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An Efficient Model for Urban Traffic Network Control

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Abstract: In order to control the urban traffic network through optimization methods, it is necessary to establish a proper urban traffic network model. This model should be both accurate in describing the network traffic and simple to compute. In this paper, an existing link model is at first improved to describe the main traffic dynamic behaviors more accurately. Then, a general urban traffic network topology is proposed. By modeling the network elements, a macroscopic model for urban traffic networks can be established. The model is compared with the microscopic traffic model CORSIM. It shows that this model well balances the accuracy and simpleness, and is thus suitable to real-time control.

1. INTRODUCTION

Urban traffic network (UTN) control is an urgent and challenging task (Papageorgiou *et al.*, 2003) due to its economical and social effects. To control an UTN with optimization methods, a suitable UTN model is always required. This UTN model should be able to handle various UTNs with different traffic scenarios. It should be elaborate enough to accurately describe the network input\output characteristics, but not contain many unnecessary details which may increase the computational complexity.

In the past few years, various macroscopic urban traffic models were developed and used for traffic control. The store-and-forward model, proposed by (Gazis *et al.*, 1963) and later used by (Diakaki *et al.*, 2002), is a simple model with continuous mathematical description, but it can only be used to the saturated traffic, i.e. the vehicle queues resulted from the red phase can not be dissolved completely at the following green phase. The model proposed by (Barisone *et al.*, 2002) and extended by (Dotoli *et al.*, 2006) takes some real traffic situations into consideration and is very intensive, but it contains many parameters, among which some is hard to obtain. And the model proposed early by (Kashani *et al.*, 1983), is recently extended by M. van den Berg *et al.* (Hegyi, 2004, Van Den Berg *et al.*, 2004, Van Den Berg *et al.*, 2003). This model can be used to any traffic scenarios (unsaturated, saturated, and over-saturated traffic conditions). It is also simple to compute, and contains a few parameters that are comparatively easy to get. However, it is not accurate enough in calculating the delay time taken by vehicles running from the beginning of a link to the tail of the waiting queues in the link. It should be also mentioned that all the above macroscopic models mainly focus on modelling the dynamics of the traffic flow on a link, and establish the model for the entire UTN by specifying the relationship of all the links. It is a laborious work to specify all the link relationship for a large UTN and the work should be done for every new UTN. Therefore, it is necessary to study the substantial structure of the UTN, and find a general way to model the structure of any UTN.

In this paper, the link model proposed by Kashani and extended by M. van den Berg, is at first improved such that the traffic dynamics in a link can be described more accurately. In order to establish a general UTN model, the UTN is decomposed into network elements, i.e. the fundamental units containing the main traffic characteristics in the UTN. The whole UTN structure can then be described by the topology composed of the network elements. In this way, the UTN model can be established by models of the network elements based on the network topology.

2. LINK MODELLING

Links are the containers in UTNs, so they are the places where most dynamic properties of traffic flow show. As a result, modelling the traffic behaviours on links plays an important role in modelling UTNs. In this section, the link model described by M. van den Berg is presented at first. Then some improvements are introduced to make the link model more accurate.

Consider a link in UTN as shown in Fig.1. We give the following notations:

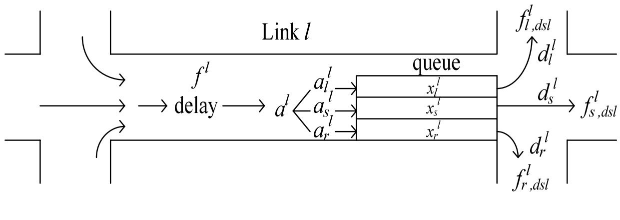

Fig. 1. The state variables in the link model

T : the sampling time interval
v^l : the free speed of the vehicles in link l
C^l : the capacity of link l expressed by the maximum number of vehicles that link l can hold
W^l : the number of lanes in link l

L_{veh} : the average vehicle length

s^l : the saturated flow rate that can depart from link l

$\beta_t^l(k)$: the ratio of the vehicles turning t (straight, left, right, i.e. $t \in \{s,l,r\}$) at the stop line in link l at time k

$d_t^l(k)$: the number of vehicles that depart from link l turning t at time k

$d_{in}^l(k)$: the number of vehicles that enter link l at time k

$d_{out}^l(k)$: the number of vehicles that depart from link l at time k

$x^l(k)$: the number of the vehicles waiting in link l at time k

$x_t^l(k)$: the number of the vehicles that wait in link l intending to turn t at time k

$a^l(k)$: the number of vehicles arriving at the tail of the waiting queue in link l at time k

$a_t^l(k)$: the number of vehicles arriving at the tail of the waiting queue in link l at time k, and the waiting queue is going to turn t

$f^l(k)$: the available free space in link l at time k expressed by the number of vehicles it can hold

$f_{t,dsl}^l(k)$: the free space in the downstream link of the departure vehicles turning t in link l at time k

$g_t^l(k)$: the signal symbol for the vehicles turning t in link l (1 when the signal is green, 0 when the signal is red).

