0 x_k \tag{7}$$

where x_k are initial conditions based on current/past inputs and outputs, and $\mathcal{P}$, H_0 are given by

$$\mathcal{P} = \begin{bmatrix} CB & 0 & \dots & 0 \\ CAB & CB & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ CA^{N_u-1}B & \dots & CAB & CB \\ CA^{N_u}B & CA^{N_u-1}B & \dots & CAB \\ \vdots & \ddots & \ddots & \vdots \\ CA^{N_y-1}B & CA^{N_y-2}B & \dots & CA^{N_y-N_u}B \end{bmatrix} \tag{8}$$

$$H_0 = \left[\, CA \; CA^2 \; \cdots \; CA^{N_u} \; CA^{N_u+1} \; \cdots \; CA^{N_y} \,\right]^T \tag{9}$$

2.2 Coupling in Model Predictive Control

A common choice of cost function in MPC takes the form:

$$J(U) = E^T \Lambda E + U^T \beta U \tag{10}$$

where both Λ and β are diagonal weighting matrices. In particular, typically, we would have

$$\Lambda = Blockdiag\,[\, \lambda \;\; \lambda \;\; \dots \;\; \lambda \,] \tag{11}$$

where $\lambda > 0$ is a diagonal weighting matrix specifying the penalty on each error variable, and similarly:

$$\beta = Blockdiag\,[\, \beta_u \;\; \beta_u \;\; \dots \;\; \beta_u \,] \tag{12}$$

where $\beta_u > 0$ is a diagonal input weighting matrix. In this case, the cost function, (10) can be rewritten as

$$J = \sum_{\ell=k+1}^{k+N_y} e_\ell^T \lambda e_\ell + \sum_{\ell=k}^{k+N_u-1} (\delta u)_\ell^T \beta_u (\delta u)_\ell. \tag{13}$$

Since the weightings are diagonal, it might be expected that the optimisation will tend to reduce cross couplings. However, in general this is not the case. In particular, the unconstrained minimisation of (10) over possible inputs, U leads to

$$U = \left(\beta + \mathcal{P}^T \Lambda \mathcal{P}\right)^{-1} \mathcal{P}^T \Lambda \,(R - H_0 x_k) \tag{14}$$

and

$$\begin{aligned} Y &= \mathcal{P}\left(\beta + \mathcal{P}^T \Lambda \mathcal{P}\right)^{-1} \mathcal{P}^T \Lambda \,(R - H_0 x_k) + H_0 x_k \\ &= \mathcal{P}\left(\beta + \mathcal{P}^T \Lambda \mathcal{P}\right)^{-1} \mathcal{P}^T \Lambda R \\ &\quad + \left(I + \mathcal{P}\beta^{-1}\mathcal{P}^T \Lambda\right)^{-1} H_0 x_k. \end{aligned} \tag{15}$$

We denote by H_{YR} the implied closed-loop transfer matrix from R to Y:

$$H_{YR} = \mathcal{P}\left(\beta + \mathcal{P}^T \Lambda \mathcal{P}\right)^{-1} \mathcal{P}^T \Lambda. \tag{16}$$

We now consider some of the implications surrounding (15) and (16). Note that in general, if β and Λ are diagonal and positive definite (that is, fully decoupled), it does not follow that H_{YR} is diagonal, which leads to cross coupling in the MPC implementation. If, however, we have equal control and output horizon, and we let the control weighting become small, then for CB invertible, (and hence $\mathcal{P}$ invertible) we obtain $H_{YR} = I$ which is therefore decoupled. More generally, however, with $\mathcal{P}$ invertible we have

$$H_{YR} = \left(I + \Lambda^{-1}\mathcal{P}^{-T}\beta\mathcal{P}^{-1}\right)^{-1}. \tag{17}$$

From (17), we see that unless the blocks of $\mathcal{P}$ are diagonal (that is, the Markov parameters $CB, CAB, CA^2B, \dots$ are diagonal, or equivalently, the plant is diagonal), then generically, the blocks of the closed loop response, H_{YR}, will be coupled.

