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Flocking of Decentralized Multi-Agent Systems with Application to Nonholonomic Multi-Robots ⋆

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Abstract: In this paper, we revisit the artificial potential based approach in the flocking control for multi-agent systems, where our main concerns are migration and trajectory tracking problems. The static destination or the tracking reference point is modeled by a virtual leader, whose information is utilized by some agents, called active agents (AA), for the controller design. We study a controller for the case where the set of AAs is fixed. By introducing dwell time for the topology-varying system, we define the solutions for the closed-loop system equations. The existence and uniqueness of the solution with any given non-singular initial condition is proved; and some results on the velocity consensus, collision avoidance, group configuration and robustness are proposed. Finally, we apply the proposed controllers to the flocking control of a team of nonholonomic mobile robots.

1. INTRODUCTION

A flock can be seen as a "loose" but connected formation which does not require the group to be in a unique geometric pattern (see Olfati-Saber (2006)). Many existing results on flocking control of multi-agent systems rely on the concept called (artificial) potential fields or potential functions. The idea based on this concept is to relate the desired geometric patterns (or configurations) to the local or global extremes of an elaborately cooked potential function of the group, and then design the gradient-based control strategy to drive the group to minimize the potential function. The problem of flocking control for particle vehicles with single or double integrator models is worthy of study not only because it can provide high level control strategies for flocking control of multi-vehicle teams with more complex dynamics, but also due to its value in determining the effects of information flow in the distributed control of coupled systems. In the early paper Leonard et al. (2001), virtual leaders of the group are introduced and pair-wise potentials not only exist between real agents in the group but also between a real agent and the virtual leaders. The aim of adding virtual leaders is to help shape the potential function for the group so that it can be stabilized at the desired geometric pattern (not only a flock). In Olfati-Saber (2006), the author describes a smooth pair-wise potential function whose gradient specifies a kind of attractive/repulsive force between neighboring agents which is continuous with respect to the relative distance. It is proved in Olfati-Saber (2006) that the control law combining the potential's gradient term with velocity matching term coincides with the Reynolds rules but will generically lead to regular fragmentation of the group. The work Tanner et al. (2007) relaxes the requirement on the smoothness of the pair-wise potentials but similar controllers as in Olfati-Saber (2006) are adopted. And the system stability is analyzed by the nonsmooth version of LaSalle Invariance Principle.

In this paper, we propose control strategies aimed at migration and trajectory tracking of a group of agents. A virtual leader is used to represent the stationary destination of the migration or a reference point on the trajectory being tracked by the group. Along the line of Leonard et al. (2001), Olfati-Saber (2006) and Tanner et al. (2007), we revisit the design of gradient-based control law in the artificial potential framework, which owns the intrinsic property of the inter-agent collision avoidance. It is assumed that some of the agents, called *active agent* (AA), in the group utilize the information of the virtual leader as well as their neighboring agents in the controllers; while the others only use the information of their neighbors. The velocity consensus and the configuration convergence of the group by the proposed controllers are analyzed.

In detail, the paper is composed of two parts. In the first part, we design the flocking controller for particle agents with double integrator model. At the current stage, we only discuss the case in which the AAs in the group are fixed. We prove the existence and uniqueness of the solutions of the differential equation describing the closed-loop dynamics of the agents. Then, we show that, by our controller, the velocities of the group reach consensus; inter-agent collision is avoided; and the configuration of the group almost converges to the local minimum of the collective potentials of the group. As a special case, we give a result on the geometric property of the group with only one AA. Also, we establish the velocity consensus and configuration convergence results for the system with some kind of disturbance.

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In the second part, the controller designed for the mass point model is applied to the flocking control of a group of unicycles. Specially, we study the case where each robot in the group cannot measure its velocity information. A passive observer is used to observe the linear and angular velocities for each agent. And the estimated data is transmitted between each pair of neighboring agents for the use of controller design.

The rest of the paper is organized as follows: In Section 2, we introduce some basics of graph theory and the properties of the potential functions used in this work. In Section 3, we present our results on the flocking control of particle model. While we describe the flocking control of multiple nonholonomic mobile robots in Section 4. Lastly, the simulation results and some brief concluding remarks will be made in Sections 5 and 6 respectively. Most of the proofs in this work are omitted for want of space, and are available from the authors upon request.

2. PRELIMINARIES

2.1 Graph theory

First, we recall some basics of graph theory from the past literature, see, e.g. Godsil et al. (2001). A directed graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ consists of a vertex set $\mathcal{V}$ and an edge set $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$. For any $i,j \in \mathcal{V}$, the ordered pair $(i,j) \in \mathcal{E}$ if and only if i is a neighbor of j. A directed path from vertex i to j is a sequence of directed edges $(v_1,v_2),(v_2,v_3),\cdots,(v_{n-2},v_{n-1}),(v_{n-1},v_n)$, where $n \geq 1, v_1 = i, v_n = j$, and $v_1,\cdots,v_n$ are distinct. A graph $\mathcal{G}(\mathcal{V},\mathcal{E})$ is undirected if and only if for any $i,j \in \mathcal{V}$, $(i,j) \in \mathcal{E}$ implies $(j,i) \in \mathcal{E}$, i.e., each edge in $\mathcal{E}$ is undirected. A path in an undirected graph is defined analogously as directed path in a directed graph. An undirected graph is said to be connected if and only if there is a path between any pair of vertices.

