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A New Subspace Identification Method for Closed-Loop Systems using Measurable Disturbance

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Abstract: In this paper, we consider a closed-loop subspace identification problem without using probing inputs; but we assume that there is a measurable disturbance which can be used as a test input for identification. Deterministic and stochastic subsystems are derived by applying the orthogonal decomposition (ORT) of the joint input-output process and realization methods. We develop a new ORT-based closed-loop subspace identification method, consisting of identification of the two subsystems. Some numerical results are included to show the applicability of the present method.

Keywords: Closed-loop identification, subspace method, orthogonal decomposition, industrial processes, measurable disturbance

1. INTRODUCTION

Closed-loop identification has been an important topic for decades (Ljung, 1999; Söderström and Stoica, 1989), because in many industrial plants, open-loop experiments are prohibited due to safety and efficiency of operation. Key issues in closed-loop identification have been discussed in the literature (Van den Hof, 1998; Forssell and Ljung, 1999). Moreover, closed-loop subspace identification problems have received much interest; see e.g. Ljung and McKelvey (1996), Jansson (2003; 2005), Chiuso (2007), Chiuso and Picci (2005*a*; 2005*b*), Qin and Ljung (2003; 2006), Wang and Qin (2006).

By using the orthogonal decomposition (ORT) method (Picci and Katayama, 1996), we have developed subspace closed-loop identification methods; one is the joint input-output ORT method (Katayama *et al.*, 2005), and the other one the two-stage ORT method (Katayama and Tanaka, 2007). In these methods, we have used the so-called deterministic component [1] of the joint input-output process, under the assumption that some test signals are available for identification. The two-stage ORT method has also been applied to identification of a waste power plant (Ase *et al.*, 2006), for which there was a mismatch between the model used and the plant, because injection of probing inputs was not allowed for data gathering. In view of this fact, we consider a closed-loop subspace identification problem without using test signals; but, we assume that there is a measurable disturbance to be used as a test input for closed-loop identification. Since control systems are usually subjected to random disturbances, we exploit information brought by both measurable and unmeasurable disturbances, i.e. both deterministic and stochastic components of the joint process.

In this situation, we can regard the plant as receiving two control inputs where the first input has feedback from the output, while the second input is feedback-free. Thus, the methods of SSARX (Jansson, 2005) or PBSID (Chiuso and Picci, 2005*a*) can be applied. But, in this paper as a continuation of Katayama and Tanaka (2007), we develop an alternative way of identifying the plant by using measurable disturbances.

The rest of the paper is organized as follows. Section 2 states the problem formulation. In Section 3, we briefly review the technique of ORT and derive the deterministic and stochastic subsystems, together with their state space realizations. In Section 4, we introduce a compatibility condition between the deterministic and stochastic subsystems. Section 5 derives a state space innovation model for the plant. Section 6 describes the basic idea of the ORT method in the present closed-loop setting. Section 7 includes two numerical results, where the first example includes a comparison with PBSID (Chiuso and Picci, 2005*a*). Section 8 concludes the paper.

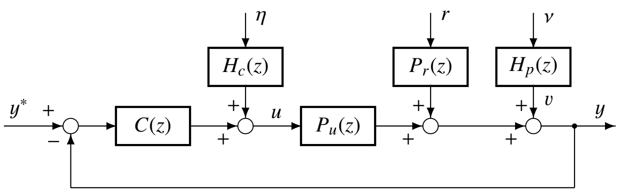


Fig. 1. Closed-loop system.

2. PROBLEM FORMULATION

We consider a closed-loop system shown in Fig. 1, where $y \in \mathbb{R}^p$ is the plant output, $u \in \mathbb{R}^m$ the control input, and $r \in \mathbb{R}^q$ the measurable disturbance. Also, the processes $\nu \in \mathbb{R}^p$ and $\eta \in \mathbb{R}^m$, generating stochastic disturbances, are mutually uncorrelated white noises with mean 0 and covariance matrices $\Lambda_\nu > 0$ and $\Lambda_\eta > 0$, respectively.

Let the LTI plant be given by

$$y(t) = P_u(z)u(t) + P_r(z)r(t) + H_p(z)\nu(t) \tag{1}$$

[1] The deterministic component is the part of the input-output process linearly related to the exogenous input, while the stochastic component is the orthogonal complement.

where $P_u(z)$ and $P_r(z)$ are the $p \times m$ and $p \times q$ transfer matrices of the plant from u to y and r to y, respectively, and the noise filter $H_p(z)$ is a minimum phase $p \times p$ transfer matrix satisfying $H_p(\infty) = I_p$. Also, the control signal is generated by

$$u(t) = C(z)(y^* - y(t)) + H_c(z)\eta(t) \tag{2}$$

where $y^* \in \mathbb{R}^p$ is the set point or desired value, which is assumed to be constant, and $C(z)$ is the $m \times p$ transfer matrix of the controller and the noise filter $H_c(z)$ is a minimum phase $m \times m$ transfer matrix satisfying $H_c(\infty) = I_m$.

The following assumptions are made on the closed-loop system, the measurable disturbance and noises.

A1: The plant P_u is strictly proper, i.e. $P_u(\infty) = 0$. This implies that the feedback system is well-posed in the sense that the joint input-output (y, u) are determined uniquely if all the external signals are given.

