

# KARNOPP FRICTION MODEL IDENTIFICATION FOR A REAL CONTROL VALVE

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**Abstract:** This paper presents an algorithm to estimate friction parameters of a real control valve. Data are collected from a valve installed in a bench, submitted to different input signals and subject to different friction forces. Two different parameter sets are obtained, based on distinct methods. These parameter sets are applied in the Karnopp friction model, generating two versions of the same model. These models are validated with different input signals and distinct friction forces. The validation tests have revealed that both models described quite well the behavior of the control valve.

## 1. INTRODUCTION

One of the main factors that affects the behavior of the control loop is friction in control valves, which are the most used final control element in industry.

It is necessary to take into account that the valve behavior will significantly change as friction increases. When a valve is affected by friction, it causes oscillations or steady-state errors in the stem position, since the valve does not respond instantaneously to the control signal. Such undesirable occurrences affect the overall profitability of the process.

The purpose of this paper is to identify the friction parameters of a control valve through bench tests, using a friction model structure proposed by Karnopp (1985). This model can be used to diagnose excessive friction, to simulate the behavior of control loops affected by friction in process valves or even to develop model based control algorithms.

There is a book (Fitzgerald, 1995) where some friction coefficients are cited, which Kayihan and Doyle (2000) used in their paper, but there is no information how they were evaluated. As far as the authors know, this is the first work that presents tests in order to identify the Karnopp friction model parameters for a real process control valve.

The paper is organized as follows: section 2 presents the Karnopp friction model applied to a pneumatic single action sliding stem valve. In section 3 the method applied to estimate the valve model parameters is briefly described. Section 4 presents the parameter values obtained from the tests performed in the valve. Section 5 shows the tests applied to validate the control valve models. Finally, in section 6, the conclusions are drawn.

#### 2. THE KARNOPP FRICTION MODEL

A dynamic model for the control valve is given by the force balance equation:

$$m \cdot \ddot{x}(t) = F_{ext}(t) - F_{spring}(t) + F_{friction}(t)$$
 (1)

where: m is the mass of the valve moving parts, x(t) is the stem position,  $F_{ext} = S_a \cdot P(t)$  is the external force applied by the actuator,  $S_a$  is the diaphragm actuator area and P(t) is the air pressure;  $F_{spring} = k \cdot x(t)$  is the spring force, k is the spring constant, and  $F_{friction}$ : is the friction force.

The Karnopp friction model includes static and moving friction, depending on the velocity of the moving parts. The expression for the moving friction is given by the first line of (2); in this case, the friction force is a static function of the stem velocity.

$$F_{friction} = \begin{cases} -F_c \cdot \operatorname{sgn}(\dot{x}) - F_v \cdot \dot{x}, & |\dot{x}| \ge DV \\ -(F_{ext} - k \cdot x), & \left\{ \begin{aligned} & \text{if } |\dot{x}| < DV \text{ and} \\ & |F_{ext} - k \cdot x| \le F_S \end{aligned} \right. \\ -F_S \cdot \operatorname{sgn}(F_{ext} - k \cdot x), & \left\{ \begin{aligned} & \text{if } |\dot{x}| < DV \text{ and} \\ & |F_{ext} - k \cdot x| > F_S \end{aligned} \right. \end{aligned}$$
 (2)

where:

 $F_c$ : Coulomb friction coefficient;

 $F_v$ : viscous friction coefficient;

 $F_s$ : static friction coefficient;

DV: limit velocity.

and the  $sgn(\cdot)$  function is defined as:

$$\operatorname{sgn}(\alpha) = \begin{cases} 1, & \alpha > 0 \\ 0, & \alpha = 0 \\ -1, & \alpha < 0 \end{cases}$$

If the magnitude of the stem velocity is smaller than the limit velocity DV, the model considers the stem velocity to be null. The second line of (2) is the case when the valve is stuck and the third one represents the situation at the instant of breakaway. In both situations, the friction force  $F_{friction}$  is a saturated version of the external force.

## 3. PARAMETERS ESTIMATION METHOD

In a previous work (Romano and Garcia, 2007), the estimation method proposed by Ravanbod-Shirazi and Besançon-Voda (2003), where Karnopp friction model parameters were estimated for an electro-pneumatic actuator, was extended for single action control valves. In addition, in this extension, the estimation of  $F_s$  is performed based on force balance and not by means of non-linear optimization, which demands excessive computational load. This method is briefly described in the sequel.