In this work, we use $\mathcal{G}_p(\mathcal{V},\mathcal{E}(t))$, sometimes simply $\mathcal{G}_p(t)$, to denote the group induced graph for a group of N agents, where the vertex set $\mathcal{V}$ and the edge set $\mathcal{E}(t), t \geq t_0$, are defined as:

$$\mathcal{V} = \{1,2,\cdots,N\}, \tag{1}$$

$$\mathcal{E}(t) = \{(i,j) : \|x_i(t) - x_j(t)\| \leq r_{nb},\ i,j \in \mathcal{V}\}, \quad t \geq t_0 \tag{2}$$

where N is the number of agents in the group, r_{nb} is a positive real number. From the definitions above, we see that the graph $\mathcal{G}_p(t)$ is an undirected graph, and $i,j \in \mathcal{V}$ are neighbors if and only if the distance between agents i and j is less than or equal to r_{nb}.

The adjacency matrix $A(t) \in \mathbb{R}^{N\times N}$ and the Laplacian $L(t) \in \mathbb{R}^{N\times N}$ of the graph $\mathcal{G}_p(t)$ are, by convention, defined as Godsil et al. (2001):

$$A_p(t) = [a_{ij}(t)],\ \text{with}\ a_{ij}(t) = \begin{cases} a_{ij}^* > 0 & \text{, if } (i,j) \in \mathcal{E}(t) \\ 0 & \text{, otherwise} \end{cases} \tag{3}$$

where $a_{ij}^* = a_{ji}^*, \forall\, i,j \in \mathcal{V}$; and

$$L_p(t) = [l_{ij}(t)],\ \text{with}\ l_{ij}(t) = \begin{cases} \sum\limits_{j\neq i} a_{ij}(t) & \text{, if } i = j \\ -a_{ij}(t) & \text{, otherwise} \end{cases} \tag{4}$$

Obviously, $A_p(t)$ and $L_p(t)$ are both symmetric.

In this paper, we call an agent, which utilizes the information of the virtual leader in its controller, an *active agent* (AA) of the group. The set of the AA's at time $t, t \geq t_0$, is denoted as $\mathcal{W}(t)$. In addition, we define matrices

$$B(t) = diag\{b_1(t),\cdots,b_N(t)\}, \tag{5}$$

$$L_a(t) = L_p(t) + B(t). \tag{6}$$

with

$$b_i(t) = \begin{cases} b_i^* > 0 & \text{, if } i \in \mathcal{W}(t) \\ 0 & \text{, otherwise} \end{cases} \tag{7}$$

2.2 Potential functions

In this subsection, we introduce potential functions that characterize, respectively, the inter-agent and leader-agent attraction and repulsion.

Inter-agent potential An inter-agent potential function $\psi_a(\cdot) : (0,+\infty) \to [0,+\infty)$ is a C^2 function with the following properties: for some positive numbers d_a, r_a satisfying $0 < d_a < r_a < r_{nb}$,

a) $\frac{d\psi_a(x)}{dx} < 0, x \in (0,d_a)$; $\frac{d\psi_a(x)}{dx} > 0, x \in (d_a,r_a)$; $\frac{d\psi_a(x)}{dx} = 0, x \in [r_a,+\infty)$;
b) $\lim_{x\to 0}\psi_a(x) = +\infty$;
c) $\psi_a(x)$ has a unique minimum at $x = d_a$.

An example of inter-agent potential is:

$$\psi_a(x) = \int_{d_a}^{x} 10 \cdot \left(-\frac{1}{\xi^2} + \frac{1}{d_a^2}\right) \varrho_h\left(\frac{\xi}{r_a}\right) d\xi, \tag{8}$$

where $\varrho_h(z)$ is a bump function defined as in Olfati-Saber (2006):

$$\varrho_h(z) = \begin{cases} 1, & z \in [0,h) \\ \frac{1}{2}\left[1 + \cos\left(\pi\frac{z-h}{1-h}\right)\right], & z \in [h,1] \\ 0, & z \in (1,+\infty) \end{cases} \tag{9}$$

Leader-agent potentials Here, we introduce two types of leader-agent potential functions ψ_l and ψ_{lr}:

- $\psi_l(\cdot) : [0,+\infty) \to (0,+\infty)$ is a C^1 function with the properties:
  a) $\frac{d\psi_l(x)}{dx} \cdot \frac{1}{x}$ is locally Lipschitz over $[0,+\infty)$, and therefore is uniformly continuous on $[0,x^*]$ for any $0 < x^* < +\infty$;
  b) $\frac{d\psi_l(x)}{dx} = 0$, for $x = 0$; $\frac{d\psi_l(x)}{dx} > 0$, for all $x > 0$;
  c) $\lim_{x\to+\infty}\psi_l(x) = +\infty$;
  d) For any given $x_* > 0$, $\exists\,\epsilon(x_*) > 0$ such that $\frac{d\psi_l(x)}{dx} > \epsilon, \forall\, x \geq x_*$.
- $\psi_{lr}(\cdot) : (0,+\infty) \to (0,+\infty)$ is a C^2 function with the properties: for some $d_l > 0$,
  a) $\frac{d\psi_{lr}(x)}{dx} < 0, x \in (0,d_l)$; $\frac{d\psi_{lr}(x)}{dx} > 0, x \in (d_l,+\infty)$; $\frac{d\psi_{lr}(x)}{dx} = 0, x = d_l$;
  b) $\lim_{x\to 0}\psi_{lr}(x) = +\infty$, $\lim_{x\to+\infty}\psi_{lr}(x) = +\infty$;
  c) $\forall\,\delta > 0$, $\exists\,\epsilon(\delta) > 0$ such that $\frac{d\psi_l(x)}{dx} > \epsilon$, $\forall\,|x - d_l| > \delta$.