Before giving a proposed solution in MPC for this cross coupling, we briefly review decoupling in multivariable control.

3. DECOUPLING IN MULTIVARIABLE CONTROL

3.1 The Method of Falb and Wolovich

In Falb and Wolovich [1967], the problem of taking a multivariable continuous-time system in state space form, and generating a static state feedback decoupled system, is considered. These results extend immediately to discrete time state space systems, (1). In particular, it is desirable to find (if possible) nonsingular $G \in \mathbb{R}^{m\times m}$ and $F \in \mathbb{R}^{m\times n}$ so that if the control is computed as

$$(\delta u)_k = Fx_k + Gw_k \tag{18}$$

where w_k is an external reference signal, then combining (18) with (1) gives a decoupled system from w_k to y_k.

In special cases, we may select F, G in a straightforward manner. In particular, suppose that

$$det(CB) \neq 0. \tag{19}$$

Under (19), one example of a decoupling set of matrices (in fact, this is the pair F^*, G^* described in Falb and Wolovich [1967]) for this case is

$$G = (CB)^{-1}, F = -(CB)^{-1}CA. \tag{20}$$

Combining (1) with (18) and (20) gives

$$x_{k+1} = \left(I - B(CB)^{-1}C\right)Ax_k + B(CB)^{-1}w_k. \tag{21}$$

Note that since $C\left(I - B(CB)^{-1}C\right) = 0$, then whenever the state dimension n exceeds the number of output variables, m, the particular decoupled state space model (21) is not observable from the output $y_k = Cx_k$. In particular, it can be shown that the eigenvalues of $A_d := \left(I - B(CB)^{-1}C\right)A$ are precisely: m eigenvalues at the origin (since C is rank m and $CA_d = 0$) together with $n - m$ eigenvalues that are transmission zeros [1] of the original transfer function matrix. Clearly therefore, the choice (20) is not suitable for non-minimum phase systems.

3.2 Alternative Methods

Alternative methods of decoupling have been described in, for example, Wittenmark *et al* [1987], where for a specific class of Multivariable ARMA Systems, a moving average (MA) input decoupler can be constructed. This technique could also be utilised in our context. However, it turns out that it may not be desirable to implement full dynamic decoupling of multivariable systems, since this suffers from robustness problems [Morari and Zafiriou, 1989, §13.3.2] and may also exacerbate problems due to non-minimum phase transmission zeros [Goodwin *et al*, 2001, Chapt. 24]. We therefore prefer to use a static decoupler, aimed at approximately decoupling the transient response of the system, rather than more complex, and potentially less robust, dynamic decoupling techniques.

[1] To see this, let ζ be a transmission zero of the system with v_1, v_2 such that $Av_1 = \zeta v_1 - Bv_2$ and $Cv_1 = 0$. Then $A_d v_1 = \zeta v_1$.

4. REFORMULATION OF MPC WITH DECOUPLING

We now wish to combine the earlier MPC formulation, of Section 2.1 with the insights into decoupling of multivariable systems described in Section 3. The key here is that by using diagonal weightings of decoupled variables, w_k and y_k, in the MPC cost function, we are able to achieve decoupling, without having to resort to zero control weighting β.

