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Yaw Control of RUAVs：an Adaptive Robust H_∞ Control Method

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Abstract—**The problem of robust H_∞ tracking control is considered for a class of linear system with time-varying uncertainties. The bounds of varying uncertainty ellipsoidal are obtained by set membership identification method. Using adaptive method, a new variable gain controller is designed to compensate the effect of uncertainty on systems. Then an application of this result to rotorcraft-based unmanned Aerial Vehicles (RUAVs) mounted on an experiment platform has demonstrated the effectiveness of the proposed method.**

I. INTRODUCTION

Research of Rotorcraft-based Unmanned Aerial Vehicles (RUAVs) has grown over the last decade as the operational requirements for such vehicles have increased in both military and civilian sectors. Potential applications of such unmanned vehicles include surveillance, reconnaissance and monitoring missions in an urban environment. RUAVs can also be used to test and validate a variety of technologies and techniques. Helicopter has complex dynamics associated with the interaction of the main rotor wake and the empennage, along with the aeroservoelastic couplings between the rotor and control system. RUAVs are typically much smaller and more agile than their full-scale counterparts, and have a particular challenge when compared to the full scale helicopters. The overall goal of the research is to determine the effectiveness of the stabilizing and robustness properties of control design techniques on RUAVs.

RUAV is a naturally unstable system with nonlinear dynamics. The complicated dynamics of helicopter lead to both parametric and dynamic uncertainty, so the controller should be designed to robust to those effects and advanced control strategies need to be used in order for a RUAV to fly autonomously. During last decade, a large number of robust controller design methods have been investigated [1-4]. Also, many robust controllers achieving some robust performances, such as H_∞ disturbance attenuation, guaranteed cost control method and so on have been presented [5-8]. If there are uncertainties in the system model, the norm H_∞ can be a desirable measure of a system's robust performance. The theoretic motivation for the H_∞ control problem and important results about output feedback control can be found in [9,10] and the references therein.

Recently, the author of this paper presented an adaptive robust H_∞ tracking control method for the helicopter control with time-invariant uncertainty [11]. Using adaptive method, a variable gain controller is designed to reduce conservatism inherent in fixed gains. Then, in [12], time-varying ellipsoidal uncertainty obtained by set membership identification method is considered in adaptive robust H_2 tracking control. To guarantee the asymptotic stability and H_2 performance of closed-loop system, an adjustable target model is introduced. Based on the error equation between state model and adjustable target model an uncertainty parameters adaptive law is obtained to construct variable gain controller. The application to the yaw control of a small-scale helicopter has shown the effectiveness of this method.

In this paper we will further consider an adaptive robust H_∞ tracking controller design with simpler structure than [12] for linear systems with time-varying uncertainty. The time-varying ellipsoidal uncertainty obtained by set membership identification method [13]. The aim of this paper is to combine robust control and adaptive control to realize their individual advantages. Without introducing adjustable target model, a simple variable gain robust controller is designed directly for augmented systems. It consists of a fixed gain and a variable gain. The fixed gain is determined by using the nominal system and the variable gain controller is designed to compensate effect of uncertainty on systems using adaptive mechanism. The resultant closed-loop systems possess the designed characteristic of robust and good performance. Sufficient conditions for the existence of adaptive robust H_∞ tracking controllers with variable gains are given in terms of LMIs and adaptive laws. An application of the proposed controller design for the rotorcraft-based unmanned Aerial Vehicles (RUAVs) mounted on an experiment platform is also given to show its effectiveness.

The paper is organized as follows. In Section II, the yaw dynamic of helicopter and the simplified model are given, followed by the robust tracking controller design in Section III. The application of the proposed controller to the yaw control of RUAV is discussed in Section IV. Finally, some conclusions are made at end of this paper.