At any simulation time interval T, the number of the departure vehicles in link l turning t can be computed as:

$$d_t^l(k) = \begin{cases} \min\left\{x_t^l(k) + a_t^l(k), f_{t,dsl}^l(k), \right. \\ \qquad \left. \beta_t^l(k)\cdot s^l \cdot T\right\} & if \quad g_t^l(k) = 1, \quad t \in \{s,l,r\} \\ 0 & if \quad g_t^l(k) = 0 \end{cases} \tag{1}$$

where the three parts in min {} correspond to unsaturated, over-saturated, and saturated traffic scenarios respectively. When the control signal is green, the number of departure vehicles should be the minimum of them.

The free space of link l which should be smaller than the link capacity is given by

$$f^l(k+1) = f^l(k) - d_{in}^l(k) + d_{out}^l(k) \tag{2}$$

In order to depict the waiting delay at the stop line, the time of the vehicles running from the beginning of the link to the tail of the waiting queues in the link is taken as:

$$\delta^l(k) = ceil\left(f^l(k)\cdot L_{veh} / v^l\right) \tag{3}$$

where $ceil(x)$ denotes the smallest integer larger than or equal to x. And the vehicles arriving at the tail of waiting queues should be added to the vehicles that arrived at link l in earlier steps but needed more time to get to the tail of the queues:

$$a^l(k+\delta^l(k))_{new} = a^l(k+\delta^l(k))_{old} + d_{in}^l(k) \tag{4}$$

The vehicle number arriving at each sub-queue can be obtained by multiplying the turning rate.

$$a_t^l(k) = \beta_t^l(k)\cdot a^l(k) \tag{5}$$

Then, the waiting vehicle number at the stop line in link l at time k can be updated like this:

$$x_t^l(k+1) = x_t^l(k) + a_t^l(k) - d_t^l(k), \quad t \in \{s,l,r\} \tag{6}$$

In the above model, the delay time of the vehicles running from the beginning of link l to the tail of the waiting queues in link l is given by (3). In this paper we suggest a new formulation for it, i.e. (7), to further improve the accuracy of the link traffic model. Similar method was used by (Dotoli *et al.*, 2006).

$$\begin{aligned} \delta^l(k) &= fix\left((C^l - x^l(k))\cdot L_{veh} / W^l \cdot v^l \cdot T\right) \\ \gamma^l(k) &= rem\left((C^l - x^l(k))\cdot L_{veh} / W^l \cdot v^l \cdot T\right), \end{aligned} \tag{7}$$

where $fix(x)$ means the largest integer that smaller than x, and $rem(x)$ is the remainder. The vehicles entering the link at time k should take $(\delta^l(k)\cdot T + \gamma^l(k))$ seconds to get to the tail of the queues. Equation (7) uses the capacity C^l minus the waiting vehicle number $x_t^l(k)$ to compute the distance from the beginning of link l to the tail of the waiting queues in link l, instead of the free space $f^l(k)$ in (3). It gets more accurate due to eliminate the impact of the vehicles running freely on the link. And the arriving vehicle number at the tail of the queues, presented by (4) early, should also be rewritten as the combination of the vehicle numbers which entered link l $(\delta^l(k)+\sigma)$ and $(\delta^l(k)+\sigma+1)$ steps ago:

$$\begin{aligned} a^l(k) = &\left((T-\gamma^l(k))/T\right)\cdot d_{in}^l(k-\delta^l(k)-\sigma) + \\ &\left(\gamma^l(k)/T\right)\cdot d_{in}^l(k-\delta^l(k)-1-\sigma) \end{aligned} \tag{8}$$

where σ is a parameter corresponding to the time for vehicles passing through the junction.

3. URBAN TRAFFIC NETWORK ELEMENT MODELLING

Any UTN can be considered to be composed of numbers of links and junctions, according to its physical structure. The variations of the traffic flows in the links are influenced by the control signals executed on the corresponding junctions. In this section, a fundamental network element characterizing above dynamics will be defined and modelled.