Using the standard dynamic decoupling relationship (18) we obtain

$$w_k = G^{-1}(\delta u)_k - G^{-1}Fx_k. \tag{22}$$

We now stack into vector form the decoupled inputs as $W^T = \left[\, w_k^T \; w_{k+1}^T \; \dots \; w_{k+N_u-1}^T \,\right]$. Then extending (22), it follows that

$$W = DU - \mathcal{H}x_k \tag{23}$$

where $D = \begin{bmatrix} G^{-1} & 0 & \dots & 0 \\ -G^{-1}FB & G^{-1} & \dots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ -G^{-1}FA^{N_u-2}B & -G^{-1}FA^{N_u-3}B & \dots & G^{-1} \end{bmatrix}$

and $\mathcal{H} = \left[\, G^{-1}F \; G^{-1}FA \; \cdots \; G^{-1}FA^{N_u-1} \,\right]^T$. We then consider optimising the modified cost function J_W defined as follows:

$$J_W(U) = E^T \Lambda E + W^T \beta_W W \tag{24}$$

where β_W in (24) is a diagonal weighting matrix to be defined ($E = R - Y$ as before). The unconstrained minimisation of J_W gives the control

$$U = \left(D^T\beta_W D + \mathcal{P}^T\Lambda\mathcal{P}\right)^{-1} \times \left(\mathcal{P}^T\Lambda R - \mathcal{P}^T\Lambda H_0 x_k + D^T\beta_W\mathcal{H}x_k\right). \tag{25}$$

Note that use of J_W as proposed in (24) is equivalent to appropriate selection of a non-diagonal β in (10), together with cost terms depending on cross coupling between control increments and states (in the case $F \neq 0$ in (22)).

From (25)),

$$H_{YR} = \left(I + \Lambda^{-1}\mathcal{P}^{-T}D^T\beta D\mathcal{P}^{-1}\right)^{-1}. \tag{26}$$

With similar analysis to that in Subsection 2.2, if we have equal control and output horizon then $\mathcal{P}$ is square; it is already assumed to be invertible, and so H_{YR} becomes diagonal due to the design of D (i.e. $D\mathcal{P}^{-1} = I$).

4.1 Selection of β_W

Given an initial selection of the control weights, β that is diagonal, we would like to select weights for β_W that preserve some of the features of the original cost term $U^T\beta U$ (see (10)) in the revised cost term $W^T\beta_W W$ (see (24)). One such selection would be to take $\beta_W = diag\left(D^{-T}\beta D^{-1}\right)$ which corresponds to considering only the quadratic term in U in the expansion of $W^T\beta_W W$, and ignoring off-diagonal terms in this expression.

4.2 'Static' Decoupling

To avoid problems with robustness and with non-minimum phase systems, it may be preferable simply to take a 'static' relationship between u_k and w_k. For example, it may make sense to use $F = 0$. We are then concerned with the choice of G. Note that we would *not* normally choose G according to the steady-state plant gain matrix, since steady-state decoupling is achieved by integral action in the controller. Therefore, it seems more important to decouple the transient response of the system. For this reason, and under the assumption that the control signal will not vary too greatly from one sample to the next, one possible selection for the decoupling matrix is to sum the first k Markov parameters of the multivariable system:

$$G^{-1} = CB + CAB + CA^2B + \ldots + CA^{k-1}B \tag{27}$$

The choice of k is dependent on the desired accuracy.

So with G^{-1} defined as in (27), and $F = 0$, D is obtained such that β_W will deliver an approximate level of decoupling for many systems.

4.3 Reduced Decoupling

If the approximate (static) decoupling scheme requires more control energy and other compromises than is desirable, then it is a simple matter to choose a convex combination of the weightings for the two cases. In particular, take $\alpha \in [0, 1]$ and define the compromise cost function:

$$J_\alpha(U) = E^T \Lambda E + \alpha W^T \beta_W W + (1-\alpha) U^T \beta U. \tag{28}$$

Unconstrained optimisation then gives the control sequence

$$U = \left(\alpha D^T \beta_W D + (1-\alpha)\beta + \mathcal{P}^T \Lambda \mathcal{P}\right)^{-1} \times \left(\mathcal{P}^T \Lambda R - \mathcal{P}^T \Lambda H_0 x_k + D^T \beta_W \mathcal{H} x_k\right). \tag{29}$$

4.4 Triangular Decoupling

Triangular decoupling refers to the case where outputs are ranked according to some criteria (with y_1 being the highest priority), and changes affecting a lower priority output (e.g. setpoint changes, output disturbances) are not transferred to any higher priority outputs.