II. MODELING YAW DYNAMIC OF HELICOPTER

In this paper a framework of the simulation model for the helicopter-platform (see Fig. 1) is set up using rigid body

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equations of motion of the helicopter fuselage. In hovering and low-velocity flight, the torque generated by main and force generated by tail rotor are dominant [14]. By simplifying the fuselage and vertical fin damping, the yaw dynamics can be rewritten as:

$$\begin{cases} \dot{\varphi} = r \\ I_{zz}\dot{r} = -Q_{mr} + T_{tr}l_{tr} + b_1 r + b_2\varphi \end{cases} \tag{1}$$

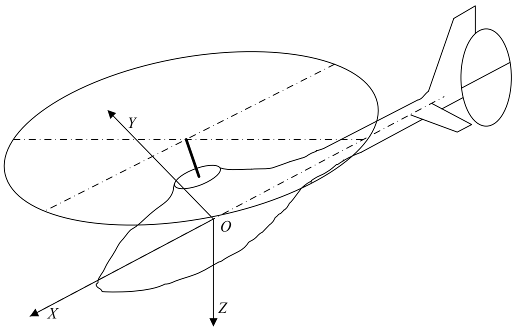

Fig.1 helicopter coordination

where Q_{mr} is the torque of main rotor, T_{tr} is the thrust of tail rotor, l_{tr} is the distance between the tail rotor and z-axis, b_1 and b_2 are damping constants. The expressions of T_{tr} and Q_{mr} has been given in [12]:

$$T_{tr} = C_1\theta_{tr} + \tfrac{1}{2}C_2(C_2 + \sqrt{C_2^2 + 4C_1\theta_{tr}}) \tag{2}$$

with

$$C_1 = \tfrac{1}{6}\rho a_{tr} b_{tr} c_{tr} \Omega_{tr}{}^2 (R_{tr}{}^3 - R_{\mathrm{tr}0}^3)$$

$$C_2 = \tfrac{1}{8}\rho a_{tr} b_{tr} c_{tr} \Omega_{tr} \sqrt{2/\rho\pi R_{tr}{}^2}\,(R_{tr}{}^2 - R_{tr0}^2)$$

where ρ, a_{tr}, b_{tr}, c_{tr}, Ω_{tr}, θ_{tr}, r_{tr}, v_{tr1} A_{tr} are respectively, density of air, slope of the lift curve, number of the rotor, chord of the blade, speed of the tail rotor, pitch angle, radial distance , induced speed of the tail rotor and area of the tail rotor disc.

$$\begin{aligned} Q_{mr} = {} & \frac{c\theta_{mr}}{48\pi R^2}\{8C_{d2}\Omega\sqrt{\rho\pi R^2(2C_3\theta_{mr} + C_4^2 - C_4\sqrt{C_4^2 + 4C_3\theta_{mr}})}(R_0^3 - R^3) \\ & + 4a\Omega\sqrt{\rho\pi R^2(2C_3\theta + C_4^2 - C_4\sqrt{C_4^2 + 4C_3\theta_{mr}})}(R^3 - R_0^3) \\ & + 6C_{d2}C_3(R^2 - R_0^2) + 6aC_3(R_0^2 - R^2) + 6C_{d1}\rho\pi\Omega^2 R^2(R^4 - R_0^4)\} \\ & + \frac{c}{48\pi R^2}\{6C_{d0}\rho\pi\Omega^2 R^2(R^4 - R_0^4) + 3aC_4^2(R_0^2 - R^2) \\ & + 3C_{d2}C_4\sqrt{C_4^2 + 4C_3\theta_{mr}}(R_0^2 - R^2) + 3aC_4\sqrt{C_4^2 + 4C_3\theta}(R^2 - R_0^2) \\ & + 4C_{d1}\Omega\sqrt{\rho\pi R^2(2C_3\theta_{mr} + C_4^2 - C_4\sqrt{C_4^2 + 4C_3\theta_{mr}})}(R_0^3 - R^3) \\ & + 3C_{d2}C_4^2(R^2 - R_0^2)\} + \tfrac{1}{8}C_{d2}\rho c\Omega^2(R^4 - R_0^4)\theta_{mr}{}^2 \end{aligned} \tag{3}$$

with $$C_3 = \tfrac{1}{6}\rho abc\Omega^2(R^3 - R_0^3)$$

$$C_4 = \tfrac{1}{8}\rho abc\Omega\sqrt{2/\rho\pi R^2}\,(R^2 - R_0^2)$$

where R, θ_{mr} are respectively, radial and pitch angle of main rotor, $a, \alpha, r, c, \phi, v_1, \Omega$ are respectively slope of the lift curve, the angle of attack of the blade element, speed radial distance, chord of the blade, inflow angle, induced speed and rotor speed of the main rotor.