In the first step, a parameter vector is defined as:

$$\theta = \begin{bmatrix} m & F_{v} & F_{c} & k \end{bmatrix} \tag{3}$$

When stem velocity is greater than DV, the friction force is given by the first line of (2) and the force balance (1) becomes:

$$F_{\text{ext}}(t) = m \cdot \ddot{x}(t) + F_c \cdot \text{sgn} \left[ \dot{x}(t) \right] + F_{\nu} \cdot \dot{x}(t) + k \cdot x(t)$$
 (4)

Note that (4) is linear with respect to the parameter vector  $\theta$ . The regression vector  $\varphi(t)$  is defined as follows:

$$\varphi(t) = \begin{bmatrix} \ddot{x}(t) & \operatorname{sgn}[\dot{x}(t)] & \dot{x}(t) & x(t) \end{bmatrix}$$
 (5)

The parameter vector can be estimated through the minimization of the following quadratic criterion:

$$\hat{\theta} = \underset{\theta}{\arg\min} \sum_{t} \left[ F_{ext}(t) - \hat{F}_{ext}(t) \right]^{2}$$

$$= \underset{\theta}{\arg\min} \sum_{t} \left[ F_{ext}(t) - \varphi(t) \cdot \theta^{T} \right]^{2}$$
(6)

The solution is given by:

$$\hat{\theta} = \left[\sum_{t} \varphi(t) \cdot \varphi^{T}(t)\right]^{-1} \sum_{t} \varphi(t) \cdot F_{ext}(t)$$
(7)

But the periods in which (4) is applicable are unknown, i.e. the limit velocity DV that characterizes the stem movement is not known a priori. To deal with this problem, a variable  $\delta v(s)$  is defined, so that:

$$\delta v(s) = s \cdot \frac{|\dot{x}|_{\text{max}}}{Z}, \quad s = 1, 2, \dots, S$$
 (8)

where: Z >> 1 and S < Z.

For each value of  $\delta v(s)$ , the regression vector  $\varphi(t)$  and  $F_{ext}(t)$  are chosen from the observed data, so that the condition  $|\dot{x}| > \delta v(s)$  is fulfilled and then the parameter vector is estimated solving (7). The behavior of these estimations, as index s increases is: [1] For  $\delta v(s) << DV$  the values of the estimated parameters vary significantly for different values of  $\delta v(s)$ , because data from periods in which the force balance (4) is not applicable are used in the estimations. [2] When  $\delta v(s)$  approaches DV, the estimations come closer to their true value and do not change significantly, even for  $\delta v(s)$  slightly greater than the limit velocity DV. This behavior was expected, because in this situation the regression model  $\varphi(t) \cdot \hat{\theta}^T$  approximates  $F_{ext}(t)$ .

The third line of the friction model (2), which corresponds to a situation where the stem is in imminence of moving, is used to estimate the static friction coefficient, i.e. since  $F_{ext}(t)$  and x(t) are measured, and the spring constant has already been estimated in (7),  $F_s$  can be obtained solving (9):

$$F_{\text{ext}}(t) - \hat{k} \cdot x(t) = F_{\text{S}} \cdot \text{sgn} \left[ F_{\text{ext}} - \hat{k} \cdot x(t) \right]$$
(9)

The procedure for estimating DV is described in Romano and Garcia (2007) and is omitted here. Additionally, the effect of this parameter in the model simulation is negligible, as stated in Karnopp (1985), hence an arbitrary value of 1% of  $|\dot{x}|_{max}$  was assigned to the limit velocity for validation purposes.

#### 4. IDENTIFICATION RESULTS

The estimation method described in the previous section was applied to estimate Karnopp friction model parameters of a single action globe valve (model *ET*, *Fisher Inc.*), with  $2.8575 \times 10^{-2}$  m travel, Teflon stem packing,  $0.0445 \text{ m}^2$  diaphragm surface area, mass of the moving parts of 1.6 kg and a nominal spring constant k of 215530 N/m ( $\pm 5\%$ ).

The valve packing was composed of Teflon gaskets, which can be tightened, in order to change the present friction forces. The tests were performed in two extreme situations: one with the gaskets lightly tightened (low friction) and the other one with them strongly tightened (high friction).

The actuator pressure and the stem position were measured using a 1 ms sampling period. Despite the fact that this high frequency oversamples the system dynamic, it was necessary to improve the numerical derivation accuracy. The derivations algorithm used to estimate  $\dot{x}(t)$  and  $\ddot{x}(t)$  is described in Appendix A.

To obtain the identification data, a voltage-pressure V/P converter was excited with a 10.4 s period square wave, switching from 15% to 75% of the input pressure range (41.74 to 206.84 kPa). The experimental data and the estimation of the stem position are shown in Fig. 1. Note that the input pressure is quite different from a square wave due to the V/P converter dynamic. Another experiment using a triangular wave was also performed in order to generate the valve signature (Kayihan and Doyle, 2000), which is shown in Fig. 2.

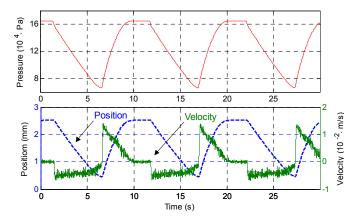


Fig. 1. Experimental input-output data used to estimate the friction model parameters.