A2: The feedback system is internally stable, and there are no pole-zero cancellations in $P_u(z)$ and $C(z)$.

A3: The exogenous input r satisfies PE condition.

A4: The exogenous input and noises are mutually uncorrelated 2nd-order jointly stationary processes with mean zero.

A5: The desired value is $y^* = 0$.

The identification problem in this paper is stated as follows.

Identification of Closed-Loop Systems: We derive a subspace method of identifying state-space models of the plant $P(z) = [P_u(z)\ P_r(z)\ H_p(z)]$ based on a finite data set $\{r(t), u(t), y(t), t = 0, 1, \cdots, T\}$, where the controller $C(z)$ is unknown.

It follows from Fig. 1 that

$$T_{yr}(z) = (I_p + P_u(z)C(z))^{-1}P_r(z) \tag{3a}$$
$$T_{ur}(z) = -C(z)(I_p + P_u(z)C(z))^{-1}P_r(z) \tag{3b}$$

and that

$$T_{y\nu}(z) = (I_p + P_u(z)C(z))^{-1}H_p(z) \tag{4a}$$
$$T_{y\eta}(z) = P_u(z)(I_m + C(z)P_u(z))^{-1}H_c(z) \tag{4b}$$
$$T_{u\nu}(z) = -C(z)(I_p + P_u(z)C(z))^{-1}H_p(z) \tag{4c}$$
$$T_{u\eta}(z) = (I_m + C(z)P_u(z))^{-1}H_c(z) \tag{4d}$$

where $T_{ab}(z)$ denote the transfer matrix from b to a.

3. DETERMINISTIC AND STOCHASTIC SUBSYSTEMS

3.1 Orthogonal decomposition

The joint input-output process and joint noise process are respectively expressed as $w = \begin{bmatrix} y \\ u \end{bmatrix} \in \mathbb{R}^l$ and $\chi = \begin{bmatrix} \nu \\ \eta \end{bmatrix} \in \mathbb{R}^l$, where $l = p + m$. Let $\mathcal{R}$ be the Hilbert space generated by exogenous input r. Let the orthogonal projection of w onto $\mathcal{R}$ and its complement $\mathcal{R}^\perp$ be given by $w_d(t) = \hat{E}\{w(t) \mid \mathcal{R}\}$ and $w_s(t) = \hat{E}\{w(t) \mid \mathcal{R}^\perp\}$, respectively. We call w_d the deterministic component and w_s the stochastic component.

Under the assumption that the exogenous input r is feedback-free, we can show (Picci and Katayama, 1996) that the joint input-output process w has the orthogonal decomposition

$$w(t) = w_d(t) + w_s(t) \tag{5}$$

where the deterministic and stochastic components are uncorrelated, i.e. $E\{w_s(t)w_d^{\mathrm{T}}(\tau)\} = 0$ holds for all $t,\ \tau = 0, \pm 1, \cdots$. We write $w_d := \begin{bmatrix} y_d \\ u_d \end{bmatrix}$ and $w_s := \begin{bmatrix} y_s \\ u_s \end{bmatrix}$. Then, we see from (5) that

$$y(t) = y_d(t) + y_s(t) \tag{6a}$$
$$u(t) = u_d(t) + u_s(t) \tag{6b}$$

where (y_d, u_d) and (y_s, u_s) are the deterministic and stochastic components of (y, u), respectively.

Applying the above decomposition results to the feedback system described by (1) and (2), we see that the deterministic and stochastic components are respectively given by

$$y_d(t) = P_u(z)u_d(t) + P_r(z)r(t) \tag{7a}$$
$$u_d(t) = -C(z)y_d(t) \tag{7b}$$

and

$$y_s(t) = P_u(z)u_s(t) + H_p(z)\nu(t) \tag{8a}$$
$$u_s(t) = -C(z)y_s(t) + H_c(z)\eta(t) \tag{8b}$$

3.2 Deterministic subsystem

We define a state vector $x_d \in \mathbb{R}^{n_d}$ for the deterministic subsystem, whose dimension is the sum of orders of $P_u(z)$ and $C(z)$ by Assumption 2. Then, a minimal state space model for (7) can be written as

$$x_d(t+1) = A_d x_d(t) + B_d r(t) \tag{9a}$$
$$w_d(t) = C_d x_d(t) + D_d r(t) \tag{9b}$$

where $A_d \in \mathbb{R}^{n_d \times n_d}$, $B_d \in \mathbb{R}^{n_d \times q}$, $C_d \in \mathbb{R}^{l \times n_d}$, $D_d \in \mathbb{R}^{l \times q}$ are constant matrices. We also define $C_d = \begin{bmatrix} C_{d1} \\ C_{d2} \end{bmatrix}$, $D_d = \begin{bmatrix} D_{d1} \\ D_{d2} \end{bmatrix}$ where $C_{d1} \in \mathbb{R}^{p \times n_d}$, $C_{d2} \in \mathbb{R}^{m \times n_d}$ and $D_{d1} \in \mathbb{R}^{p \times q}$, $D_{d2} \in \mathbb{R}^{m \times q}$.