From (1) we can see that there exist couplings between main rotor torque Q_{mr} and tail rotor thrust T_{tr}. And (2) and (3) further demonstrate that the models are highly nonlinear and too complex to be used for control design. Instead of the dynamics described by (2) and (3), a simplified model is proposed for control design :

By plotting the torque vs pitch angle, we can find that relation between Q_{mr} and θ_{mr} approximated with quadratic polynomial (see Fig.2)

$$Q_{mr} = k_{Q_2}\theta_{mr}^2 + k_{Q_1}\theta_{mr} + k_{Q_0}$$

where k_{Q_2} , k_{Q_1} and k_{Q_0} depend on the shape of the blades and the speed of main rotor Ω , while Ω are constant. So, k_{Q_0} , k_{Q_2} and k_{Q_1} are constants.

Similarly, the lift of tail rotor, T_{tr} (see Fig.3), can be written:

$$T_{tr} = k_{T2}\theta_{tr}^2 + k_{T1}\theta_{tr} + k_{T0}$$

Then we can obtain the following nonlinear model:

$$\begin{cases} \dot{\varphi} = r \\ I_{zz}\dot{r} = -(k_{Q_2}\theta_{mr}^2 + k_{Q_1}\theta_{mr} + k_{Q_0}) \\ \qquad + (k_{T2}\theta_{tr}{}^2 + k_{T1}\theta_{tr} + k_{T0})l_{tr} + b_1 r + b_2\varphi \end{cases} \tag{4}$$

For the yaw control , the force generated by main rotor can be treated with disturbance, so the yaw dynamic can be described by :

$$\begin{cases} \dot{\varphi} = r \\ \dot{r} = k_1 r + k_2\theta_{tr} + k_3\theta_{tr}^2 + k_4\Omega\,\theta_{tr} + k_5\varphi \end{cases} \tag{5}$$

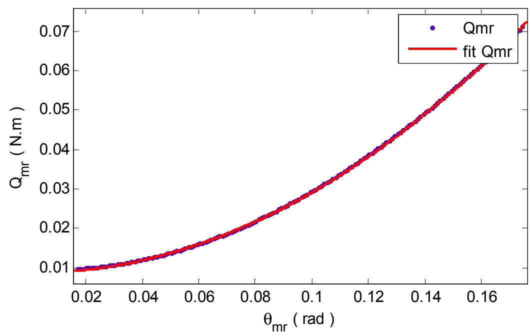

Fig. 2 Torque of main rotor with Quadratic Polynomial Fitting

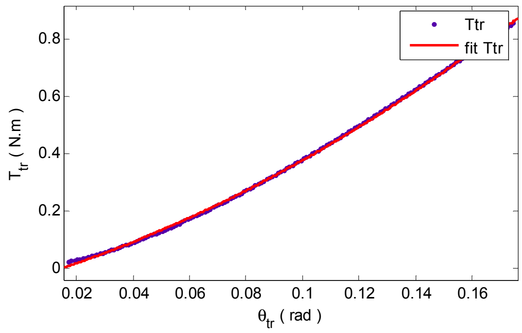

Fig. 3 Thrust of tail rotor with Quadratic Polynomial Fitting

The nonlinear dynamic can be presented by the following state space description

$$\dot{x} = f(x,u)$$

where, $x = [\varphi \quad r]^T, u = \theta_{tr}$.

Furthermore (4) can be linearized at a trim point (x_0, u_0)

$$\dot{x} = Ax + Bu \tag{6}$$

with

$$\mathbf{A}=\frac{\partial f}{\partial x}|_{x_0,u_0} = \begin{bmatrix} 0 & 1 \\ k_5 & k_1 \end{bmatrix}, \mathbf{B}=\frac{\partial f}{\partial u}|_{x_0,u_0} = \begin{bmatrix} 0 \\ a \end{bmatrix}$$

where, $a = 2k_3\theta_{tr0} + k_2 + k_4\Omega$.

III. ADAPTIVE ROBUST TRACKING CONTROLLER DESIGN

In this section, we propose the control method for linear systems with time-varying ellipsoidal uncertainty. This result can not only be used in this paper to solve the adaptive robust H_∞ tracking control for the rotorcraft-based unmanned Aerial Vehicles (RUAVs) mounted on an experiment platform, but also can be applied to other related problems due to its general formulation.