It is easy to see that $\psi_l(x)$ (resp. $\psi_{lr}(x)$) has a unique minimum at $x = 0$ (resp. $x = d_l$). Examples of functions ψ_l and ψ_{lr} are, respectively, $\frac{x^2}{2} + 1$ and $x + \frac{1}{x} - 1$.

Throughout this paper, we use $\mathbb{N}, \mathbb{R}^+, \mathbb{Z}^+$ to denote, respectively, the set of natural numbers, nonnegative real numbers and nonnegative integers. $\mathcal{L}_p^m[t_0, +\infty)$ is used to denote the set of all piecewise continuous functions $u : [t_0, +\infty) \to \mathbb{R}^m$ such that $\left(\int_{t_0}^{+\infty} \|u(t)\|^p dt\right)^{1/p} < +\infty$, Khalil (2002). In addition, we use $\mathbf{1}_N$ and $\mathbf{0}_N$ to represent the $N \times 1$ vectors with all the elements being 1 and 0, respectively.

3. FLOCKING CONTROLLER FOR DOUBLE-INTEGRATOR MODEL

In this section, we consider the model of each agent in the group as:

$$\dot{x}_i(t) = v_i(t), \quad \dot{v}_i(t) = u_i(t), \quad i \in \mathcal{V}, \tag{10}$$

where $x_i(t) \in \mathbb{R}^n$ and $v_i(t) \in \mathbb{R}^n$ ($n = 2, 3$) are the position and velocity of the ith robot respectively; and $u_i(t)$ is the control input (acceleration) of the ith robot. The model for the virtual leader is in the same form as that of the agent, i.e.,

$$\dot{x}_l(t) = v_l(t), \quad \dot{v}_l(t) = u_l(t) \tag{11}$$

where "l" stands for the word "leader".

We emphasize that in this work, we only discuss the flocking behavior of a group of robots with fixed AAs, i.e. we make the assumption:

Assumption 1. The set of active agents in the group $\mathcal{W}$ is nonempty and time-invariant.

Since, under Assumption 1, the set $\mathcal{W}(t)$ and the matrix $B(t)$ in (5) are time-invariant, we drop the argument t in their expressions.

Throughout this section, we will focus our discussion on the flocking control strategy which involves the leader-agent potential ψ_l. Some remarks will be made at the end to clear the differences incurred by substituting ψ_l with ψ_{lr}.

It is known that the mobility and limited sensing range of the agents in the group raises the issue that the neighboring relationship of the group may be time-varying. For this reason, to start with our discussion, we need to define the following time-dependent agent sets:

Definition 1. Agent sets $\mathcal{S}_i(t), \mathcal{N}_i(t), \mathcal{I}_i(t)$, $i \in \mathcal{V}, t \in [t_0, +\infty)$ are defined as

$$\mathcal{S}_i(t) = \{j \in \mathcal{V} : \|x_i(t) - x_j(t)\| < r_s\}, \tag{12}$$

$$\mathcal{N}_i(t) = \{j \in \mathcal{V} : \|x_i(t) - x_j(t)\| < r_{nb}\}, \tag{13}$$

$$\mathcal{I}_i(t) = \{j \in \mathcal{V} : \|x_i(t) - x_j(t)\| < r_a\}, \tag{14}$$

where $r_s > r_{nb} > 0$ is the physical sensing range of each agent; r_{nb} is defined in (2); and r_a is as in subsection 2.2. Obviously, we have the relation: $r_a < r_{nb} < r_s$.

Note that in Tanner et al. (2007), the solutions of the switching closed-loop system is discussed using the tool of differential inclusion. But in this way, one cannot specify the single rate of change of the state when system switches since it is only can be said to lie in a set. In view of this, in the analysis of the closed-loop system, we introduce dwell time in the system dynamics. Indeed, our control strategy is that each agent determines its neighbor set at every moment in the time sequence

$$\mathcal{T} := \{t_0, t_1, \cdots\} \quad \text{with} \quad t_{k+1} - t_k = \tau_d > 0, \tag{15}$$

and for all $t \in [t_k, t_{k+1}), k \in \mathbb{Z}^+$, agent $i, i \in \mathcal{V}$ implements the control law

$$\begin{aligned} u_i^{af}(t) = &\sum_{j \in \mathcal{N}_i(t_k)} f_a(d_{ij}) n_{ji} + g(b_i) f_l(d_{il}) n_{li} \\ &- \sum_{j \in \mathcal{N}_i(t_k) \bigcap \mathcal{S}_i(t)} a_{ij}^*(v_i - v_j) - b_i(v_i - v_l) + u_l, \end{aligned} \tag{16}$$