3.3 Stochastic subsystem

For the stochastic subsystem, we can define a state vector $x_s \in \mathbb{R}^{n_s}$, whose dimension is also the sum of orders of $P_u(z)$ and $C(z)$. Thus, a minimal state space model for (8) can be written as

$$x_s(t+1) = A_s x_s(t) + K_s \xi(t) \tag{10a}$$
$$w_s(t) = C_s x_s(t) + \xi(t) \tag{10b}$$

where $A_s \in \mathbb{R}^{n_s \times n_s}$, $C_s \in \mathbb{R}^{l \times n_s}$ are constant matrices, and $K_s = \begin{bmatrix} K_{s1} & K_{s2} \end{bmatrix}$ is the steady-state Kalman gain with $K_{s1} \in \mathbb{R}^{n_s \times p}$, $K_{s2} \in \mathbb{R}^{n_s \times m}$, and $\xi \in \mathbb{R}^l$ is the innovation process of w_s defined by $\xi(t) = w_s(t) - \hat{E}\{w_s(t) \mid w_s(\tau),\ \tau < t\}$.

According to the spectral factorization theory for feedback stochastic systems (Ng *et al.*, 1977; Anderson and Gevers, 1982), we can show that $\xi := \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}$ is related to $\chi := \begin{bmatrix} \nu \\ \eta \end{bmatrix}$ as

$$\begin{bmatrix} \xi_1(t) \\ \xi_2(t) \end{bmatrix} = \begin{bmatrix} I_p & 0 \\ \Delta_{21} & I_m \end{bmatrix} \begin{bmatrix} \nu(t) \\ \eta(t) \end{bmatrix} \tag{11}$$

where the lower triangular form is due to the fact that there exists a delay from u to y, and $\Delta_{21} \in \mathbb{R}^{m \times p}$ denotes a possible correlation of ν to ξ_2. Define $C_s = \begin{bmatrix} C_{s1} \\ C_{s2} \end{bmatrix}$ with $C_{s1} \in \mathbb{R}^{p \times n_s}$ and $C_{s2} \in \mathbb{R}^{m \times n_s}$. It then follows from (10) and (11) that the state space model of stochastic subsystem can be expressed as

$$x_s(t+1) = A_s x_s(t) + B_s \nu(t) + K_{s2}\eta(t) \tag{12a}$$
$$y_s(t) = C_{s1}x_s(t) + \nu(t) \tag{12b}$$
$$u_s(t) = C_{s2}x_s(t) + \Delta_{21}\nu(t) + \eta(t) \tag{12c}$$

where $B_s := K_{s1} + K_{s2}\Delta_{21} \in \mathbb{R}^{n_s \times p}$.

4. CONNECTING DETERMINISTIC AND STOCHASTIC SUBSYSTEMS

In Section 3, we have derived state-space realizations for deterministic and stochastic subsystems. We now put these two subsystems together to obtain a state space model of the plant (1). In view of (5), the outputs w_d and w_s of two subsystems can be added to recover the joint input-output. However, we need to align two state vectors by a suitable similarity transformation T before adding two state vectors x_d and x_s of two subsystems, because they are defined up to similarity transformations. To obtain a similarity transformation, we must find some relations connecting the deterministic and stochastic subsystems.

We see from (3) that

$$C(z)T_{yr}(z) + T_{ur}(z) = 0 \tag{13}$$

Also, since the noise filters are nonsingular, we see from (4a) and (4c) that

$$C(z)T_{y\nu}(z) + T_{u\nu}(z) = 0 \tag{14}$$

Hence, from (13) and (14), we have the following lemma that connects the deterministic and stochastic subsystems. All the proofs are omitted due to space limitation.

Lemma 1. For the compatibility of the two subsystems, we have

$$\left[\begin{array}{c|c} A_d & B_d \\ \hline C_{d2} & D_{d2} \end{array}\right] = \left[\begin{array}{c|c} \bar{A}_s & B_s \\ \hline \bar{C}_s & \Delta_{21} \end{array}\right]\left[\begin{array}{c|c} A_d & B_d \\ \hline C_{d1} & D_{d1} \end{array}\right] \tag{15}$$

where $\bar{A}_s := A_s - B_s C_{s1} \in \mathbb{R}^{n_s\times n_s}$, $\bar{C}_s := C_{s2} - \Delta_{21}C_{s1} \in \mathbb{R}^{m\times n_s}$.

Lemma 2. The condition of (15) holds if and only if $D_{d2} = \Delta_{21}D_{d1}$ and

$$\begin{aligned} C_d(zI_{n_d} - A_d)^{-1}B_d - C_s(zI_{n_s} - A_s)^{-1}B_s D_{d1} \\ = \left(C_s(zI_{n_s} - A_s)^{-1}B_s + \begin{bmatrix} I_p \\ \Delta_{21} \end{bmatrix}\right)F(z) \end{aligned} \tag{16}$$

where $F(z)$ is a strictly proper $p \times q$ matrix.