A. Problem statement and preliminaries

Consider the following linear uncertainty model described by

$$\begin{aligned} \dot{x}(t) &= A(\theta(t))x(t) + B(\theta(t))u(t) + B_\omega(\theta(t))\omega(t) \\ y(t) &= C_1 x(t) \end{aligned} \tag{7}$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input , $y(t) \in R^p$ is the measured output and $\omega(t) \in R^l$ is an exogenous disturbance which belongs to $L_2[0,\infty)$, respectively. The system matrices have the following time-varying structure

$$A(\theta(t)) = A_0 + \sum_{i=1}^{N} \theta_i(t) A_i$$

$$B(\theta(t)) = B_0 + \sum_{i=1}^{N} \theta_i(t) B_i$$

$$B_\omega(\theta(t)) = B_{\omega 0} + \sum_{i=1}^{N} \theta_i(t) B_{\omega i}$$

where $A_0, A_1 \cdots A_N, B_0, B_1 \cdots B_N, B_{\omega 0}, B_{\omega 1} \cdots B_{\omega N}$ are known constant matrices. The time-varying parameter vector $\theta(t) \in R^N$ represents unknown parameters which belong to the N-dimensional ellipsoidal set expressed as

$$\Delta \equiv \{\theta \in R^N \mid \theta^T(t)\Sigma^{-2}\theta(t) \le 1\} \tag{8}$$

$$\Sigma = \mathrm{diag}(\sigma_1, \cdots \sigma_N) \tag{9}$$

where $\Sigma \in R^{N \times N}$ represents the size of the ellipsoid.

The ellipsoidal set can be obtained by set membership identification method. Set membership identification is one of the identification technique that use a priori assumptions about a parametric model to constrain the solutions to certain sets. . In this approach, uncertainty is described by means of an additive noise which is known only to have given bounds. The motivation for this approach is that in many practical cases the Unknown but Bounded (UBB) error description is more realistic and less demanding than the statistical description. In Section IV, the Fogel-Huang Algorithm [15] is used for the parameter identification.

Control objective: design a robust controller such that:

1. The closed-loop system is stable for all $\theta(t) \in \Delta$ with a guaranteed level of disturbance attenuation.
2. The output $y(t)$ tracks the reference signal $r_d(t)$ with zero steady-state error, that is $\lim_{t\to\infty} e(t) = 0$ where $e(t) = r_d(t) - y(t)$.
3. Satisfactory transient performance in time-response by adding a controller with adaptation mechanism.

It is well known that integral control can effectively eliminate the steady tracking error. In order to obtain a robust tracking controller with state feedback plus tracking error integral, the following augmented state-space description is introduced.

$$\dot{\bar{x}}(t) = \bar{A}(\theta(t))\bar{x}(t) + \bar{B}(\theta(t))u(t) + \bar{B}_\omega(\theta(t))\omega_a(t) \tag{10}$$

where

$$\bar{x}(t) = \left[(\int_0^t e(\tau)\mathrm{d}\tau)^T \quad x^T(t) \right]^T, \omega_a(t) = [r_d^T(t) \quad \omega^T(t)]^T$$

And

$$\bar{A}(\theta(t)) = \begin{bmatrix} 0 & -C_1 \\ 0 & A(\theta(t)) \end{bmatrix}, \bar{B}(\theta(t)) = \begin{bmatrix} 0 \\ B(\theta(t)) \end{bmatrix}, \bar{B}_\omega(\theta(t)) = \begin{bmatrix} I & 0 \\ 0 & B_\omega(\theta(t)) \end{bmatrix}$$

Choose the controlled output $\bar{z}(t) \in R^q$, defined by

$$\bar{z}(t) = C\bar{x}(t) + Du(t) + \Gamma\omega_a(t) \tag{11}$$

where C and D are constant weighting matrices which can be adjusted to achieve satisfactory response.

Then the design problem can be reduced to the following: Find a robust controller $u(t)$ such that:

1. The augmented closed-loop system is robust for stable

for all $\theta(t) \in \Delta$.

2. Transient performance improves in time-response.

Next, we will propose the robust control method with adaptation mechanism.

B. Controller design

In order to obtain on-line information on the parameter uncertainty, we introduce the vector $\hat{\theta}(t) \in R^N$, denotes the adjustable parameter vector, and let the matrices $\bar{A}(\hat{\theta})$ and $\bar{B}(\hat{\theta})$ have the same structure as the system matrices of (10).

$$A(\hat{\theta}(t)) = A_0 + \sum_{i=1}^{N} \hat{\theta}_i(t) A_i$$

$$B(\hat{\theta}(t)) = B_0 + \sum_{i=1}^{N} \hat{\theta}_i(t) B_i$$

The input $u(t)$ is determined so as to improve the output $\bar{z}(t)$ according to the adjustable parameter $\hat{\theta}(t)$.