If triangular decoupling is desired, then it is sufficient to let $\beta_W = tril\left(D^{-T}\beta D^{-1}\right)$, where $tril(.)$ is an operation that selects the lower triangular terms of $D^{-T}\beta D^{-1}$. With $H_{YR} = (\Lambda + \beta_W)^{-1}\Lambda$ (from considering $D = P$), the relationship from R to Y is triangular, which is the desired result.

5. EXAMPLES

We consider three examples that illustrate the performance of the proposed algorithm.

5.1 Static Coupling, Ill-Conditioned Plant

We first consider a simple 2×2 example of a plant with very simple dynamics and an ill-conditioned static cross-coupling.

$$y(t) = \frac{1}{(s+0.1)} \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} u(t). \tag{30}$$

Note that the static coupling matrix in (30) is almost singular and has condition number 65.98. This precludes the use of a simple inversion of the D.C. gain as a static decoupling element.

The following simulations were performed using the algorithm suggested in Section 4.2, with parameters: sampling time, 1 second; $N_u = 5$, $N_y = 20$, $\beta = 300I$ and $\lambda = I$. Results for these simulations are shown in Figures 1 and 2.

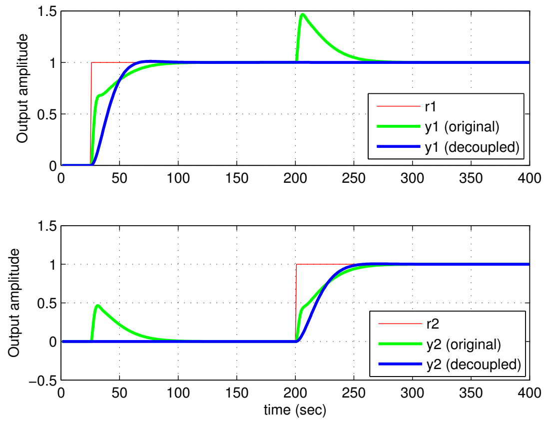

Fig. 1. Output responses for Example 5.1, with and without decoupling.

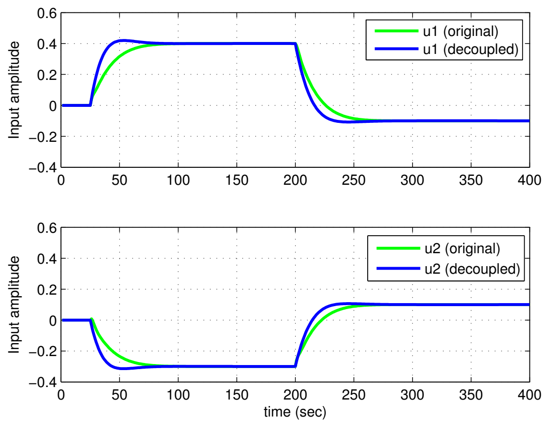

Fig. 2. Control responses for Example 5.1, with and without decoupling.

Note from Figures 1 and 2 that we are able to achieve complete decoupling in this case, with very little increase in control effort, a slower initial rise time, but similar, if not superior, settling time behaviour.

The triangular (partial) results are shown below in Figures 3 and 4. The triangular decoupling is clearly evident; even though the diagonal responses are similar, the main benefit is in the reduced control effort for input 1.

