

## Short and Long-Term Dynamic Voltage Instability

M. J. Hossain\*, H. R. Pota\*, V. Ugrinovskii\*,

*\* University of New South Wales at the Australian Defence Force Academy, Australia (Tel: +61432155461; e-mail: mj.hossain, h-pota and v.ougrinovski@adfa.edu.au.*

---

### Abstract:

This paper presents a novel approach to capture the development of dynamic voltage instability caused by the dynamics of different power system devices, such as loads, generators, automatic voltage regulators (AVR), overexcitation limiters (OXL), power system stabilizers (PSS), and on-load tap changing (OLTC) transformers using an accurate time-domain analysis. A small power system model is presented which allows one to analyse combinations of these effects, showing how different major forms of long-term and short-term dynamic voltage instability occur. Effects of line tripping, sudden change of load, and fault clearing time on dynamic voltage instability will also be discussed. Finally, advantages of the dynamic analysis over the static analysis will be investigated.

---

### 1. INTRODUCTION

Dynamic operating modes in interconnected power systems are initiated whenever abrupt changes occur to otherwise steady operating conditions. They arise from momentary imbalances in system operation, which can project the system or individual items of a plant within the system into unplanned operating regions. Continued and safe operation is then momentarily at risk. The nature of the risk is one of operating instability. There are two approaches to the study of the voltage instability problem, namely the static approach and the dynamic approach. The static approach using power-flow analysis and sensitivity studies has been extensively studied over the past two decades, whereas the dynamic approach where power system components are modelled by appropriate dynamic equations is still an active area of research.

For a general power system, static voltage stability involves the determination of the system load ability limit under pre-disturbance conditions and post-disturbance conditions, the identification of weak buses from the P-V curves, and the determination of the amount of corrective measures required at some of these weak buses for a specified system Lof et al. [1995], Gao et al. [1992], Custem and Vournas [1998], Kundur [1994]. The amount of corrective measures and application time to be applied for a specified system following a large disturbance cannot be obtained using static analysis. The aim of the dynamic analysis is to obtain the critical control application time for the corrective measures. Many researchers have dealt with the problem of voltage collapse as a loss of equilibria or noting the singularity of the Jacobian at the onset of the phenomenon. In these approaches, the problem is characterized as a quasi-static bifurcation occurring in response to a slowly varying increase or decrease in voltage depended load. Lee and lee [1991] considered the voltage stability problem using the synchronous motor as load and investigated the eigenvalues of the linearized system

matrix for the dynamic voltage stability. Thomas and Tiranuchit [1987] analysed the dynamic voltage instability as a quasi-bifurcation using an induction motor (IM) load. Tripathy [2000] suggested to use Hopf bifurcation method to determine oscillatory voltage instability. It was shown that as the reactive power load is increased slowly from a small value, the eigenvalues, which were originally in the left-half  $s$ -plane move to the right-half  $s$ -plane and again return to the left-half plane.

However, in many voltage incidents 38-02-10 [1993] experienced so far, instability occurred following large disturbance, such as short circuits, line outages, or generator tripping, which cannot be analysed accurately using the bifurcation theory. Time domain analysis is mainly used to investigate the instability mechanisms induced by abrupt, large variations in the structure and the parameters of the system. Some research has been done in the area of voltage instability caused by large disturbance. Vu and Liu [1990] explained the dynamics of voltage collapse using generator excitation limit, load dynamics and on-load tap-changer. The modelling of generator dynamics was neglected in Vu and Liu [1990] on the assumption that generators do not lose synchronism; rather, the subsequent voltage decreases to a low level over a considerably long period. But this is not always true. Begovic and Phadke [1990] investigated the dynamic voltage stability using a slightly modified transient stability program. The general structure of the system model used in that reference is similar to that for transient stability analysis. Sekine and Ohtsuki [1990] analysed the dynamic phenomena of voltage collapse using induction motor models. Potamianakis and Vournas [2006] presented different voltage stability scenarios for short-term voltage instability caused by synchronous and induction machines. The existing papers on dynamic voltage instability analysis mainly focus on voltage instability caused by the loss of post-disturbance equilibrium and use static load in the long-term voltage analysis.