By considering the control input

$$u(t) = K(\hat{\theta})\bar{x}(t)$$

with

$$K(\hat{\theta}) = K_0 + \sum_{i=1}^{\mathrm{N}} \hat{\theta}_i K_i$$

then the closed-loop system (10) is written as

$$\dot{\bar{x}} = (\bar{A}(\theta) + \bar{B}(\theta)K(\hat{\theta}))\bar{x} + \bar{B}_{\omega}(\theta)\omega_a(t) \tag{12}$$

Furthermore,

$$\begin{aligned}
&\bar{A}(\theta) + \bar{B}(\theta)K(\hat{\theta}) \\
&= \bar{A}(\hat{\theta}) + \sum_{i=1}^{N}(\theta_i - \hat{\theta}_i)\bar{A}_i + (\bar{B}_0 + \sum_{i=1}^{N}\theta_i \bar{B}_i)(K_0 + \sum_{i=1}^{N}\hat{\theta}_i K_i) \\
&= \bar{A}(\hat{\theta}) + \sum_{i=1}^{N}(\theta_i - \hat{\theta}_i)\bar{A}_i + \bar{B}(\hat{\theta})K_0 + \sum_{i=1}^{N}(\theta_i - \hat{\theta}_i)\bar{B}_i K_0 + \bar{B}(\theta)\sum_{i=1}^{N}\hat{\theta}_i K_i \\
&= \bar{A}(\hat{\theta}) + \bar{B}(\hat{\theta})K_0 + \bar{B}(\theta)\sum_{i=1}^{N}\hat{\theta}_i K_i + \sum_{i=1}^{N}(\theta_i - \hat{\theta}_i)(\bar{A}_i + \bar{B}_i K_0)
\end{aligned} \tag{13}$$

So, the system (12) can be written as

$$\begin{aligned}
\dot{\bar{x}} &= (\bar{A}(\hat{\theta}) + \bar{B}(\hat{\theta})K_0 + \bar{B}(\theta)\sum_{i=1}^{N}\hat{\theta}_i K_i + \sum_{i=1}^{N}(\theta_i - \hat{\theta}_i)(\bar{A}_i + \bar{B}_i K_0))\bar{x} + \bar{B}_{\omega}\omega_a(t) \\
&= (\bar{A}(\hat{\theta}) + \bar{B}(\hat{\theta})K_0 + \bar{B}(\theta)\sum_{i=1}^{N}\hat{\theta}_i K_i)\bar{x} + \sum_{i=1}^{N}(\theta_i - \hat{\theta}_i)(\bar{A}_i + \bar{B}_i K_0)\bar{x} + \bar{B}_{\omega}\omega_a(t) \\
&= (\bar{A}(\hat{\theta}) + \bar{B}(\hat{\theta})K_0 + \bar{B}(\theta)\sum_{i=1}^{N}\hat{\theta}_i K_i)\bar{x} + (E_1 + E_2)(\theta - \hat{\theta}) + \bar{B}_{\omega}\omega_a(t)
\end{aligned} \tag{14}$$

with $E_1 \in R^{(n+p)\times N}$, $E_2 \in R^{(n+p)\times N}$, are given by

$E_1 = \left[\bar{A}_1\bar{x} \mid \cdots \mid \bar{A}_N\bar{x}\right]$, $E_2 = \left[\bar{B}_1 K_0\bar{x} \mid \cdots \mid \bar{B}_N K_0\bar{x}\right]$

Here, for the system(14), we determine the parameter vector $\hat{\theta}(t)$ and the gain matrix $K(\hat{\theta})$ so as to ensure quadratic stability and an L_2 gain bound $\gamma > 0$ from the exogenous signal ω_a to the output error signal $\bar{z}$, i.e.

$$\int_0^{\infty} \bar{z}^T\bar{z}\,\mathrm{d}t \le \gamma^2 \int_0^{\infty} \omega_a^T\omega_a\,\mathrm{d}t \ \text{ for all } \theta \in \Delta \text{ and } \omega_a \in L_2[0,\infty) \tag{15}$$

for zero-state initial conditions.