where a_{ij}^* and b_i have been defined in (3) and (7); and

$$f_a(d_{ij}) = \frac{d\psi_a(d_{ij})}{dd_{ij}}, \quad f_l(d_{il}) = \frac{d\psi_l(d_{il})}{dd_{il}}, \tag{17}$$

$$\begin{aligned} &d_{ij} = \|x_i - x_j\|, \ \ d_{il} = \|x_i - x_l\|, \\ &n_{ji} = \frac{x_j - x_i}{d_{ij}}, \ \ n_{li} = \frac{x_l - x_i}{d_{il}}, \end{aligned} \tag{18}$$

$$g(y) = \begin{cases} 1 \ , \ y > 0, \\ 0 \ , \ y = 0. \end{cases} \tag{19}$$

Note that $f_a(d_{ij}), f_l(d_{il})$ are sometimes called, respectively, the virtual force applied on agent i by agent j and the virtual leader.

In the third term of (16), we must add $j \in \mathcal{S}_i(t)$ since, taking the sensing capability of the agents into consideration, it is possible that some agent in the set $\mathcal{N}_i(t_k)$ moves out of the sensing range of agent i at some $t \in [t_k, t_{k+1})$. However, $j \in \mathcal{S}_i(t)$ is not needed for the first term due to the property of the function $f_a(\cdot)$ that $f_a(d_{ij}) = 0$ for $d_{ij} \geq r_{nb}$.

From (16), it can be seen that if for all $t \in [t_k, t_{k+1})$, $j \in \mathcal{S}_i(t)$ for any $j \in \mathcal{N}_i(t_k)$, and $j \notin \mathcal{I}_i(t)$ for any $j \notin \mathcal{N}_i(t_k)$, the control law in (16) can be put into the form: $\forall\, i \in \mathcal{V}$, $\forall\, t \in [t_k, t_{k+1})$,

$$\begin{aligned} u_i^{df}(t) = &-\sum_{j \neq i} \nabla_{x_i} \psi_a(d_{ij}) - g(b_i) \nabla_{x_i} \psi_l(d_{il}) - \\ &\sum_{j \in \mathcal{N}_i(t_k)} a_{ij}^*(v_i - v_j) - b_i(v_i - v_l) + u_l, \end{aligned} \tag{20}$$

or compactly,

$$\begin{aligned} u^{df} = &-\nabla_x V_a - \nabla_x V_l - (L_a(t_k) \otimes I_n)(v - \mathbf{1}_N \otimes v_l) \\ &+ \mathbf{1}_N \otimes u_l, \end{aligned} \tag{21}$$

where $u^{df} = [(u_1^{df})^\top, \cdots, (u_N^{df})^\top]^\top$, $x = [x_1^\top, \cdots, x_N^\top]^\top$, $v = [v_1^\top, \cdots, v_N^\top]^\top$, and $V_a(x), V_l(x, x_l)$ are the collective inter-agent and leader-agent potentials for the group which are defined as

$$V_a(x) = \frac{1}{2} \sum_{i=1}^{N} \sum_{j \neq i} \psi_a(d_{ij}), \quad V_l(x, x_l) = \sum_{i \in \mathcal{W}} \psi_l(d_{il}). \tag{22}$$

To make our analysis mathematically rigorous, we should first define the solutions of the differential equations (10), (16), which will be discussed in the rest of this section.

Definition 2. A solution of differential equations (10), (16) with initial value $(x_0^i, v_0^i), i \in \mathcal{V}$, is a continuous function $(x_i(t), v_i(t)) : [t_0, +\infty) \to \mathbb{R}^{2n}$ that satisfies

$$(x(t_0), v(t_0)) = (x_0, v_0), \tag{23}$$

$$\dot{x}(t) = v(t), \quad \dot{v}(t) = u^{df}(t), \quad \forall\, t \in [t_k, t_{k+1}),\ k \in \mathbb{Z}^+, \tag{24}$$

where $x_0 = [x_0^1, x_0^2, \cdots, x_0^N]^\top$, $v_0 = [v_0^1, v_0^2, \cdots, v_0^N]^\top$.

In the following, by choosing the dwell time τ_d to be small enough, we show that the closed-loop system (10) and (16) has a unique solution over $[t_0, +\infty)$ in the sense of Definition 2.

Define functions $H(v, v_l) : \mathbb{R}^{(N+1)n} \to \mathbb{R}^+$, $V(x, x_l) : \mathbb{R}^{(N+1)n} \to \mathbb{R}^+$, $J(x, x_l, v, v_l) : \mathbb{R}^{2(N+1)n} \to \mathbb{R}^+$ are defined as

$$V(x, x_l) = V_a(x) + V_l(x, x_l), \tag{25}$$

$$H(v, v_l) = \frac{1}{2}\|v - \mathbf{1}_N \otimes v_l\|^2, \tag{26}$$

$$J(x, x_l, v, v_l) = V(x, x_l) + H(v, v_l), \tag{27}$$

where V_a, V_l are defined in (22).