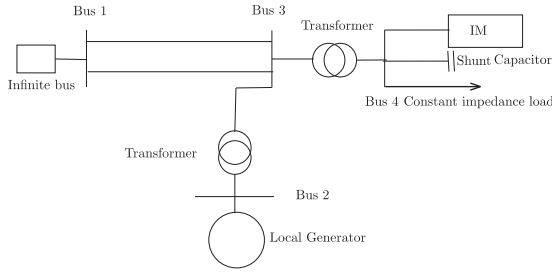


Fig. 1. Power system model

However, for voltage stability analysis, special attention should be paid to the instability mechanism of the power system caused by (i) the loss of post-disturbance equilibrium, i.e., post fault system has no equilibrium point, (ii) lack of attraction towards the stable post-disturbance equilibrium of short and long-term dynamics, which occurs due to slow fault clearing or delayed corrective action to restore a stable equilibrium but not soon enough for the system to be attracted by the stable post-control equilibrium, and (iii) loss of short-term equilibrium caused by long-term dynamics i.e., the slow degradation due to long-term instability leads to a sudden transition in the form of a collapse. Although different approaches have been proposed and employed for voltage collapse analysis till now, the literature dealing with the latter two mechanisms of voltage instability in large interconnected power systems is scarce.

The objective of this paper is to develop a power system model to allow one to capture all possible forms of instability mechanism caused by dynamics of the different devices, comprising a typical power system, such as, loads, generators, automatic voltage regulators, power system stabilizers, overexcitation limiters, and OLTCs with high accuracy using an accurate time-domain technique. In this paper, long-term voltage analysis will be carried out using both static and dynamic loads. Comparisons between the static and dynamic analysis will also be presented. An understanding of the dynamic voltage instability should enable the development of appropriate analytical tools to study this phenomenon and provide corrective control strategies. The paper is organized as follows: Section 2 describes briefly the model of the power system devices comprising the model and Section 3 presents main results of the paper as different voltage instability scenarios and their interpretations. Finally Section 4 presents the conclusions.

## 2. POWER SYSTEM MODEL

Fig.1 shows the power system model, which will be used to analyse the dynamics of different systems under consideration, such as, generator, loads, automatic voltage regulator, power system stabilizer, overexcitation limiter, nominal transformer, and on load tap changer. The local generator is equipped with AVR, OXL, and PSS. Voltage collapse studies and their related tools are typically based on the following general mathematical description of the system consisting of a set of algebraic and differential equations:

$$\dot{x} = f(x, y, z),$$

$$\dot{z} = h(x, y, z), \quad (1)$$

$$0 = g(x, y, z),$$

where  $x \in R^m$  represents the short-term state variables corresponding to fast dynamic states of generators, IM loads, FACTS (Flexible AC Transmission System) and HVDC (High Voltage DC) controllers, etc;  $y \in R^n$  corresponds to the algebraic variables, usually associated with the transmission system and steady-state element models, such as voltage magnitudes and phases at nodes, some generating sources and loads in the network;  $z \in R^k$  represents the long-term dynamic state variables of slow acting devices, such as OLTC, OXL, and secondary voltage controls (if any), etc. The differential equations represent the dynamic behaviour of the system, while algebraic equations represent the interaction of dynamic elements.

The remote system is characterized by its short-circuit level at Bus 1 and is represented by its Thevenin equivalent. Basically, there are two kinds of load models: static model and dynamic model. In this paper the load is made of: (i) one part represented by an exponential load; (ii) another part represented by an equivalent induction motor including rotor dynamics, and (iii) a shunt capacitor, for compensation purposes. The function of the AVR is to maintain the generator terminal voltage at the preset value. Any change in the terminal voltage from the desired value is detected and is used as the actuating signal to control the excitation. The primary objective of a PSS is to introduce, via the AVR, a component of electrical torque in the synchronous machine rotor that is proportional to the deviation of actual speed from the synchronous speed. When the rotor oscillates, this torque acts as a damping torque to counter the oscillation. The overexcitation limiter protects the field winding of a synchronous machine from overheating. Under a sufficiently stressed state, the loss of the transmission line and subsequent OXL action can cause machines to reach excitation limits. This action, along with other control actions and the characteristics of the system loads, can drive the system into a voltage collapse, which will be analysed in this paper. The limiter used in this paper allows excitation overload as an inverse function of time Vu and Liu [1990]. OLTCs are used in the power system to maintain bus voltages near a constant value.