Theorem 1: The closed-loop system (14) is stable and its H_∞ disturbance attenuation is no more than γ if there exist $Y_i, i = 0 \cdots N$ and $P > 0$ such that

$$\begin{bmatrix} M + M^T & \bar{B}_{\omega}(\theta) & (CX + DY(\hat{\theta}))^T \\ * & -\gamma^2 I & \Gamma^T \\ * & * & -I \end{bmatrix} < 0$$

$$\text{for all } \theta(t),\ \hat{\theta}(t) \in \Delta \tag{16}$$

where, * denotes the symmetric part ,

$$X = P^{-1},\ Y(\hat{\theta}) = Y_0 + \sum_{i=1}^{N} Y_i\hat{\theta}_i,$$

$$Y_0 = K_0 X,\ Y_i = K_i X,\ \ i = 1 \cdots N.$$

$$M = \bar{A}(\hat{\theta})X + \bar{B}(\hat{\theta})Y_0 + \sum_{i=1}^{N} \bar{B}(\theta)Y_i\hat{\theta}_i$$

and also if $\hat{\theta}(t)$ is determined according to the adjustment law

$$\hat{\theta}(t) = \begin{cases} \dfrac{\Sigma^2[E_1^{\mathrm{T}} + E_2^{\mathrm{T}}]P\bar{x}}{\left\|\Sigma[E_1^{\mathrm{T}} + E_2^{\mathrm{T}}]P\bar{x}\right\|} & \text{if } [E_1^{\mathrm{T}} + E_2^{\mathrm{T}}]P\bar{x} \ne 0 \\ \hat{\theta}(t^-) & \text{if } [E_1^{\mathrm{T}} + E_2^{\mathrm{T}}]P\bar{x} = 0 \end{cases} \tag{17}$$

where $t^- = \lim_{\tau>0,\tau\to 0}(t-\tau)$ and the initial guess of the parameter $\hat{\theta}(t)$, denoted by $\hat{\theta}(0)$, is supposed to be chosen from on the boundary surface of the ellipsoidal set Δ.

Proof:

Choose the following candidate Lyapunov function

$$V = x^T(t)Px(t) \tag{18}$$

Then from the derivative of V along the system (14) with $\omega_a = 0$, we can get

$$\begin{aligned}
\dot{V} &= \bar{x}^{\mathrm{T}}[(\bar{A}(\hat{\theta}) + \bar{B}(\hat{\theta})K_0 + \bar{B}(\theta)\sum_{i=1}^{N}\hat{\theta}_i K_i)^T P \\
&\quad + P(\bar{A}(\hat{\theta}) + \bar{B}(\hat{\theta})K_0 + \bar{B}(\theta)\sum_{i=1}^{N}\hat{\theta}_i K_i)]\bar{x} \\
&\quad + 2\bar{x}^T P(E_1 + E_2)(\theta - \hat{\theta}) \\
&\le -\alpha\|\bar{x}\|^2 + 2\bar{x}^T P(E_1 + E_2)(\theta - \hat{\theta})
\end{aligned} \tag{19}$$

where

$$\begin{aligned}
\alpha = -\lambda_{\substack{\max\\ \theta\in\Delta}}&[(\bar{A}(\hat{\theta}) + \bar{B}(\hat{\theta})K_0 + \bar{B}(\theta)\sum_{i=1}^{N}\hat{\theta}_i K_i)^T P \\
&+ P(\bar{A}(\hat{\theta}) + \bar{B}(\hat{\theta})K_0 + \bar{B}(\theta)\sum_{i=1}^{N}\hat{\theta}_i K_i)] > 0
\end{aligned}$$

Setting the parameter adjustment law as (17) results in $\dot{V} \le -\alpha \|\bar{x}\|^2$, because using (8) the following relation holds

$$\begin{aligned}\left|\bar{x}^T P(E_1+E_2)\theta\right| &\le \left\|\bar{x}^T P(E_1+E_2)\Sigma\right\| \left\|\Sigma^{-1}\theta\right\| \\ &\le \left\|\bar{x}^T P(E_1+E_2)\Sigma\right\| \\ &= \bar{x}^T P(E_1+E_2)\hat{\theta}\end{aligned}$$