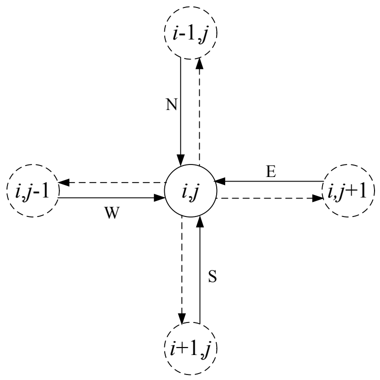

Fig. 2. Network element

The network element is defined as a junction with all four links running into it, as the solid line part shown in Fig. 2. The junction of the network element is identified by its line number i and column number j, marked as J (i, j). Other junctions connecting with it have relevant coordinates as shown in the dash line circles. The links running into J (i, j) are labelled by their orientations, i.e. west link (W), north link (N), east link (E) and south link (S). The whole network element is marked as E (i, j). In fact, any UTN can be decomposed into such network elements.

There are four links in the network element E (i, j). The link model proposed in section 2 can be directly used to establish their models respectively. In order to give an uniform expression of the link models for all four links, denote D as the orientation of the link in the network element, $D \in \{W, N, E, S\}$; t as the direction of the traffic turn movement in a link of the network element, $t \in \{s, l, r\}$. Therefore, any link can be identified by the coordinate of the network element E (i, j) and the orientation D. For any link D in E (i, j), the number of the departure vehicles turning t is given by

$$d_{Dt}(i,j,k) = \begin{cases} \min\{x_{Dt}(i,j,k) + a_{Dt}(i,j,k), \\ \quad f_{Dt,dsl}(k), & if \quad g_{Dt}(i,j,k) = 1 \\ \quad \beta_{Dt}(i,j,k) \cdot s_D(i,j) \cdot T\} \\ 0 & if \quad g_{Dt}(i,j,k) = 0 \end{cases} \quad (9)$$

where the free space $f_{Dt,dsl}(k)$ in the downstream link receiving the departure vehicles is given by Table 1.

Table 1. The free spaces in the downstream links of *E* (*i*, *j*)

	W (West)	N (North)	E (East)	S (South)
s(straight)	$f_W(i, j+1, k)$	$f_N(i+1, j, k)$	$f_E(i, j-1, k)$	$f_S(i-1, j, k)$
l (left)	$f_S(i-1, j, k)$	$f_W(i, j+1, k)$	$f_N(i+1, j, k)$	$f_E(i, j-1, k)$
r (right)	$f_N(i+1, j, k)$	$f_E(i, j-1, k)$	$f_S(i-1, j, k)$	$f_W(i, j+1, k)$

Table 2. The formation of the inputs for the links in *E* (*i*, *j*)

	W (West)	N (North)	E (East)	S (South)
s (straight)	$d_{Ws}(i, j-1, k)$	$d_{Ns}(i-1, j, k)$	$d_{Es}(i, j+1, k)$	$d_{Ss}(i+1, j, k)$
l (left)	$d_{Nl}(i, j-1, k)$	$d_{El}(i-1, j, k)$	$d_{Sl}(i, j+1, k)$	$d_{Wl}(i+1, j, k)$
r (right)	$d_{Sr}(i, j-1, k)$	$d_{Wr}(i-1, j, k)$	$d_{Nr}(i, j+1, k)$	$d_{Er}(i+1, j, k)$
SUM	$d_{in,W}(i, j, k)$	$d_{in,N}(i, j, k)$	$d_{in,E}(i, j, k)$	$d_{in,S}(i, j, k)$

The free space of link D in E (i, j) is updated by

$$f_D(i,j,k+1) = f_D(i,j,k) - d_{in,D}(i,j,k) + d_{out,D}(i,j,k) , \quad (10)$$

$$d_{out,D}(i,j,k) = d_{Ds}(i,j,k) + d_{Dl}(i,j,k) + d_{Dr}(i,j,k) , \quad (11)$$

where the formation of the inputs for link D $d_{in,D}(i,j,k)$ is given in Table 2.