### 2.1 Generator Dynamics

Under typical assumptions, the synchronous generator with a AVR can be modelled by the following set of nonlinear differential equations Kundur [1994], Bergen [1986]

$$\dot{\delta}_1 = \omega_1 \omega_s - \omega_s, \quad (2)$$

$$\dot{\omega}_1 = \frac{1}{2H_1} [P_{m1} - E'_{q1} I_q - D_1 \omega_1], \quad (3)$$

$$\begin{aligned} \dot{E}'_{q1} = \frac{1}{T'_{d01}} [K_a (V_{ref} - V_{o1} + V_{s1}) \\ - E'_{q1} - (X_{d1} - X'_{d1}) I_d], \end{aligned} \quad (4)$$

$$\dot{V}_{o1} = \frac{1}{T_{r1}} [V_{t1} - V_{o1}], \quad (5)$$

where  $\delta_1$  is the power angle of the generator,  $\omega_1$  is the rotor speed with respect to a synchronous reference,  $E'_{q1}$

is the quadrature-axis transient voltage behind transient reactance,  $V_{o1}$  is the output of the terminal voltage transducers, and  $V_{s1}$  is the stabilizing input signal for the AVR. The mechanical power  $P_{m1}$  and the reference voltage  $V_{ref}$  are parameters that set the operating point, and therefore are constant for each operating condition considered in the design. Furthermore,  $\omega_s$  is a constant representing the absolute value of the synchronous speed in radians per second,  $H_1$  is the inertia of the generator,  $D_1$  is the damping,  $T_{do1}$  and  $T_{r1}$  are the time constants of the rotor, stator circuit and terminal voltage transducer, respectively,  $K_a$  is the AVR gain, and  $X_{d1}$  and  $X'_{d1}$  are the synchronous and transient reactance respectively.

## 2.2 PSS Dynamics

The dynamic response of the PSS is modelled by the following equation

$$V_{pssi} = K_{pssi} \frac{sT_w}{1 + sT_w} \frac{(1 + sT_1)(1 + sT_3)}{(1 + sT_2)(1 + sT_4)} \quad (6)$$

where the notation carries their standard meaning as in Sauer and Pai [1998].

## 2.3 On-Load Tap-Changer Dynamics

A tap changer is governed by its step size, time constant, reference voltage, and deadband. In this model, a tap changing takes place (after some built-in time delay) if the load voltage rms  $V$  falls beyond a voltage range  $[V_{ref} - D - \epsilon, V_{ref} + D + \epsilon]$

$$n_{k+1} = n_k + d \times (V_{ref} - V), \quad (7)$$

where  $n_{k+1}$  and  $n_k$  are the turns-ratios before and after a tap change,  $\epsilon$ ,  $D$  and  $d$  are the hysteresis band, deadband and step size of tap respectively.

## 2.4 Load Model

The static load in this paper is modelled as

$$P(V_4) = zP_0 \left(\frac{V_4}{V_{40}}\right)^\alpha \quad (8)$$

$$Q(V_4) = zQ_0 \left(\frac{V_4}{V_{40}}\right)^\beta \quad (9)$$

where  $\alpha = \beta = \text{constant} = 2$ ,  $P$ ,  $Q$ ,  $z$ ,  $V_4$  are real power, reactive power, load demand and voltage at bus 4 respectively 0 represents initial value.