5.2 Non-minimum Phase Example

We now consider a 2×2 non-minimum phase example. This is based on a continuous-time plant model, $P_c(s)$, composed of individual SISO transfer functions

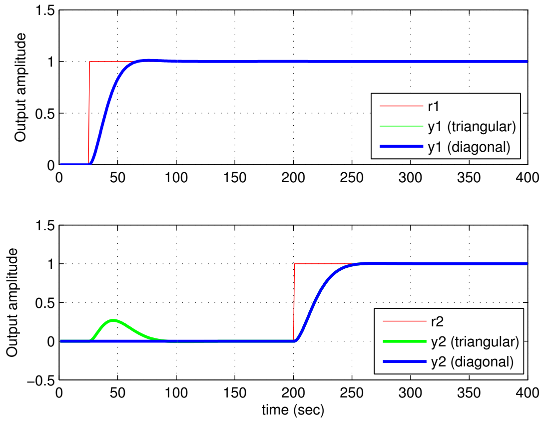

Fig. 3. Output responses for Example 5.1, with triangular and diagonal decoupling.

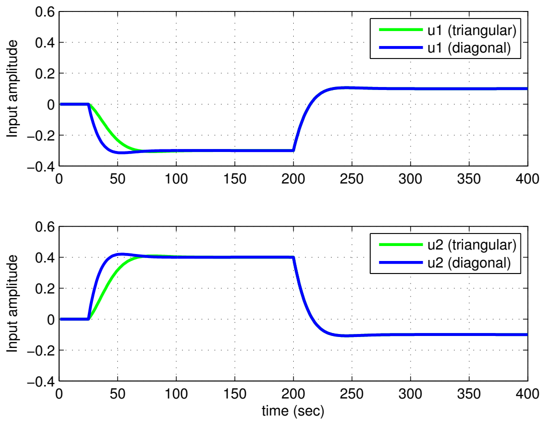

Fig. 4. Control responses for Example 5.1, with triangular and diagonal decoupling.

$$P_c(s) = \begin{bmatrix} \dfrac{-5}{25s+1} & \dfrac{0.995s-0.005}{s^2+s} \\ \dfrac{1}{25s+1} & \dfrac{-0.0023s-0.0023}{s^2+s} \end{bmatrix}. \quad (31)$$

The McMillan degree of $P_c(s)$ is 3, with poles at $s = 0, -0.04, -1$ and a transmission zero at $s = +0.0168$, which is therefore non-minimum phase. We use a 20 second sampling period, control horizon $N_u = 20$ samples, output horizon $N_y = 50$ samples, input weights $\beta = diag\{500, 2000\}$ and output weights $\lambda = diag\{1, 5\}$. The results are shown below in Figures 5 and 6.

In this case, we have not attempted to obtain perfect dynamic decoupling. Note that we obtain a substantial reduction in cross-coupling (for example, the peak coupling in the (1,2) element dropping from approximately 73% to 21%) with almost no change in control effort used, and a similar response time.

With absolute input limits of ± 0.2 units introduced, the results are shown below in Figures 7 and 8. Observe how the decoupling is preserved when the controls are constrained.

The triangular (partial) results are not shown due to the minor reduction in overall control effort.

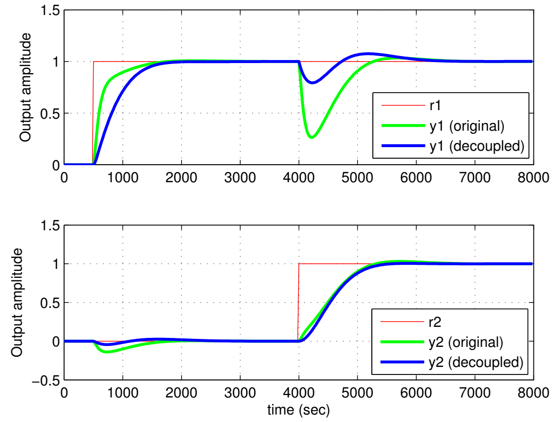

Fig. 5. Output responses for Example 5.2, with and without decoupling.