The number of vehicles arriving at the tail of the waiting queues in link D in E (i, j) is given by

$$\begin{aligned} a_D(i,j,k) = & \left((T - \gamma_D(i,j,k))/T\right) \cdot d_{in,D}(i,j,k - \delta_D(i,j,k) - \sigma) \\ & + \left(\gamma_D(i,j,k)/T\right) \cdot d_{in,D}(i,j,k - \delta_D(i,j,k) - 1 - \sigma) \end{aligned} \quad (12)$$

where

$$\delta_D(i,j,k) = fix\left(\frac{(C_D(i,j) - x_D(i,j,k)) \cdot L_{veh}}{W_D(i,j) \cdot v_D(i,j) \cdot T}\right),$$
$$\gamma_D(i,j,k) = rem\left(\frac{(C_D(i,j) - x_D(i,j,k)) \cdot L_{veh}}{W_D(i,j) \cdot v_D(i,j) \cdot T}\right). \quad (13)$$

Then, the number of vehicles arriving at each sub-queue in link D is obtained by multiplying the turning rate.

$$a_{Dt}(i,j,k) = \beta_{Dt}(i,j,k) \cdot a_D(i,j,k) \quad (14)$$

Lastly, the number of vehicles at each sub-queue waiting at the stop line of link D in E (i, j) is updated by

$$x_{Dt}(i,j,k+1) = x_{Dt}(i,j,k) + a_{Dt}(i,j,k) - d_{Dt}(i,j,k) \quad (15)$$

Write (9)-(15) over all four orientations D of E (i, j), we can get the model for the network element. Typically the network element is composed of one junction with four links running into it. However, the above model is also available if the network element is somewhat simplified, which will be shown in the later sections.

4. URBAN TRAFFIC NETWORK MODELLING

Based on the network element model given above, the whole UTN model can be established by defining the UTN topology and justifying the network element model.

Table 3. network elements comparison

Element Type	Symbol	Absent Links	Free Space	Departure
Standard	+	none	discussed	discussed
West T-shape	$\triangleright$	W	$f_W = 0$	$d_{Wt} = 0$, $d_{Es} = d_{Sl} = d_{Nr} = 0$
North T-shape	∇	N	$f_N = 0$	$d_{Nt} = 0$, $d_{Ss} = d_{Wl} = d_{Er} = 0$
East T-shape	$\triangleleft$	E	$f_E = 0$	$d_{Et} = 0$, $d_{Ws} = d_{Nl} = d_{Sr} = 0$
South T-shape	Δ	S	$f_S = 0$	$d_{St} = 0$, $d_{Ns} = d_{El} = d_{Wr} = 0$
West Source	S_W	W, N, S	$f_E = \infty$	$d_{Ws}(i,j,k) = e(i,j,k) \cdot T$
North Source	S_N	W, N, E	$f_S = \infty$	$d_{Ns}(i,j,k) = e(i,j,k) \cdot T$
East Source	S_E	N, E, S	$f_W = \infty$	$d_{Es}(i,j,k) = e(i,j,k) \cdot T$
South Source	S_S	W, E, S	$f_N = \infty$	$d_{Ss}(i,j,k) = e(i,j,k) \cdot T$
None	0	W, N, E, S	none	none

4.1 Urban Traffic Network Topology

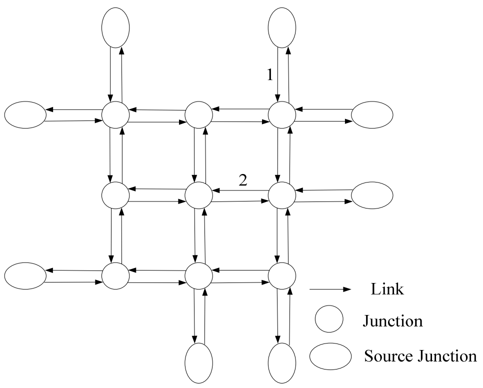

Fig. 3. An urban traffic network

The streets in most of the cities generally have the form of grid, so almost any UTN or a part of it can be characterized by a general network topology (GNT). The GNT is composed of some kinds of network elements in association with directions. Taking the UTN in Fig. 3 for example, it is a network with 5 lines and 5 columns, and is composed of 4 kinds of network elements, i.e. "Standard", "T-shape", "Source", and "None". Among them, "T-shape" and "Source" are classified by their directions again. All these network elements and their symbols are listed in Table 3.

As a result, the GNT of the UTN in Fig. 3 can be described by the matrix

$$\boldsymbol{A} = \begin{bmatrix} 0 & S_N & 0 & S_N & 0 \\ S_W & + & \nabla & + & S_E \\ 0 & \triangleright & + & + & S_E \\ S_W & \Delta & + & \triangleleft & 0 \\ 0 & 0 & S_S & S_S & 0 \end{bmatrix}, \quad (16)$$

which can be easily resulted from the physical structure of the UTN.