Theorem 1. Suppose the differential equation (11) has a unique solution with any given initial condition $(x_l(t_0), v_l(t_0)) = (x_0^l, v_0^l)$ over $[t_0, +\infty)$. If the dwell time τ_d satisfies $\tau_d < \min\{r_s - r_{nb}, r_{nb} - r_a\}/(2\sqrt{2J(t_0)})$, then, $\forall\, i \in \mathcal{V}$, differential equations (10) and (16), with $(x_i(t_0), v_i(t_0)) = (x_0^i, v_0^i)$, $x_0^i \neq x_0^j, \forall\, i, j \in \mathcal{V}$, have a unique solution over $[t_0, +\infty)$.

Before presenting the main results in this section, we make a connectivity assumption of the group, which says that any non-AA agent has a direct or indirect link with some AA at all times.

Assumption 2. For all $t \geq t_0$, there is a path connecting any agent in $\mathcal{V} \backslash \mathcal{W}$ to some agent in $\mathcal{W}$.

In Hong et al. (2006), it has been proved that under Assumption 2, the symmetric matrix $L_a(t)$, defined in (6), is positive definite for any $t \geq t_0$. Since the group can only has finite interconnection topologies, we can define a positive constant

$$\lambda_m = \min\{\lambda_{min}(L_a(t)), \forall\, t \geq t_0 : \text{Assumption 2 holds } \forall\, t \geq t_0\}. \tag{28}$$

Lemma 2 below is a slight extension of celebrated Barbalat Lemma, which is useful in the proofs of some following results.

Definition 3. Function $f(\cdot) : \mathbb{R} \to \mathbb{R}$ is said to be piecewise uniformly continuous over $[t_0, +\infty)$ w.r.t. an infinite sequence $\{\hat{t}_i\}_{i=0}^{\infty}$, with $\hat{t}_0 = t_0$ and $\inf \hat{t}_i - \hat{t}_{i-1} \geq \hat{\tau} > 0$, if $\forall\, t \in [\hat{t}_{i-1}, \hat{t}_i), i \in \mathbb{N}$, $\forall\, \epsilon > 0$, $\exists\, \hat{\delta}_\epsilon > 0$, $|f(\tilde{t}) - f(t)| < \epsilon$, $\forall\, \tilde{t} \in B_{\hat{\delta}_\epsilon}(t) \bigcap [\hat{t}_{i-1}, \hat{t}_i)$, where $B_{\hat{\delta}_\epsilon}(t)$ is the open ball centered at t with the radius $\hat{\delta}_\epsilon$.

Lemma 2. Let $f(\cdot) : \mathbb{R} \to \mathbb{R}$ be piecewise uniformly continuous over $[t_0, +\infty)$ w.r.t. $\{\hat{t}_i\}_{i=0}^{\infty}$, and $h(\cdot) : \mathbb{R} \to \mathbb{R}$ satisfy $\lim_{t\to+\infty} h(t) = 0$. Suppose that $\lim_{t\to+\infty} \int_{t_0}^{t} (f(s) + h(s))ds$ exists and finite. Then $\lim_{t\to+\infty} f(t) = 0$.

Remark 1. If the function f is uniformly continuous over $[t_0, +\infty)$, then the conclusion in Lemma 2 naturally follows.

Theorem 3. If Assumptions 1, 2 hold, the solutions of (10), (16) and (11) satisfy $\lim_{t\to+\infty} \|v_i(t) - v_l(t)\| = 0$, $\forall\, i \in \mathcal{V}$; the inter-agent collision is avoided; and the configuration $(x(t), x_l(t))$ a.e. converges to some local minimum of the potential $V(x, x_l)$.

Sketch of proof: Consider the function J defined in (27). The derivative of J along the solutions of (10), (16) and (11), is

$$\begin{aligned} \dot{J} &= -(v - \mathbf{1}_N \otimes v_l)^\top (L_a(t) \otimes I_N)(v - \mathbf{1}_N \otimes v_l) \\ &\leq -\lambda_m \|(v - \mathbf{1}_N \otimes v_l)\|^2 \end{aligned} \tag{29}$$

where λ_m is defined in (28). On the other hand, it can be shown that $H(t) = \frac{1}{2}\|(v - \mathbf{1}_N \otimes v_l)\|^2$ is uniformly continuous w.r.t. t for all $t \geq t_0$. By Barbalat Lemma Khalil (2002), we conclude from (29) that $\forall\, i \in \mathcal{V}, \|v_i(t) - v_l(t)\| \to 0$ as $t \to +\infty$. The avoidance of inter-agent collision can be easily seen from the boundedness of $J(t)$ over $[t_0, +\infty)$ and the property b) of the inter-agent potential ψ_a. Finally, in light of Lemma 2, it follows from (21) and the consensus of v that $\lim_{t\to+\infty} \nabla_x (V_a + V_l) = \mathbf{0}_n$, which leads to the last argument of the theorem $\square$

Now, we give a result on the configuration of the group achieved by controller (16) when there is only one AA in the group.

Proposition 4. If Assumptions 1, 2 hold, and the leader set $\mathcal{W} = \{i\}, i \in \mathcal{V}$ is a singleton, then the solutions of (10), (16) and (11) satisfy $\lim_{t\to+\infty} \|x_i(t) - x_l(t)\| = 0$.