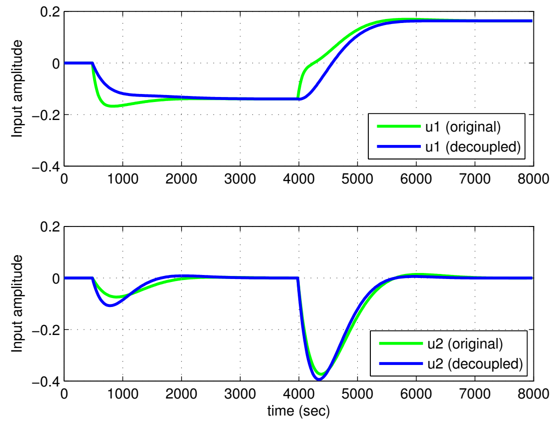

Fig. 6. Unconstrained control responses for Example 5.2, with and without decoupling.

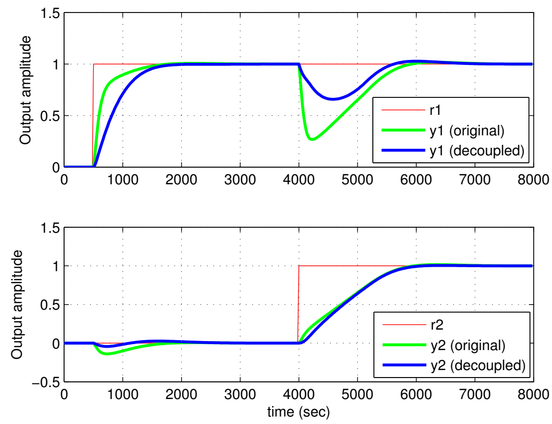

Fig. 7. Output responses for Example 5.2, with and without decoupling (constrained inputs).

5.3 3×3 *Example*

We lastly consider a 3×3 example, which exhibits significant time delay elements. The plant model is

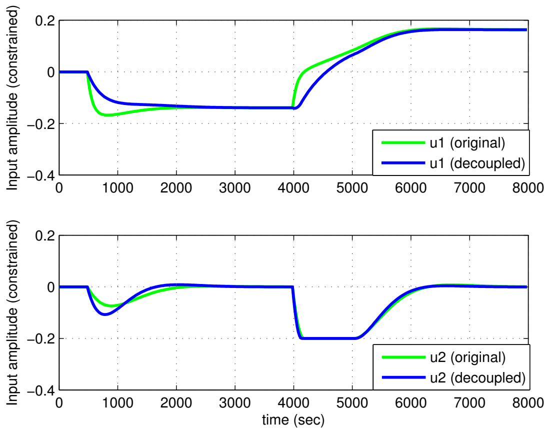

Fig. 8. Constrained control responses for Example 5.2, with and without decoupling.

$$P_c(s) = \begin{bmatrix} \frac{4.05e^{-27s}}{50s+1} & \frac{1.77e^{-27s}}{60s+1} & \frac{5.88e^{-27s}}{50s+1} \\ \frac{5.39e^{-16s}}{50s+1} & \frac{5.72e^{-16s}}{60s+1} & \frac{6.9e^{-16s}}{40s+1} \\ \frac{4.38e^{-11s}}{33s+1} & \frac{4.42e^{-11s}}{44s+1} & \frac{7.2e^{-11s}}{19s+1} \end{bmatrix}. \quad (32)$$

We use a 1 second sampling period, control horizon $N_u = 3$ samples, output horizon $N_y = 40$ samples, input weights $\beta = I$ and output weights $\lambda = I$. There are rate limits on the inputs of ± 0.05 units per second, and absolute input limits of ± 0.5 units. The results are shown below in Figures 9 and 10.