With the stator transients neglected and the rotor windings shorted, a simplified transient model of a single cage induction machine is described by the following algebraic-differential equations written in a synchronously-rotating reference frame Taylor [1994]

$$(v_d + jv_q) = (R_s + jX')(i_{dm} + ji_{qm}) + j(e'_{qm} - je'_{dm}),$$

$$\dot{s} = \frac{1}{2H_m} [T_e - T_L], \quad (10)$$

$$T'_{dom} e'_{qm} = -e'_{qm} + (X - X')i_{dm} - T'_{dom} \omega_s (s - 1)e'_{dm}, \quad (11)$$

$$T'_{dom} e'_{dm} = -e'_{dm} - (X - X')i_{qm} + T'_{dom} \omega_s (s - 1)e'_{qm}, \quad (12)$$

where

$$X' = X_s + \frac{X_m X_r}{X_m + X_r},$$

$$X = X_s + X_m,$$

$$T'_{dom} = \frac{X_r + X_m}{\omega_0 R_r},$$

$$T_e = e'_{qm} i_{qm} + e'_{dm} i_{dm},$$

In (10-12)  $\delta_m$  is the angle of q-axis w.r.t. system reference,  $s$  is the slip,  $E'_{dm}$  and  $E'_{qm}$  is the motor quadrature-axis transient voltage referred to system axis,  $T_L$  is the output load torque,  $T'_{dom}$  is the time constant,  $X'$ ,  $X_s$ , and  $X_m$  is the transient reactance, stator reactance and magnetizing reactance respectively,  $H_m$  is the inertia of the motor.

## 3. DYNAMIC VOLTAGE INSTABILITY

To test if the proposed model can serve as a benchmark for various voltage instability scenarios, several analyses were performed. These analyses involved (a) outage of one transmission line, (b) change in the mix between static and dynamic loads, (c) analysis with nominal transformer and OLTC, (d) analysis with and without AVR and PSS dynamics taken into account, and (e) sudden change of load.

### 3.1 Scenarios I: Short-term voltage instability

In the case of short-term voltage instability, the driving force of instability is the tendency of dynamic load to restore consumed power in the time frame of a second after a voltage drop caused by a contingency. A typical load component of this type is the induction motor. In this scenario the OLTC is considered as a nominal transformer and the OXL is switched off. The proposed power system model is simulated first with only constant impedance load and then with combinations of constant impedance and induction motor load. Fig.2 shows the effect of tripping one circuit only with (a) constant impedance and (b) 50% constant and 50% induction motor load. It is clear from Fig.2 that the system reaches new stable equilibrium at a reduced voltage in case (a) but the equilibrium disappears when 50% IM load is incorporated in case (b). The instability occurs due to the dynamics of induction motor. When the motor is subjected to a voltage dip, the motor demands reactive power at a certain rate to maintain the voltage as shown in Fig.3. If the reactive power demand is not met, the deficit of power results in a decline in voltage and the motor stalls, which can be seen from Fig.4 in which it's mechanical and electrical torque curve do not intersect after the disturbance, leaving the system without a post-disturbance equilibrium.

The short-term voltage instability can also occur for delay in fault clearing. The motor mechanical and electrical torque curves intersect in this case, but at fault clearing the motor slip exceeds the unstable equilibrium value. Next a three phase short circuit fault is considered at bus 3. Fig.5 shows the effect of fault clearing time on voltage stability. If the fault is cleared rapidly the system is attracted by the equilibrium but for a delayed clearance, the stability will be lost because the motor will decelerate beyond the stable region and will be unable to reaccelerate even after

the fault is cleared. The slip response of IM as shown in Fig.6 proves the instability of IM.

From the above analysis, it is clear that short-term voltage instability mainly occurs due to the loss of post-disturbance equilibrium and lack of attraction towards the stable post-disturbance equilibrium of short-term dynamics.

### 3.2 Scenarios II: Long-term voltage instability

In the long-term voltage analysis, it is assumed that the system has survived the short-term period following the initial disturbance. From there on the system is driven by the long-term dynamics captured by the  $z$  variables in equation (1). The contingency in this scenario is the outage of one transmission line and the local generator is equipped with AVR, PSS and OXL. The outage of one transmission line results in an increased reactive loss on the transmission line and thus largely reduces the transmission capability. As a consequence, system voltages drop. To keep the terminal voltage magnitude constant, the AVR of local generator boost its field current to increase their reactive power output. With the increase in reactive power, the system becomes transient stable. Fig.7 depicts the effects of OLTC on voltage stability with and without AVR dynamics, where OXL is not activated. The initial fast transient caused by the disturbance dies out, showing that short-term dynamics are stable. Thus a short-term equilibrium is established, with  $V_4$  settling down to 0.96 pu. After this point the mechanism driving the system response is the OLTC, which tries to restore the load-side voltage by lowering its tap ratio  $n$ . The operation of the OLTC starts after an initial time delay of 40s. After several tap changes, it succeeds in bringing the voltage back into dead-band. The required reactive power is supplied by the local generator. It can be concluded that a disturbance to initially stable operating conditions initiates a dynamic transition from an initial state towards a final state. In the transient period immediately subsequent to the disturbance, a restoring action is released by the deviation from steady operation. When stability is maintained, the restoring action returns operation to a steady equilibrium condition. Without the dynamics of AVR, the OLTC is unable to restore the voltage to its pre-contingency value, which can be seen from Fig.7.