Therefore the stability of the e system (14) is ensured. Furthermore, suppose that $\bar{x}(0)=0$ we have

$$\begin{aligned}J &= \int_0^\infty (\mathrm{z}^T\mathrm{z} - \gamma^2 \omega_a^T \omega_a)\mathrm{d}t \\ &= \int_0^\infty (\mathrm{z}^T\mathrm{z} - \gamma^2 \omega_a^T \omega_a + \frac{\mathrm{d}}{\mathrm{d}t}(\bar{x}^T P \bar{x}))\mathrm{d}t - \bar{x}^T(\infty) P \bar{x}(\infty)\end{aligned}$$

$$\le \int_0^\infty \left[(\bar{x}^T \quad \omega_a^T)\phi(\theta)\begin{pmatrix}\bar{x}\\ \omega_a\end{pmatrix} + 2\bar{x}^T P(E_1+E_2)(\theta-\hat{\theta})\right]\mathrm{d}t \tag{20}$$

where

$$\phi(\theta) = \begin{bmatrix}\Lambda & P\bar{B}_\omega(\theta) + (C+DK(\hat{\theta}))^T\Gamma \\ * & -\gamma^2 I + \Gamma^T\Gamma\end{bmatrix}$$

with

$$\Lambda = (\bar{A}(\hat{\theta}) + \bar{B}(\hat{\theta})K_0 + \bar{B}(\theta)\sum_{i=1}^N \hat{\theta}_i K_i)^T P + P(\bar{A}(\hat{\theta}) + \bar{B}(\hat{\theta})K_0 + \bar{B}(\theta)\sum_{i=1}^N \hat{\theta}_i K_i) + (C+DK(\hat{\theta}))^T(C+DK(\hat{\theta}))$$

By using Schur complement formula, $\phi(\theta)<0$ is equivalent to

$$\begin{bmatrix}\Psi & P\bar{B}_\omega(\theta) & (C+DK(\hat{\theta}))^T \\ * & -\gamma^2 I & \Gamma^T \\ * & * & -I\end{bmatrix} < 0 \tag{21}$$

with

$$\Psi = (\bar{A}(\hat{\theta}) + \bar{B}(\hat{\theta})K_0 + \bar{B}(\theta)\sum_{i=1}^N \hat{\theta}_i K_i)^T P + P(\bar{A}(\hat{\theta}) + \bar{B}(\hat{\theta})K_0 + \bar{B}(\theta)\sum_{i=1}^N \hat{\theta}_i K_i)$$

By pre-and post-multiplying (21) by $\mathrm{diag}\left(P^{-1}, \ I, \ I\right)$, we can get that $\phi(\theta)<0$ is equivalent to (16).

Remark 1: The adjustable parameter $\hat{\theta}(t)$ satisfies $\hat{\theta}(t)\Sigma^{-2}\hat{\theta}(t)=1$, which means that $\hat{\theta}(t)$ is adjusted on the boundary surface of the prespecified ellipsoidal set Δ.

Remark 2: In order to transform (14) to a convex problem, a substitute set for the ellipsoidal set Δ can be used in Theorem 1, that is

$$\bar{\Delta} = \{\theta(t) \in R^N \mid \theta_i(t) \le \sigma_i, i = 1\cdots N\}$$

Then since $\theta(t), \hat{\theta}(t)$ appear affinely in (14), the problem can be reduced to check (16) for all $\theta(t), \hat{\theta}(t) \in \bar{\Delta}_{vex}$, where $\bar{\Delta}_{vex} = \{\theta \in R^N \mid \theta_i = \sigma_i \text{ or } \theta_i = -\sigma_i\}$ denotes the set of 2^N vertices of $\bar{\Delta}$.

Next, a algorithm is given to choose the control gains .

Algorithm:

Step 1: Design K_0 by using the standard H_∞ control theory [16] for the nominal augmented system

$$\dot{\bar{x}}(t) = \bar{A}_0\bar{x}(t) + \bar{B}_0 u(t) + \bar{B}_{\omega 0}\omega(t)$$

Step 2: Solving the following optimization

$\min \ \delta$ **s.t.** (16) and $M>0$

where $\delta = \gamma^2$.

The feedback gains $K_1 \cdots K_N$ are obtained by $K_i = Y_i X, i = 1\cdots N$.