Lemma 3. In (16), we get $F(z) = 0$ if and only if

$$C_d(zI_{n_d} - A_d)^{-1}B_d = C_s(zI_{n_s} - A_s)^{-1}B_s D_{d1} \tag{17}$$

5. STATE SPACE MODEL OF PLANT

For the deterministic subsystem (A_d, B_d, C_d) and stochastic subsystem $(A_s, B_s D_{d1}, C_s)$, we respectively define extended observability matrices O_k^d and O_k^s as

$$O_k^d = \begin{bmatrix} C_d \\ C_d A_d \\ \vdots \\ C_d A_d^{k-1} \end{bmatrix} \in \mathbb{R}^{kl\times n_d}, \quad O_k^s = \begin{bmatrix} C_s \\ C_s A_s \\ \vdots \\ C_s A_s^{k-1} \end{bmatrix} \in \mathbb{R}^{kl\times n_s}$$

Similarly, extended reachability matrices C_k^d and $\bar{C}_k^s$ are defined as

$$C_k^d = \begin{bmatrix} B_d & A_d B_d & \cdots & A_d^{k-1}B_d \end{bmatrix} \in \mathbb{R}^{n_d\times qk}$$
$$\bar{C}_k^s = \begin{bmatrix} B_s D_{d1} & A_s B_s D_{d1} & \cdots & A_s^{k-1}B_s D_{d1} \end{bmatrix} \in \mathbb{R}^{n_s\times qk}$$

As shown in Subsections 3.2 and 3.3, the dimensions of the deterministic and stochastic subsystems are the same. Thus we write $n_{cl} = n_d = n_s$ in the following.

Lemma 4. Suppose that $D_{d1} \neq 0$. Then, (17) holds if and only if there exists a nonsingular matrix $T \in \mathbb{R}^{n_{cl}\times n_{cl}}$ satisfying

$$C_d = C_s T, \quad TA_d = A_s T, \quad TB_d = B_s D_{d1} \tag{18}$$

Moreover, if $k > n_{cl}$, the similarity transformation is given by $T = (O_k^s)^\dagger O_k^d$, where (†) denotes the pseudo inverse.

Proof: See Kailath (1980). □

Theorem 5. Suppose that there exists a transformation $T \in \mathbb{R}^{n_{cl}\times n_{cl}}$ that satisfies (18). Then, by defining $x = x_s + Tx_d$, the plant output y is expressed as

$$\begin{aligned} x(t+1) =& (A_s - K_{s2}C_{s2})x(t) + K_{s2}u(t) \\ &+ (TB_d - K_{s2}D_{d2})r(t) + K_{s1}\nu(t) \end{aligned} \tag{19a}$$
$$y(t) = C_{s1}x(t) + D_{d1}r(t) + \nu(t) \tag{19b}$$

Equation (19) is easily derived by using (9) and (12). It is a state space realization of the output process y of (1) in terms of the combined state vector x, the input u, the exogenous input r and the white noise ν. This theorem implies that a state space model of the plant can be obtained by identifying both the deterministic subsystem (9) and the stochastic subsystem (10), followed by suitably combining parameters of the two subsystems by a similarity transformation.

It should be noted that Theorem 5 is derived under the condition $F(z) = 0$. If, however, $D_{d1} = 0$ in (16), we get

$$C_d(zI_{n_d} - A_d)^{-1}B_d = \left(C_s(zI_{n_s} - A_s)^{-1}B_s + \begin{bmatrix} I_p \\ \Delta_{21} \end{bmatrix}\right)F(z) \tag{20}$$

Thus, obviously $F(z) \neq 0$, so that in this case, Lemmas 3 and 4 are not valid. But, by setting $D_{d1} = 0$ and hence $D_{d2} = 0$ in (19), we have

$$\begin{aligned} x(t+1) =& (A_s - K_{s2}C_{s2})x(t) + K_{s2}u(t) \\ &+ TB_d r(t) + K_{s1}\nu(t) \end{aligned} \tag{21a}$$
$$y(t) = C_{s1}x(t) + \nu(t) \tag{21b}$$

Thus, from (21), we have a strictly proper transfer matrix as expected, i.e.

$$P_r(z) = C_{s1}(zI_{n_s} - A_s + K_{s2}C_{s2})^{-1}TB_d \tag{22}$$

This implies that the state space model of (19) is valid even if $P_r(z)$ is strictly proper.

The following lemma also verifies (22).

Lemma 6. For $D_{d1} = 0$, we can show that $F(z)$ satisfying (20) is given by

$$F(z) = C_{s1}(zI_{n_s} - A_s + B_s C_{s1})^{-1}TB_d \tag{23}$$

under the first two conditions of (18). By using (23), we can prove that $P_r(z)$ is given by (22). □

6. SUBSPACE IDENTIFICATION METHOD

The deterministic and stochastic subsystems (9) and (10) are derived under the assumption that infinite history of joint process w is given. In this section, for given finite data, we briefly describe the basic procedure of ORT-based identification according to Katayama (2005).

Suppose that the data $\{y(t), u(t), r(t), t = 0, 1, \cdots, N + 2k - 2\}$ are given, where $k > n_{cl}$, and N sufficiently large. As usual we define a block Hankel matrix $R_{0|k-1} \in \mathbb{R}^{kq\times N}$ as

$$R_{0|k-1} = \begin{bmatrix} r(0) & r(1) & \cdots & r(N-1) \\ r(1) & r(2) & \cdots & r(N) \\ \vdots & \vdots & & \vdots \\ r(k-1) & r(k) & \cdots & r(k+N-2) \end{bmatrix}$$

and similarly for $R_{k|2k-1}$. Also, we define $W_{0|k-1}, W_{k|2k-1} \in \mathbb{R}^{kl\times N}$ by using the joint input-output process w.

A key step is the use of LQ decomposition to compute the deterministic and stochastic components (w_d, w_s) of the joint process w.