The effect of OXL on voltage stability is shown in Fig.8. At about  $t = 120s$ , the generator OXL is activated which causes the voltage to drop with the subsequent change of tap. If the system voltage goes to very low level, the system may be unstable. The instability is occurred due to the field current limitation of local generator. It is clear that the initial departure from planned operation is counter-balanced by the inherent recovery capabilities of stable operating conditions. But as these must have upper limits, circumstances may arise in which stable operating conditions are exceeded thereby leading to unstable operating modes.

The time available for taking a corrective measure aimed at restoring a long-term equilibrium is limited by attraction considered. Fig.9 shows the effect of delayed corrective actions, where the system becomes stable if the load is shed after 1s of generator tripping and equilibrium condition is

lost if it is delayed more 0.1s. From the above scenarios, it can be concluded that long-term voltage instability occurs due to the attempt of recovering load to their pre-disturbance value through OLTC action or delayed corrective action, which restores a stable equilibrium but not soon enough for the system to be attracted by the stable post-control equilibrium.

### 3.3 Scenarios III: Short-term voltage instability caused by long-term dynamics

We now consider the case where the evolution of long-term variables, usually after a long-term instability, leads to a short-term instability. In this case, the long-term instability is the cause, the short term instability being the ultimate result. The system initial conditions are modified by increasing the local generator active production to full rated turbine power. At first the load is considered as constant impedance type and the contingency is the outage of one transmission line. Fig.10 shows the response of the transmission-side voltage  $V_3$  as a function of time. This time the generator field current gets limited at about  $t = 100s$ . As the OLTC keeps reducing the tap ratio, the generator eventually loses synchronism at about  $t = 220s$  as shown in Fig.10. In the second case the dynamic induction motor load is included and same fault is applied. The transmission-side voltage response is shown in Fig.11, where 33.33% of the load is made of IM. The local active generation is reduced below 50% of rated turbine power. The circuit tripping causes a more severe generator over excitation problem due to the increased reactive consumption of the induction motor at lower voltage. The increased overload forces the OXL to act faster. The loss of short-term equilibrium takes on the form of motor stalling as shown in Fig. 11. So the short-term instability caused by long-term dynamics may result in both motor stalling and generator loss of synchronism.

### 3.4 Comparison between static and dynamic analysis

To compare the dynamic analysis and static analysis, two cases are considered. In first case the load is modelled as constant impedance load and dynamic IM load and only constant impedance load in the second case. The local generator is equipped with AVR and PSS, and contingency in first case is sudden change of load and outage of one transmission line in the second case. The responses of load voltage using static and dynamic analysis with sudden 5% change in load power are shown in Fig.12 and Fig.13 respectively. From the static analysis, it can be seen that the final steady voltage reduces somewhat but the system is stable. A different scenario is obtained from dynamic analysis where the system is unstable due to the dynamics of the load. The response of load voltage by static and dynamic simulation due to the outage of one transmission line is shown in Fig.14 and Fig.15 respectively. The outage is stable for dynamic simulation and unstable by  $V - Q$  analysis using conventional power-flow models. Because there is no operating point for the for the unstable  $V - Q$  curve cases, results at the end of stable dynamic simulation cannot be compared with the power flow results. The difference in results in both cases occurs because dynamics of generator, AVR, PSS and IM load is neglected in static

analysis. We can judge the acceptability of the dynamic simulations by the post-fault voltage levels, the remaining reactive power reserves at generating plants, and the time available for operation. Dynamic simulation results provide more information to judge acceptability.