IV. SIMULATIONS

The proposed control algorithm is verified by the simulation model obtained from the helicopter-on-arm platform, shown as Fig.4. A small-scale electrical helicopter is mounted at the end of a two-DOF arm, while the weight of the helicopter is perfectly balanced at the other side of the arm. First, the parameters of the nonlinear yaw dynamic model are identified and followings are the result:

$$\begin{cases}\dot{\varphi} = r \\ \dot{r} = k_1 r + k_2\theta_{tr} + k_3\theta_{tr}^2 + k_4\Omega\theta_{tr} + k_5\varphi\end{cases} \tag{22}$$

with, k_1= -1.38, k_2 = 63.09, k_3= 11.65, k_4= -0.14, k_5= -3.33, $\Omega = 1200$. System (22) can be linearized, and system matrices are as follow:

$$A_0 = \begin{bmatrix}0 & 1 \\ -3.33 & -1.38\end{bmatrix}, A_1 = \begin{bmatrix}0 & 0 \\ -3.33 & 0\end{bmatrix},$$

$$A_2 = \begin{bmatrix}0 & 0 \\ 0 & -1.38\end{bmatrix}, A_3 = \begin{bmatrix}0 & 0 \\ 0 & 0\end{bmatrix}$$

$$B_0 = \begin{bmatrix}0 \\ 72.32\end{bmatrix}, B_1 = \begin{bmatrix}0 \\ 0\end{bmatrix}, B_2 = \begin{bmatrix}0 \\ 0\end{bmatrix}, B_3 = \begin{bmatrix}0 \\ 72.32\end{bmatrix}$$

Parameter uncertainty

$\theta^T\Sigma^{-2}\theta \le 1, \Sigma = diag(0.5, 0.3, 0.2)$

We choose the controlled output matrices as

$$C = \begin{bmatrix}4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{bmatrix}, D = \begin{bmatrix}0 \\ 0 \\ 1\end{bmatrix}$$

In the following simulations, the initial conditions are: $\varphi(0)=0$, $r(0)=0$. The tracking command of φ is

$$\varphi_d = 20, \quad 0 \le t \le t_{off}$$

and the following disturbance is used:

$$\zeta_1 = \begin{cases} 5 & 3 \le t \le 4(s) \\ 0 & \text{else} \end{cases}$$

A robust tracking controller with adaptive compensation input is designed to control yaw model of the helicopter using the proposed approach in Section III. We can get the gains of robust controller:

$$K_0 = [39.18 \quad -22.72 \quad -1.29]$$
$$K_1 = [0.56 \quad -0.31 \quad -0.02]$$
$$K_2 = [-0.88 \quad 0.54 \quad 0.03]$$
$$K_3 = [-6.46 \quad 3.75 \quad 0.20]$$

Fig.5 and Fig.6 are the response curves of the yaw and the yaw velocity with the proposed controller and standard H_∞ controller for nominal system, respectively. It is easy to see that using the proposed robust controller the closed-loop system is stable even in presence of disturbance. The robust controller with adaptive compensation input can improve the performance the system with uncertainty.

V. CONCLUSIONS

In this paper, we have first solved an adaptive robust H_∞ tracking control problem for a class of linear systems with the varying uncertainty. A new variable gains controller with simple structure is proposed to guarantee the asymptotic stability and H_∞ tracking performance of closed-loop system. Then we apply this result to solve the yaw tracking control problem of RUAV. Simulation results have demonstrated the effectiveness.

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Fig.4 Helicopter on arm

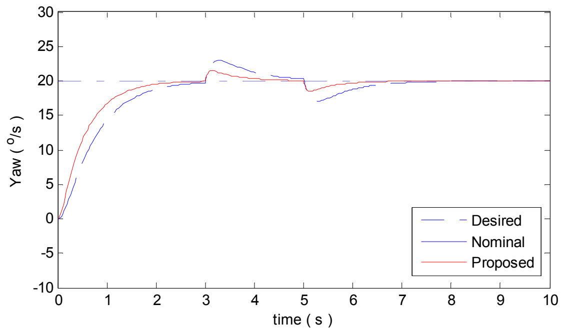

Fig. 5 yaw response curve with nominal controller and the proposed robust controller

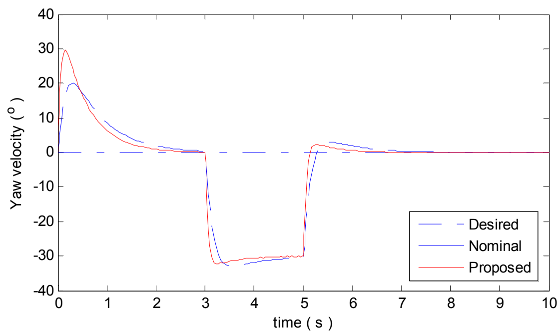

Fig.6 yaw velocity response curve with nominal controller and the proposed robust controller