We compute the LQ decomposition

$$\begin{bmatrix} R_{k|2k-1} \\ R_{0|k-1} \\ W_{0|k-1} \\ W_{k|2k-1} \end{bmatrix} = \begin{bmatrix} L_{11} & 0 & 0 & 0 \\ L_{21} & L_{22} & 0 & 0 \\ L_{31} & L_{32} & L_{33} & 0 \\ L_{41} & L_{42} & L_{43} & L_{44} \end{bmatrix} \begin{bmatrix} Q_1^{\mathrm{T}} \\ Q_2^{\mathrm{T}} \\ Q_3^{\mathrm{T}} \\ Q_4^{\mathrm{T}} \end{bmatrix} \quad (24)$$

where $L_{11}, L_{22}, L_{33}, L_{44}$ are lower triangular matrices, and Q_is are orthogonal, i.e. $Q_i^{\mathrm{T}} Q_j = I\delta_{ij}$. From the theory of subspace identification, we see that $\mathrm{rank}(R_{0|2k-1}) = 2kq$ should hold in (24), so that we assume that the disturbance r satisfies PE condition of order $2k$.

6.1 Identification of deterministic subsystem

We see from (24) that the deterministic component $W^d_{0|2k-1} := \hat{E}\{W_{0|2k-1} \mid R_{0|2k-1}\}$ is given by

$$\begin{bmatrix} W^d_{0|k-1} \\ W^d_{k|2k-1} \end{bmatrix} = \begin{bmatrix} L_{31} & L_{32} \\ L_{41} & L_{42} \end{bmatrix} \begin{bmatrix} Q_1^{\mathrm{T}} \\ Q_2^{\mathrm{T}} \end{bmatrix} \quad (25)$$

and that $\mathrm{Im}(O^d_k) = \mathrm{Im}(L_{42})$. Let the SVD of L_{42} be given by

$$L_{42} = [\hat{U} \ \ \tilde{U}] \begin{bmatrix} \hat{S} & 0 \\ 0 & \tilde{S} \end{bmatrix} \simeq \hat{U}\hat{S}\hat{V}^{\mathrm{T}}$$

where $\dim(\hat{S}) = n_d$ and the term with less significant singular values are discarded. Then, the extended observability matrix is given by $O^d_k = \hat{U}\hat{S}^{1/2}$, so that A_d and C_d are

$$A_d = (O^d_{k-1})^{\dagger}\bar{O}^d_k, \quad C_d = \begin{bmatrix} C_{d1} \\ C_{d2} \end{bmatrix} = O^d_k(1:l,:)$$

where $\bar{O}^d_k := O^d_k(l+1:kl,:)$.

Moreover, the matrices (B_d, D_d) can be obtained by solving

$$\tilde{U}^{\mathrm{T}} L_{41} = \tilde{U}^{\mathrm{T}} \Psi_k(B_d, D_d) L_{11} \quad (26)$$

where $\Psi(B_d, D_d)$ is the well-known block Toeplitz matrix formed by the Markov parameters $g_d(0) = D_d$ and $g_d(j) = C_d A_d^{j-1} B_d,\ j = 1, 2, \cdots, k-1$.

6.2 Identification of stochastic subsystem

We see from (24) that the stochastic component is given by

$$\begin{bmatrix} W^s_{0|k-1} \\ W^s_{k|2k-1} \end{bmatrix} = \begin{bmatrix} L_{33} & 0 \\ L_{43} & L_{44} \end{bmatrix} \begin{bmatrix} Q_3^{\mathrm{T}} \\ Q_4^{\mathrm{T}} \end{bmatrix} \quad (27)$$

Define the covariance matrices of stochastic component as

$$\begin{bmatrix} \Sigma_{pp} & \Sigma_{pf} \\ \Sigma_{fp} & \Sigma_{ff} \end{bmatrix} := \frac{1}{N} \begin{bmatrix} W^s_{0|k-1} \\ W^s_{k|2k-1} \end{bmatrix} \begin{bmatrix} W^s_{0|k-1} \\ W^s_{k|2k-1} \end{bmatrix}^{\mathrm{T}}$$

Then, we have

$$\Sigma_{pp} = \frac{1}{N} L_{33} L_{33}^{\mathrm{T}}, \ \Sigma_{fp} = \frac{1}{N} L_{43} L_{33}^{\mathrm{T}}, \ \Sigma_{ff} = \frac{1}{N}(L_{43} L_{43}^{\mathrm{T}} + L_{44} L_{44}^{\mathrm{T}})$$

Computing the normalized SVD

$$\Sigma_{ff}^{-1/2} \Sigma_{fp} \Sigma_{pp}^{-\mathrm{T}/2} = U \Sigma V^{\mathrm{T}} \simeq \hat{U}\hat{\Sigma}\hat{V}^{\mathrm{T}}$$

we get the extended observability matrix $O^s_k = \Sigma_{ff}^{1/2} \hat{U} \hat{\Sigma}^{1/2}$.