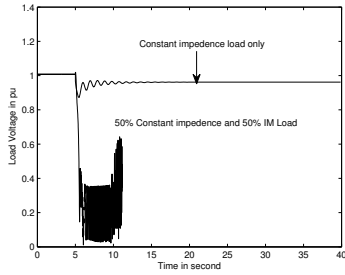


Fig. 2. Load voltage with static and dynamic load

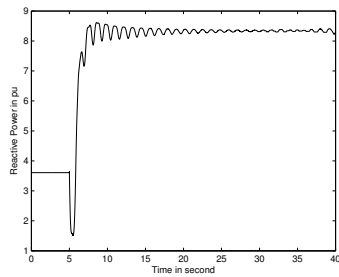


Fig. 3. Reactive power drawn by IM load

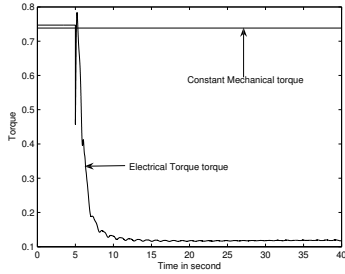


Fig. 4. Torque response of IM

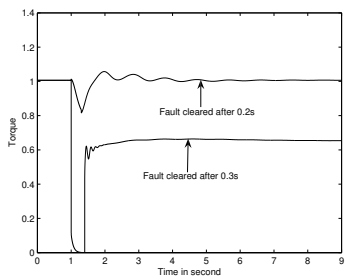


Fig. 5. Motor terminal voltage after a fault

#### 4. CONCLUSION

In this paper, we have discussed different aspects of the voltage instability problem, both static ( $P-V$  and  $Q-V$  analysis) and dynamic analyses. We have explored the effects of the inclusion of dynamics of generator, AVR,