Note that Proposition 4 tells us that if the group has one fixed AA and is connected all the time, the control law (16) can drive the group to track, or migrate to, the virtual leader in the sense that the AA converges asymptotically to the virtual leader.

Now we investigate a robustness property of the proposed control law (16). Consider the control law

$$\tilde{u}^{af} = u^{af} + \delta_u, \tag{30}$$

where u^{af} is as in (16); and $\delta_u(t) : \mathbb{R} \to \mathbb{R}^{Nn}$ denotes the disturbance which, in this paper, is restricted to satisfy $\delta_u(t) \in \mathcal{L}_1^{Nn}[t_0, +\infty) \bigcap \mathcal{L}_2^{Nn}[t_0, +\infty) \bigcap \mathcal{L}_\infty^{Nn}[t_0, +\infty)$.

Remark 2. Here we can define the solutions of the differential equations (10), (30) in a mimic way as in Definition 2. And the existence and uniqueness of the solutions of (10), (30) can be proved by trivially modifying the proof for Theorem 1 with some additional assumptions on the smoothness of $\delta_u(t)$. In this work, we omit these parts and just assume that the differential equations (10), (30) own a unique solution over $[t_0, +\infty)$ with any given initial condition $(x(t_0), v(t_0))$, where $x_i(t_0) \neq x_j(t_0), \forall\, i \neq j$.

Theorem 5. Suppose Assumptions 1, 2 hold. Then the solutions of (10), (30) and (11) satisfy $\lim_{t\to+\infty} \|v_i(t) - v_l(t)\| = 0, \forall\, i \in \mathcal{V}$; and inter-agent collision is avoided. Futhermore, if $\delta_u(t) \to 0$ as $t \to +\infty$, then the configuration (x, x_l) a.e. converges to some local minimum of the potential $V(x, x_l)$.

Remark 3. If we substitute the leader-agent function ψ_l with ψ_{lr}, the counterparts of Theorems 1, 3, 5 and

Proposition 4 can be obtained with simple modifications. Indeed, in Theorem 1, the initial condition is required to further satisfy $x_0^i \neq x_0^l, \forall i \in \mathcal{W}$; in Proposition 4, "$\lim_{t\to+\infty} \|x_i(t) - x_l(t)\| = 0$" should be changed to "$\lim_{t\to+\infty} \|x_i(t) - x_l(t)\| = d_l$"; and in Theorems 3, 5, the collision between the virtual leader and the AA can be avoided, which may be of interest in some scenarios where the virtual leader is substituted by a real one.

4. APPLICATION TO FLOCKING CONTROL OF NONHOLONOMIC ROBOTS

In this section, we apply the control laws discussed above to the flocking control of a group of unicycles. Here, we study the case where each robot can directly obtain its position and orientation, but cannot measure its velocity information. Instead, an observer is used to give the estimate, for each robot, of the velocity information, which can be transmitted between neighboring robots.

Dynamic model of the robot The dynamic model of the unicycle is given as in Do et al. (2004)

$$\begin{aligned} \dot{\eta} &= J(\eta_i) z_i \\ M\dot{z}_i + C(\dot{\eta}_i) z_i + D z_i &= \tau_i, \quad \forall\, i \in \mathcal{V} \end{aligned} \tag{31}$$

with

$$\eta_i = [x_i, y_i, \phi_i], \quad z_i = [z_i^r, z_i^l], \quad \tau_i = [\tau_i^r, \tau_i^l],$$
$$J(\eta_i) = \frac{r}{2}\begin{bmatrix} \cos(\phi_i) & \cos(\phi_i) \\ \sin(\phi_i) & \sin(\phi_i) \\ b^{-1} & b^{-1} \end{bmatrix}, \quad M = \begin{bmatrix} m_{11} & m_{12} \\ m_{12} & m_{11} \end{bmatrix},$$
$$D = \begin{bmatrix} d_{11} & 0 \\ 0 & d_{22} \end{bmatrix}, \quad C(\dot{\eta}_i) = \begin{bmatrix} 0 & c\dot{\phi}_i \\ -c\dot{\phi}_i & 0 \end{bmatrix} \tag{32}$$

where (x, y, ϕ) is the position and orientation of the robot; z_i^r and z_i^l are the angular velocities of, respectively, the right and left wheels; and τ_i^r and τ_i^l are the torques applied to the wheels. The relation between z_i^r, z_i^l and the linear and angular velocities of the robot i, denoted by v_i, ω_i, is

$$[z_i^r, z_i^l]^\top = B[v_i, \omega_i]^\top, \text{ with } B = \frac{1}{r}\begin{bmatrix} 1 & b \\ 1 & -b \end{bmatrix} \tag{33}$$

Observer The observer proposed in Do et al. (2004) is used here to estimate, for each robot, of the velocity information v_i, ω_i (or z_i^r, z_i^l). For each robot in the team, the variables directly estimated by the observer are

$$X_i = Q(\eta_i) z_i,$$
$$Q(\eta_i) = \begin{bmatrix} n_{11}\cos(c\Delta\phi_i) & \Delta\sin(c\Delta\phi_i) - n_{12}\cos(c\Delta\phi_i) \\ n_{11}\sin(c\Delta\phi_i) & -n_{12}\sin(c\Delta\phi_i) - \Delta\cos(c\Delta\phi_i) \end{bmatrix}$$

where

$$n_{11} = m_{11}(m_{11}^2 - m_{12}^2)^{-1}, \quad n_{12} = -m_{12}(m_{11}^2 - m_{12}^2)^{-1},$$
$$\Delta = \sqrt{n_{11}^2 - n_{12}^2}$$

It is straightforward to check that $Q(\eta_i)$ is globally invertible and its elements are bounded.