Now we define the estimate of the state vector of the stochastic subsystem as $\bar{X}^s_k = \hat{\Sigma}^{1/2} \hat{V}^{\mathrm{T}} \Sigma_{pp}^{-1/2} W^s_{0|k-1} \in \mathbb{R}^{n_s \times N}$, and define

$$\hat{X}^s_{k+1} := \bar{X}^s_k(:, 2:N), \quad \hat{X}^s_k := \bar{X}^s_k(:, 1:N-1)$$
$$\hat{W}^s_{k|k} := W^s_{k|k}(:, 1:N-1)$$

Then we have the regression equation

$$\begin{bmatrix} \hat{X}_{k+1} \\ \hat{W}^s_{k|k} \end{bmatrix} = \begin{bmatrix} A_s \\ C_s \end{bmatrix} \hat{X}_k + \begin{bmatrix} \rho_x \\ \rho_w \end{bmatrix} \quad (28)$$

where $\rho_x \in \mathbb{R}^{n_s \times (N-1)}$ and $\rho_w \in \mathbb{R}^{l \times (N-1)}$ are residual errors. Applying the least-squares method to (28), we have the estimate of (A_s, C_s) together with the covariance matrices of residuals

$$\begin{bmatrix} \hat{Q} & \hat{S} \\ \hat{S}^{\mathrm{T}} & \hat{R} \end{bmatrix} = \frac{1}{N-1} \begin{bmatrix} \rho_x \rho_x^{\mathrm{T}} & \rho_x \rho_w^{\mathrm{T}} \\ \rho_w \rho_x^{\mathrm{T}} & \rho_w \rho_w^{T} \end{bmatrix}$$

By solving the algebraic Riccati equation (ARE), we compute the Kalman gain $K = [K_{s1} \ \ K_{s2}]$. The covariance matrix of the innovation process is then given by $\Xi = (\Xi_{ij}) = C_s P C_s^{\mathrm{T}} + \hat{R}$, where P is the solution of ARE. Thus, we have $\Delta_{21} = \Xi_{21}\Xi_{11}^{-1}$.

Define $T = (O^s_k)^{\dagger} O^d_k$, and $A = A_s - K_{s2} C_{s2}$, $B_u = K_{s2}$, $C = C_{s1}$, $B_r = T B_d - K_{s2} D_{d2}$, $D_r = D_{d1}$. Then, a plant model is given by (19).

7. NUMERICAL EXAMPLES

7.1 Example

Consider a 2nd-order system described by

$$x(t+1) = Ax(t) + B_u u(t) + B_r r(t) + K\nu(t)$$
$$y(t) = Cx(t) + D_r r(t) + \nu(t)$$

where

$$A = \begin{bmatrix} 1.80 & -0.85 \\ 1.00 & 0.00 \end{bmatrix}, \ B_u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ B_r = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \ K = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$
$$C = [-0.3 \ 1.0], \ D_r = 1,$$

and where r and ν are Gaussian white noises with mean zero and variances $\sigma_r^2 = 0.25$ and $\sigma_\nu^2 = 0.36$, respectively. Also, the feedback control is given by

$$u(t) = -C(z)y(t) + \eta(t)$$

where $C(z)$ is a PI controller of the form

$$C(z) = 0.04 \left[1 + \frac{z}{30(z-1)} \right], \quad H_c(z) = 1$$

and where η is a Gaussian white noise with mean zero and variance $\sigma_\eta^2 = 0.04$.

For simulation studies, we choose $N = 1000$, $n_d = 3, n_s = 3$ and $k = 21$ [2]. Also, the order of identified models is reduced from $n_{cl} = \min(n_d, n_s) = 3$ to $n = 2$, where it is assumed that the true model order n is known.

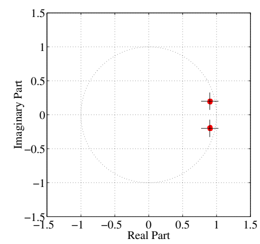


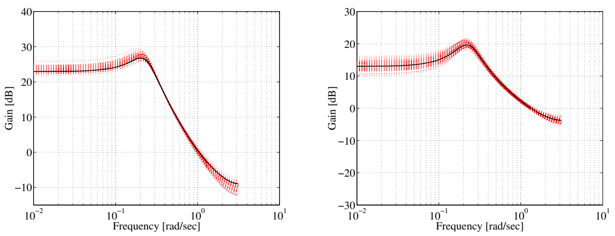
Fig. 2. Pole plots of the plants estimated by ORT method, where the true poles are denoted by +.

Table 1 displays the performance of the ORT-based method by Monte Carlo simulation. For comparison, the result by the PBSID method (Chiuso and Picci, 2005*a*) is also included. We see that both methods give quite similar estimates of the

[2] The orders (n_d, n_s) are obtained by computing the prediction errors, so that they may be different. If $n_d \geq n_s$, the present algorithm works with $T \in \mathbb{R}^{n_s \times n_d}$. If $n_d < n_s$, then we must define a different transformation matrix $\tilde{T} \in \mathbb{R}^{n_d \times n_s}$. Details are omitted due to space limitation.

Table 1. Simulation results for the 2nd-order plant $P_u(z) = N_u(z)/d(z)$ and $P_r(z) = N_r(z)/d(z)$, where the means and standard deviations (s.d.) are computed based on 50 Monte Carlo runs.