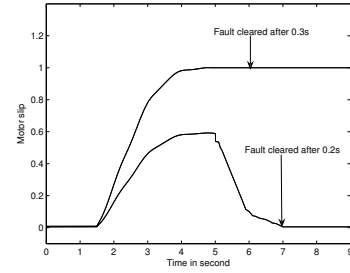


Fig. 6. Slip response of IM

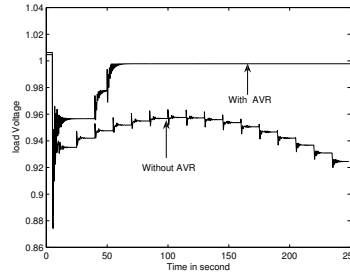


Fig. 7. Load voltage with OLTC effect

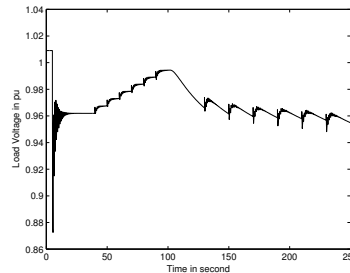


Fig. 8. Load voltage with OLTC and OXL effect

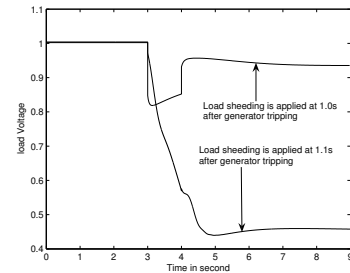


Fig. 9. Load voltage due to local generator tripping

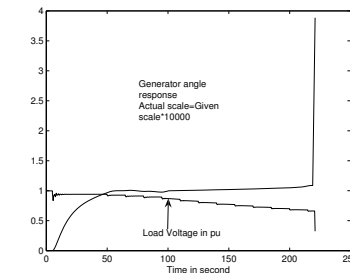


Fig. 10. Voltage and angle response of local generator

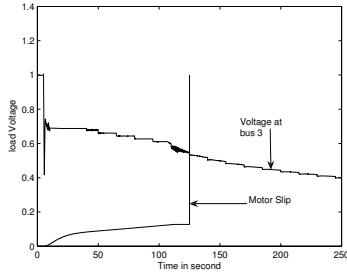


Fig. 11. OLTC response with IM Load

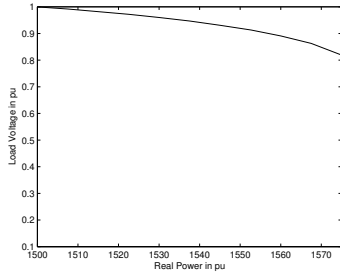


Fig. 12. PV relation at load bus due to change in load power

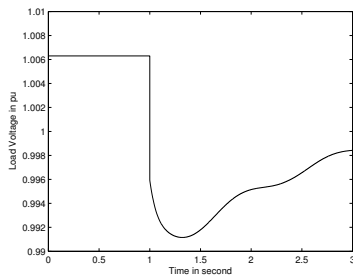


Fig. 13. Load voltage with dynamic analysis due to change in load power

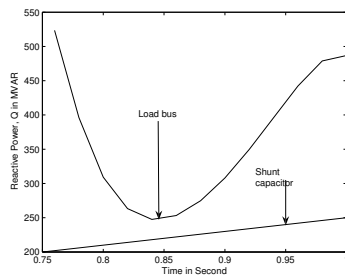


Fig. 14. QV relation at load bus due to outage of one transmission line

PSS, OXL, IM and OLTC on the voltage stability characteristics of power system using time domain analysis. In particular in this paper, the mechanism of voltage collapse phenomenon was analysed from the physical point of view rather than from the mathematical point of view, and some meaningful physical interpretations are given.

This paper presents a number of possible voltage collapse mechanisms to provide a deeper insight into the dynamical mechanism of voltage collapse phenomenon. Although

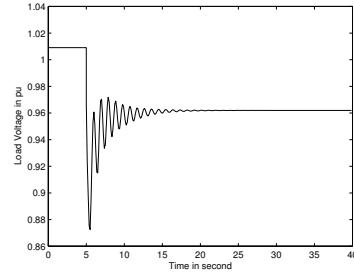


Fig. 15. Load voltage with dynamic analysis due to outage of one transmission line

static methods based on the power flow analysis is very suitable for screening loadability, final decisions regarding the system planning and operation should be confirmed by more accurate time-domain simulation in which different characteristics of multiple controllers, protection relays must be taken into account.

### REFERENCES

CIGRE Task Force 38-02-10. Modelling of voltage collapse including dynamic phenomena. *Summary in Electra*, 1993.

M. M. Begovic and A. G. Phadke. Dynamic simulation of voltage collapse. *IEEE Trans. on Power Systems*, 5: 1529–1534, 1990.

A. R. Bergen. *Power System Stability*. Prentice-Hall, New Jersey, 1986.

T. V. Cutsem and C. D. Vournas. *Voltage stability of the electric power systems*. Kluwer Academic, Norwell, 1998.

B. Gao, G. K. Morison, and P. Kundur. Voltage stability evaluation using modal analysis. *IEEE Trans. on Power Systems*, 7:1529–1542, 1992.

P. Kundur. *Power System Stability and Control*. McGraw-Hill, New York, 1994.

B. H. Lee and K. Y. Lee. A study on voltage collapse mechanism in electric power systems. *IEEE Trans. on Power Systems*, 6:966–974, 1991.

P. A. Lof, G. Andersson, and D. J. Hill. Voltage dependent reactive power limits for voltage stability studies. *IEEE Trans. on Power Systems*, 10:220–228, 1995.

E. G. Potamianakis and C. D. Vournas. Short-term voltage instability: effects on synchronous and induction machines. *IEEE Trans. on Power Systems*, 21:791–798, 2006.

P. W. Sauer and M. A. Pai. *Power System Dynamics and Stability*. Prentice-Hall, USA, 1998.

Y. Sekine and H. Ohtsuki. Cascaded voltage collapse. *IEEE Trans. on Power Systems*, 5:250–256, 1990.

C. W. Taylor. *Power System Voltage Stability*. McGraw-Hill, New York, 1994.

R. J. Thomas and A. Tiranuchit. Dynamic voltage instability. In *26th Conference on Decision and Control*, 1987.

S. C. Tripathy. Study of dynamic voltage stability of power systems. *International Journal of Electrical Engineering Education*, 37:374–383, 2000.

K. T. Vu and C. C. Liu. Dynamic mechanisms of voltage collapse. *Systems and Control*, 15:329–338, 1990.