4.2 Network Element Model Extending

In Table 3, "Standard" is a network element whose modelling has been discussed above. Here we will show that all the other network elements in Table 3 can be considered as special cases of "Standard". So the models of these network elements can have the same formulations as in the

"Standard" model, but should be adjusted because information in some links may be missed. For network elements with "T-shape", one link in corresponding direction is missed from the "Standard" model. So the states in the absent link do not exist, and the inputs and outputs of this link also equal to zero. For the network element "Source", there is only one link connecting it with other elements, through which the outside traffic flows get into the UTN. Actually, this kind of network elements does not really exist. It is only used to describe the delay at the entries of the UTN. As a result, for this kind of network elements, only the inputs and the output spaces of the UTN should be depicted just as shown in Table 3, where $e(i, j, k)$ denotes the traffic flow rate entering the source network element E (i, j) at time k. Finally, "None" means that no network element exists at this coordinate in the UTN.

5. MODEL EVALUATION

To evaluate the effectiveness of the proposed UTN model, the microscopic model CORSIM exploited by (FHWA, 2001) is employed to simulate the real traffic. The comparisons are performed with two MOEs (Measure of Effectiveness) defined in CORSIM. One is the vehicle trips, i.e. the number of vehicles that have been discharged from the link since simulation begins. The other is the link content, i.e. the cumulative count of vehicles on a link, accumulated every time step. The vehicle trips can describe the utility of the control inputs (splits, cycle time, offset, phase), and the link content is the output to estimate the control performance. For control purpose, how close the model inputs and outputs are to real ones, is particularly interested. So both MOEs are chosen to evaluate the UTN model.

The simulated UTN is given in Fig. 3 with 5 lines and 5 columns. In the UTN model, all the turn movement percentages, the lane numbers and the capacities of the links are considered to be definite and known. The free speed is 30 km/h, the average vehicle length is 5 m, and the saturated flow rate for all lanes in any link is 2000 veh/h. The time for vehicles passing through a junction is set to be σ =3 s. The fixed-time control strategy is executed in this UTN, where the phases, the cycle times, and the splits are all constant, and the offsets are set to be zero. The network input flow rates in every source network elements are set to be equal to each other and constant in time. Set the sampling time interval T =1s and the simulation step 900. All simulations are performed using CORSIM, the model given by M. van den Berg and our UTN model respectively. The results are given in Fig. 4, Fig. 5, Fig. 6, and Fig. 7 for comparison.

As the figures show, the MOEs of our UTN model are consistent with that of CORSIM, no matter for different links (link 1 and link 2 shown in Fig. 3), or under different network input flow rates (100veh/h, 1000veh/h and 2000 veh/h). But the MOEs of the model by M. van den Berg cannot follow the MOEs of CORSIM accurately as the time grows.

For our model, the results with network input flow rate 100 veh/h seem mostly close to the results of CORSIM. This implies that the formulation for computing the arriving vehicles which can depart in T is accurate in unsaturated scenario. But the error increases with the growing of the network input flow rate. The reason is that our UTN model is a macroscopic traffic model in which many details have been ignored, such as the start-up delay, the non-clear boundary between the free driving vehicles and the stopping vehicles, etc. So simpleness is achieved in our UTN model but with the cost of sacrificing some accuracy. Moreover, the figures also show that link 1 (the link near the edge of the network) achieves better performance than link 2 (the link inside the network). The reason may be that link 2 accumulates more errors resulted from its upstream links, i.e. the more inner the link position is in the UTN, the less accurate the link model would be. Nevertheless, the computing speed of our model is about 13 times faster than CORSIM, when simulating the UTN in Fig. 3. Thus, as a macroscopic model, this UTN model shows great capability for real-time control.