		True	ORT	PBSID
$d(z)$	$\hat{a}_1$	-1.8	-1.8073	-1.7962
	(s.d.)		(0.0109)	(0.0121)
	$\hat{a}_2$	0.85	0.8550	0.8466
	(s.d.)		(0.0102)	(0.0176)
$N_u(z)$	$\hat{b}_{u1}$	-0.3	-0.2219	-0.2932
	(s.d.)		(0.0822)	(0.0936)
	$\hat{b}_{u2}$	1.0	0.9367	1.0002
	(s.d.)		(0.0778)	(0.1252)
$N_r(z)$	$\hat{b}_{r0}$	1.0	0.9975	1.0040
	(s.d.)		(0.0530)	(0.0443)
	$\hat{b}_{r1}$	-1.3	-1.2972	-1.2954
	(s.d.)		(0.0936)	(0.0780)
	$\hat{b}_{r2}$	0.0775	0.0801	0.0825
	(s.d)		(0.0701)	(0.0529)



(a) $P_u(z)$ (b) $P_r(z)$

Fig. 3. Bode plots of the plant identification results by ORT method, where the true one is shown in a thick line.

denominator $d(z)$, and the PBSID gives unbiased estimates of the numerator $N_u(z)$ but with somewhat larger variance errors, while the ORT-based method gives biased estimates. Also, both methods give comparable estimates of the numerator $N_r(z)$.

Figs. 2 and 3 respectively display estimates of plant poles ($0.9 \pm 0.2j$) and Bode plots of $P_u(z)$ and $P_r(z)$ by the present method.

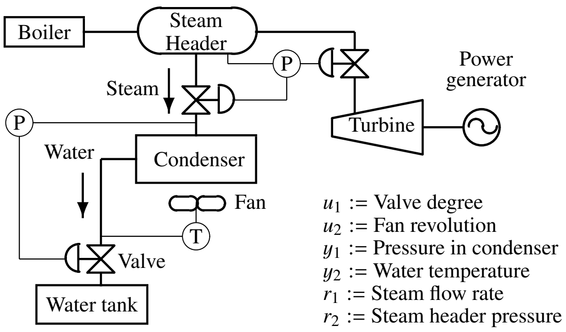


Fig. 4. High pressure steam condenser

7.2 Example 2

We consider a high pressure steam condenser as an industrial application; see Fig. 4. Steam from boiler is accumulated in a high pressure steam header, where a fixed amount of it is continuously fed to a steam turbine to generate electricity. On the other hand, the high pressure steam condenser receives excess steam, converting it into water for re-circulation. Due to large fluctuations of the flow rate r_1 and pressure r_2 of incoming steam, we see large variations in the pressure y_1 and water temperature y_2 of the high pressure steam condenser, so that the condenser is regulated by two PID controllers, i.e. the valve degree u_1 and fan revolution u_2.

Table 2. Identification result: Simulation errors for 14 days.

Day	Standard deviations (%)	
	y_1	y_2
12/10/2005	3.62	1.48
13/10/2005	3.53	5.14
14/10/2005	3.43	4.22
15/10/2005	15.97*	18.15*
16/10/2005	2.93	3.91
17/10/2005	3.65	1.17
18/10/2005	2.82	1.52
19/10/2005	2.99	1.70
20/10/2005	6.99	6.98
21/10/2005	2.53	1.69
22/10/2005	3.26	2.61
23/10/2005	3.64	2.79
24/10/2005	3.01	3.10
25/10/2005	2.33	1.42

We have collected data for 14 days from October 12 to 25, 2005, where the sampling time is 20s, so that the number of data for one day is $N = 4,320$. By applying the present method, we have obtained a set of 14 identification results as shown in Table 2, where the performance is evaluated in terms of simulation errors rather than prediction errors (Ljung, 1999). As shown in Table 2, the results are nearly the same except for the data of October 15 marked by asterisks.

The results for October 21, 2005 are shown in Figs. 5 – 8, where we have chosen $(n_d, k_d) = (8, 17)$, $(n_s, k_s) = (7, 14)$. Figs. 5 and 6 display the poles and Bode plots of an identified plant model with $n_p = 5$. Also, we see from Fig. 7 that the simulated output is very close to measured outputs. The step responses of the derived model are displayed in Fig. 8, showing that these responses are good agreement with our physical knowledge acquired from past experiences. Though not shown here, other results are quite similar to the one shown above. Especially, the identification results are quite stable in the sense that if two data sets of 24 hours are similar, so are the identification results.

8. CONCLUSIONS

In this paper, we have developed a new identification method for closed-loop system without using test signals under the assumption that there exists a measurable disturbance satisfying a certain PE condition. By introducing a compatibility condition between realizations of deterministic and stochastic subsystems, we have derived a state space realization of the plant, based on which an ORT-based algorithm of identifying the closed-loop system is obtained. Numerical results for a simple system and an industrial plant are included to show the applicability of the proposed method. Since there are many real plants similar to the one treated here, the present ORT-based algorithm has many applications in the future.