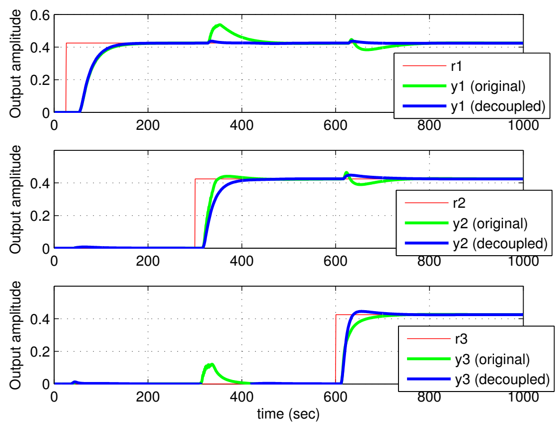

Fig. 9. Output responses for Example 5.3, with and without decoupling.

Note that the 3×3 system has been decoupled significantly, with little change in the closed-loop response for diagonal terms, and only small increases in control effort.

Triangular (partial) decoupling is also possible with this system, but the reduction in overall control effort is minor. Thus the results are not shown.

6. CONCLUSIONS

This note has shown how to modify the cost functions used in MPC in order to achieve approximate output decoupling. Using the concept of an input decoupling matrix, a simple modification of the MPC cost function used yields a decoupled (or less coupled) closed-loop response. The examples illustrate that this decoupling is achieved with little additional cost in terms of control energy, and with similar overall closed-loop response times. The scheme is applicable to a broad range of plants including those that are ill-conditioned and non-minimum phase. It is possible to achieve greater levels of decoupling in MPC, at the cost of less robustness.

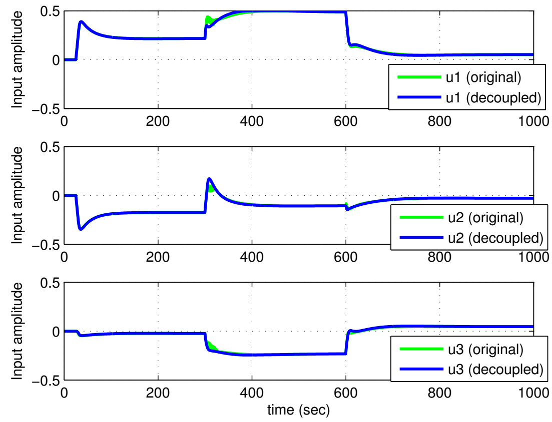

Fig. 10. Control responses for Example 5.3, with and without decoupling.

REFERENCES

T. Chai, K. Mao, X. Qin, "Decoupling design of multivariable generalised predictive control", *IEE Proceedings - Control Theory and Applications*, V141, N3, pp197-201, 1994.

E. Comacho and C. Bordons, "*Model Predictive Control in the Process Industry*", Springer, London, 1995.

H. Demircioglu and P. J. Gawthrop, "Multivariable continuous-time generalized predictive control (MCGPC)", *Automatica*, V28, N4, pp697-713, 1992.

P.L. Falb and W.A. Wolovich, "Decoupling in the design and synthesis of multivariable control systems", *IEEE Trans. on Automatic Control*, V12, N6, pp651-659, 1967.

G.C. Goodwin, S.F. Graebe, and M.E. Salgado, "*Control System Design*", Prentice-Hall, 2001.

C.H. Lu and C.C. Tsai, "Adaptive decoupling predictive temperature control for an extrusion barrel in a plastic injection molding process", *IEEE Trans. on Industrial Electronics*, V48, N5, pp968-975, 2001.

J.M. Maciejowski, "*Multivariable Feedback Design*", Addison-Wesley, 1989.

M. Morari and E. Zafiriou, "*Robust Process Control*", Prentice-Hall, 1989.

A.J. Niemi, L. Tian and R. Ylinen, "Model predictive control for grinding systems", *Control Engineering Practice*, V5, N2, pp271-278, 1997.

J.B. Rawlings, "Tutorial overview of model predictive control", *IEEE Control Systems Magazine*, V20, N3, pp38-52, 2000.

B. Wittenmark, R.H. Middleton and G.C. Goodwin, "Adaptive decoupling of multivariable systems", *International Journal of Control*, V46, N6, pp1993-2009, 1987.