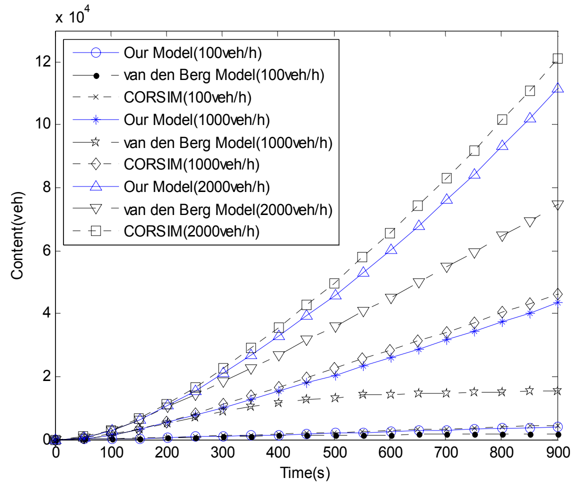

Fig. 4. The contents of link 1 for different network input flow rates

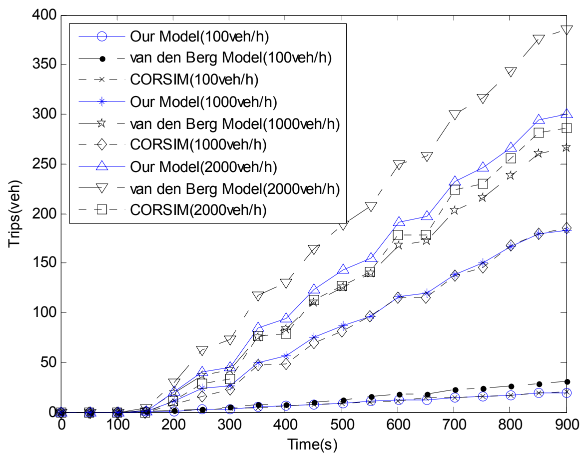

Fig. 5. The trips of link 1 for different network input flow rates

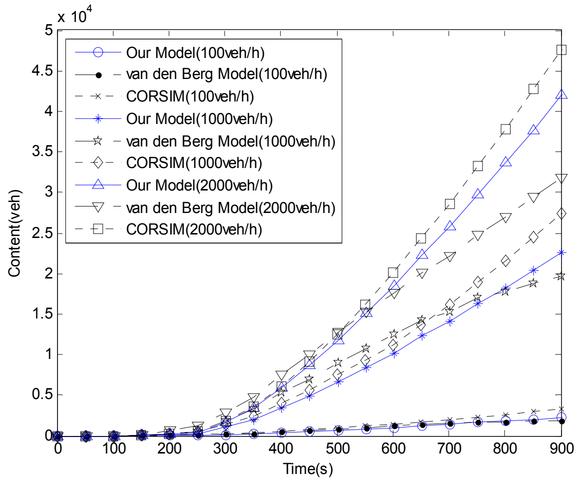

Fig. 6. The contents of link 2 for different network input flow rates

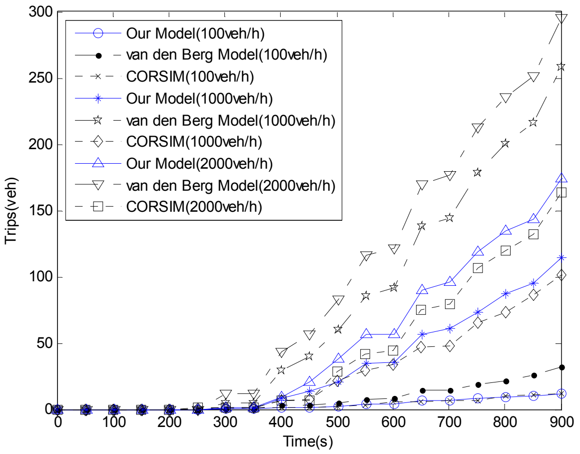

Fig. 7. The trips of link 2 for different network input flow rates

6. CONCLUSIONS

In this paper, the link model proposed by Kashani and extended by M. van den Berg is at first improved by revising the delay time of the vehicles running freely on a link. In order to establish the whole UTN model, we decomposed it into network elements, which are the fundamental units containing the main traffic dynamics and the control actions in the UTN. The model of the network element is established based on the improved link model. Then we gave a straightforward way to describe the general network topology and all the network elements in it can be modelled uniformly just with some specifications. In this way, the model of any UTN can be easily established.

This UTN model is general for two reasons: First, it takes all the main traffic scenarios (unsaturated, saturated, and over-saturated traffics) into consideration; second, it gives a new uniform UTN modelling method instead of the troublesome work of specifying all the link relationships in UTNs. This macroscopic model is also simple to compute, hence be able to be used in on-line control. Simulation results show that although this UTN model is somewhat rough, it is still accurate enough to have the similar input\output characteristics as the microscopic model (CORSIM) has. Both traffic optimization and prediction can be realized based on the global information over the whole traffic network provided by this model. Our future research work will focus on the effective control strategies for the UTN.

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