In the rest of this section, we denote the estimated value by adding "$\wedge$" on the corresponding original variables. The observer dynamics is given by Do et al. (2004)

$$\begin{aligned} \dot{\hat{\eta}}_i &= J(\eta_i) Q^{-1}(\eta_i) \hat{X}_i + K_{1i}(\eta_i - \hat{\eta}_i) \\ \dot{\hat{X}}_i &= -G(\eta_i)\hat{X}_i + Q(\eta_i) M^{-1}\tau_i + K_{2i}(\eta_i - \hat{\eta}_i) \end{aligned} \tag{34}$$

where $G(\eta_i) = Q(\eta_i) M^{-1} D Q^{-1}(\eta_i)$. The feedback gain matrices K_{1i} and K_{2i} are chosen to satisfy

$$K_{1i}^\top P_1 + P_1 K_{1i} = R_1, \quad G(\eta_i)^\top P_2 + P_2 G(\eta_i) = R_2,$$
$$\left(J(\eta_i) Q^{-1}(\eta_i)\right)^\top P_1 - P_2 K_{2i} = 0$$

where R_1, R_2, P_1, P_2 are positive definite matrices.

Using the observer (34), the estimate errors $\tilde{\eta}_i = \eta_i - \hat{\eta}_i, \tilde{X}_i = X_i - \hat{X}_i$ decay exponentially to zero, i.e., there exist positive constants k_i and γ_i such that

$$\|(\tilde{\eta}_i(t), \tilde{X}_i(t))\| \le k_i \|(\tilde{\eta}_i(t_0), \tilde{X}_i(t_0))\| e^{-\gamma_i(t-t_0)}, \ \ \forall\, t \ge t_0$$

Controller To avoid the non-holonomic constraint in the model (31), for robot $i, i \in \mathcal{V}$, consider a control reference point CRP_i, which is off the wheel axis by the distance μ Ren et al. (2007). The position of CRP_i is given by

$$q_i^h = \begin{bmatrix} x_i \\ y_i \end{bmatrix} + \mu \begin{bmatrix} \cos(\phi_i) \\ \sin(\phi_i) \end{bmatrix}, \quad \mu > 0 \tag{35}$$

Now, we consider the flocking control problem of CRP_is. In the rest of this section, by "group", we mean the the group composed of $CRP_i, i \in \mathcal{V}$, and the associated notions should be redefined accordingly.

By (31) and (33), the velocity and acceleration of CRP_i are

$$p_i^h := \dot{q}_i^h = \begin{bmatrix} v_i\cos(\phi_i) - \mu\omega_i\sin(\phi_i) \\ v_i\sin(\phi_i) + \mu\omega_i\cos(\phi_i) \end{bmatrix},$$
$$u_i^h := \dot{p}_i^h = S(\phi_i)[\tau_i - DB\zeta_i - C(\dot{\eta})B\zeta_i] - \xi(v_i, \omega_i, \phi_i)$$

where $\zeta_i = [v_i, \omega_i]^\top$ and

$$S(\phi_i) = \begin{bmatrix} \cos(\phi_i) & -\mu\sin(\phi_i) \\ \sin(\phi_i) & \mu\cos(\phi_i) \end{bmatrix} B^{-1}M^{-1},$$
$$\xi(v_i, \omega_i, \phi_i) = \begin{bmatrix} v_i\omega_i\sin(\phi_i) + \mu\omega_i^2\cos(\phi_i) \\ -v_i\omega_i\cos(\phi_i) + \mu\omega_i^2\sin(\phi_i) \end{bmatrix}$$

The virtual leader we use is a moving point with the dynamics

$$\dot{q}_l = p_l, \quad \dot{p}_l = u_l. \tag{36}$$

Firstly, following the idea in Section 3, we propose the decentralized control law for the group with fixed AA set: $\forall\, i \in \mathcal{V}, \forall\, t \in [t_0, +\infty)$:

$$\tau_i^{af}(t) = S^{-1}(\phi_i)\left(\chi_i^{af} + \xi(\hat{v}_i, \hat{\omega}_i, \phi_i)\right) + (D + C(\hat{\omega}_i))B\hat{\zeta}_i \tag{37}$$

with

$$\hat{\zeta}_i = \begin{bmatrix} \hat{v}_i \\ \hat{\omega}_i \end{bmatrix} = B^{-1}Q^{-1}(\eta_i)\hat{X}_i,$$
$$\chi_i^{af}(t) = \sum_{j\in\mathcal{N}_i(t_k)} f_a(d_{ij}^h) n_{ji}^h + g(b_i) f_l(d_{il}^h) n_{li}^h -$$
$$\sum_{j\in\mathcal{N}_i(t_k)\bigcap\mathcal{S}_i(t)} a_{ij}^*(\hat{p}_i^h - \hat{p}_j^h) - b_i(\hat{p}_i^h - p_l) + u_l,$$
$$\forall\, t \in [t_k, t_{k+1}), k \in \mathbb{Z}^+, \tag{38}$$