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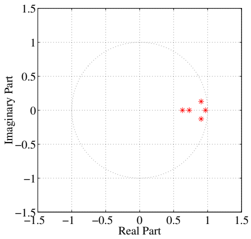

Fig. 5. Pole plots of an estimated plant model ($n_p = 5$)

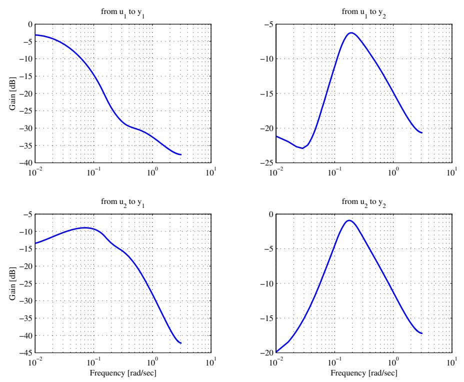

Fig. 6. Bode plots of an estimated plant model

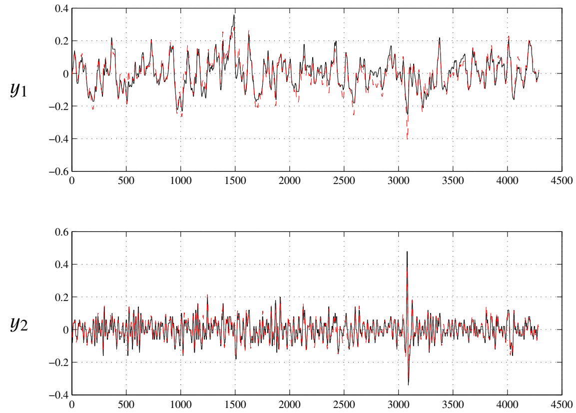

Fig. 7. Simulation of outputs y_1 and y_2, where "black" denotes measured outputs and "red" predicted outputs

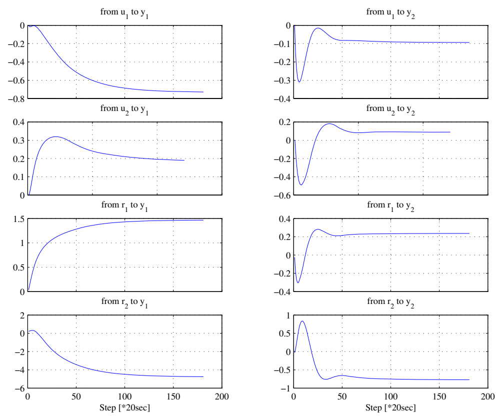

Fig. 8. Step responses of the estimated plant model

REFERENCES

Anderson, B. D. O. and M. R. Gevers (1982). Identifiability of linear stochastic systems operating under linear feedback. *Automatica* **18**(2), 195–213.

Ase, H., K. Takaba and T. Katayama (2006). Subspace identification of gasifying and direct melting plant for control. In: *Proc. 14th IFAC Symposium on System Identification*. Newcastle, Australia.

Chiuso, A. (2007). The role of vector autoregressive modeling in predictor-based subspace identification. *Automatica* **43**(6), 1034–1048.

Chiuso, A. and G. Picci (2005*a*). Consistency analysis of some closed-loop subspace identification methods. *Automatica* **41**(3), 377–391.

Chiuso, A. and G. Picci (2005*b*). Prediction error vs subspace methods in closed-loop identification. In: *Preprints 16th IFAC World Congress*. Prague. Paper No. Tu-E02-TO/3.

Forssell, U. and L. Ljung (1999). Closed-loop identification revisited. *Automatica* **35**(7), 1215–1241.

Jansson, M. (2003). Subspace identification and ARX modeling. In: *Proc. 13th IFAC Symposium on Systems Identification*. Rotterdam. pp. 1625–1630.

Jansson, M. (2005). A new subspace identification method for open and closed loop data. In: *Preprints 16th IFAC World Congress*. Prague. Paper No. Tu-E02-TO/2.

Kailath, T. (1980). *Linear Systems*. Prentice-Hall. New York.

Katayama, T. (2005). *Subspace Methods for System Identification*. Springer-Verlag. London.

Katayama, T. and H. Tanaka (2007). An approach to closed-loop subspace identification by orthogonal decomposition. *Automatica* **43**(9), 1623–1630.

Katayama, T., H. Kawauchi and G. Picci (2005). Subspace identification of closed-loop system by the orthogonal decomposition method. *Automatica* **41**(5), 863–872.

Ljung, L. (1999). *System Identification – Theory for the User (2nd ed.)*. Prentice-Hall. Englewood Cliffs, NJ.

Ljung, L. and T. McKelvey (1996). Subspace identification from closed loop data. *Signal Processing* **52**(2), 209–216.

Ng, T. S., G. C. Goodwin and B. D. O. Anderson (1977). Identifiability of MIMO linear dynamic systems operating in closed loop. *Automatica* **13**(5), 477–485.

Picci, G. and T. Katayama (1996). Stochastic realization with exogenous inputs and 'subspace methods' identification. *Signal Processing* **52**(2), 145–160.

Qin, S. J. and L. Ljung (2003). Closed-loop subspace identification with innovation estimation. In: *Proc. 13th IFAC Symposium on Systems Identification*. Rotterdam. pp. 887–892.

Qin, S. J. and L. Ljung (2006). On the role of future horizon in closed-loop subspace identification. In: *Preprints 14th IFAC Symposium on System Identification*. Newcastle, Australia. pp. 1080–1084.

Söderström, T. and P. Stoica (1989). *System Identification*. Prentice-Hall. Englewood Cliffs, NJ.

Van den Hof, P. M. J. (1998). Closed-loop issues in system identification. *Annual Reviews in Control* **22**, 173–186.

Wang, J. and S. J. Qin (2006). Closed-loop subspace identification using the parity space. *Automatica* **42**(2), 315–320.