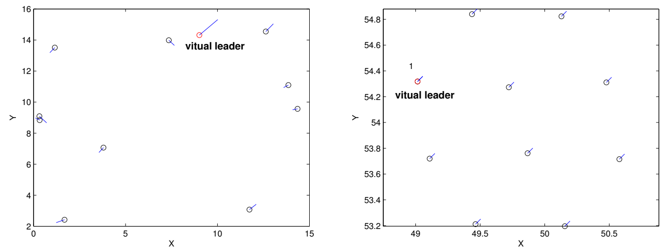
where $\hat{p}_i^h = [\hat{v}_i \cos(\phi_i) - \mu\hat{\omega}_i \sin(\phi_i), \hat{v}_i \sin(\phi_i) + \mu\hat{\omega}_i \cos(\phi_i)]^\top$; and the definitions of $d_{ij}^h, d_{il}^h, n_{ij}^h$ and n_{li}^h mimic those of d_{ij}, d_{il}, n_{ij} and n_{li} in Section 3 by substituting $x_i, i \in \mathcal{V}$ with q_i^h.

Theorem 6. Under Assumptions 1, 2, the solutions of the system (31), (34), (37) and (36) satisfies that $\lim_{t\to+\infty} \|p_i^h(t) - p_l(t)\| = 0, \forall\, i \in \mathcal{V}$; and the configuration $(q_1^h, \cdots, q_N^h, q_l)$ a.e. converges to the local minimum of the potential $V = V_a^h + V_l^h$, where V_a^h and V_a^l are defined as

$$V_a^h = \frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}\psi_a(d_{ij}^h), \quad V_l^h = \sum_{i\in\mathcal{W}}\psi_l(d_{il}^h).$$

5. SIMULATIONS

First, the flocking of 10 mass point agents by the controller (16) is shown in Figure 1, where agent 1 is the only AA of the group. The inter-agent potential is the one defined in (8) with $d_a = 1$ and, to ensure the Assumption 2 can hold, $r_a = 30$. The leader-agent potential is chosen as $\psi_l(x) = 10\left(\frac{1}{2}x^2 + 1\right)$. In addition, we let $a_{ij}^* = b_i^* = 10$, $\forall\, i, j \in \mathcal{V}$. The initial positions of the group are randomly chosen in the square $[0, 20]m \times [0, 20]m$; and the velocities are randomly chosen in $[-0.5, 0.5]m/s \times [-0.5, 0.5]m/s$. The position for the virtual leader is also randomly chosen in $[0, 20]m \times [0, 20]m$, but its velocity is fixed to $[1, 1]^\top m/s$.



(a) $t = t_0$ (b) $t = t_0 + 40s$

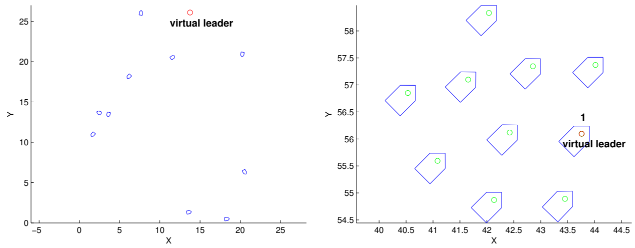
Fig. 1. Flocking of mass point group, $N = 10$.

Next, we simulate the flocking control for a group of unicycles. The model parameters of the robots are: $m_{11} = m_{22} = 1.2356$, $c = 0.2250$, $b = 0.2$ and $d_{11} = d_{22} = 10$. And the offset of the control reference point $\mu = 0.2$.

The observer-controller (34)-(37) is applied to a group of 10 unicycles. Also, agent 1 is the only AA in the group. The inter-agent potential is also in the form of (8) but with $d_a = 2, r_a = 100$. The leader-agent potential is chosen the same as for the mass point case. The initial positions and headings of the group are randomly chosen in the square $[0, 30]m \times [0, 30]m$ and $[0, 2\pi]rad$ respectively. And the linear and angular velocity are randomly chosen, respectively, in the intervals $[-1, 1]m/s$ and $[-0.5, 0.5]rad/s$. The position of the virtual leader is also selected randomly in $[0, 30]m \times [0, 30]m$, while its velocity is fixed to $[0.3, 0.3]m/s$.

6. CONCLUSION

In this paper, we discuss the migration and trajectory tracking of a group of agents by the use of artificial potential based approach. The leader-agent potential is responsible for attracting the active agents to the virtual leader, while the inter-agent potential takes effect to generate the attraction and repulsion between neighboring agents. The velocity consensus of the group is due to the involvement of the linear velocity feedback term in the controller. In the framework of potential field, the key issue of the gradient-based controller is that it is hard to specify the steady state configuration of the group, even if the pairwise potential between any pair of neighboring agents has unique local minimum at some desired relative distance. Another important problem which needs attention in the future work is that the designed controller should be able to ensure Assumption 2 being satisfied by the group.



(a) $t = t_0$ (b) $t = t_0 + 100s$

Fig. 2. Flocking of unicycle group, $N = 10$.

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