It is possible to see in Fig. 2 that the dead zone of the valve with high friction is at least three times the one with low friction. It is also noted that the higher friction limits the maximum stroke of the valve stem. Furthermore, there is no slip jump (Choudhury *et al.*, 2005) in any of the situations, then  $F_s$  is supposed to be equal to  $F_c$ .

Before applying the identification method to the valve model parameter estimation, it is necessary to solve a practical issue: in the model (1), the spring force is supposed to be null when x(t) = 0, but for this assumption to be true, the measured stem position,  $x_{meas}(t)$  must be biased, i.e.:

$$x(t) = x_{meas}(t) + \overline{x} \tag{10}$$

where:  $\overline{x}$  is the bias.

The procedure to evaluate this bias is based on force balance equations in breakaway instants (denoted in Fig. 2 as  $P_{IL}$  and  $P_{2L}$  in the low friction situation and  $P_{IH}$  and  $P_{2H}$  in the high one) and is described as follows:

The force balance in  $P_1$  ( $P_{1L}$  or  $P_{1H}$ ) and in  $P_2$  ( $P_{2L}$  or  $P_{2H}$ ) are respectively given by (11) and (12):

$$F_{ext,1} = F_S + k \cdot x_1 \tag{11}$$

$$k \cdot x_2 = F_{ext,2} + F_S \tag{12}$$

Adding (11) with (12) gives:

$$F_{ext,1} + F_{ext,2} = k \cdot (x_1 + x_2) \tag{13}$$

Substituting (10) in (13) and isolating  $\bar{x}$  results:

$$\overline{x} = \frac{1}{2} \left[ \frac{1}{k} \left( F_{ext,1} + F_{ext,2} \right) - \left( x_{meas,1} + x_{meas,2} \right) \right]$$
 (14)

Notice that the spring constant k is unknown at this moment. To overcome this problem, we use the valve signature that was generated with a triangular excitation. In this case, when the valve is sliding, the stem velocity is constant and the acceleration is null, hence from (1) and (2), it follows that:

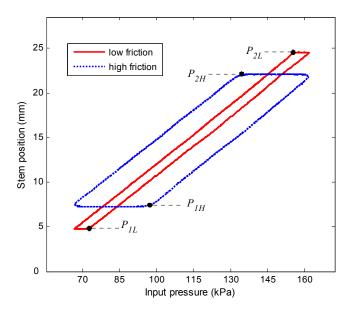


Fig. 2. Valve signature of two extreme situations: low and high friction.

$$k = \frac{dF_{\text{ext}}}{dx} \tag{15}$$

Evaluating the mean of (15) on instants where the valve stem is sliding and substituting k in (14) results:

$$k_{\text{low}} = 2.0293 \times 10^5 \,\text{N/m} \Rightarrow \overline{x} = 1.12 \,\text{mm (low friction)}$$
  
 $k_{\text{high}} = 2.0391 \times 10^5 \,\text{N/m} \Rightarrow \overline{x} = 1.14 \,\text{mm (high friction)}$  (16)

Thus, a bias of 1.13 mm (the mean value between the low and high friction estimation data) is considered in x(t) for estimation purposes.

The values of S and Z, used in the procedure, defined in (8) to characterize  $\delta v(s)$ , are 330 and 350, respectively. The behavior of the estimated parameter vector for different values of  $\delta v(s)$ , when the identification algorithm is applied to the valve on the low friction experiment, is shown in Fig. 3.

As mentioned earlier, the estimation vector comes closer to its true value when it does not change significantly. But the region where the estimates approach the true values is not obvious from Fig. 3. Firstly, for small values of the auxiliary variable  $\delta v(s)$ , the force balance (4) is not applicable. On the other hand, as higher values of the index s are considered, fewer points are used in the parameter estimation, due to the condition  $|\dot{x}| > \delta v(s)$ . As can be seen in Fig. 1, few points are available so that  $|\dot{x}| > 0.01$  m/s. This explains the high variance on the parameters estimation for  $\delta v(s) > 0.012$  m/s.

In order to find the interval for which the model parameters best describe the valve behavior, the standard deviation ( $\sigma_s$ )

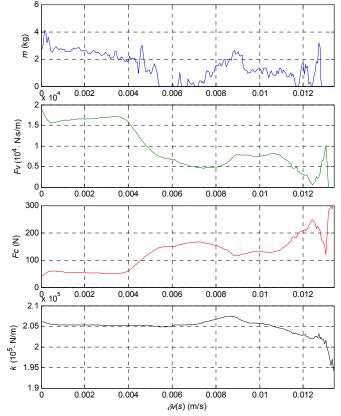


Fig. 3. Behavior of the parameter vector estimates defined in (3), for the experiment with low friction.

of estimations is defined as:

$$\sigma_{S} = \sqrt{\frac{\sum_{t=1}^{n_{S}} \left[ F_{ext} \left( t \right) - \hat{F}_{ext}^{(S)} \left( t \right) \right]^{2}}{n_{S} - n_{P}}}$$

$$(17)$$

where  $n_s$  is the amount of data points in  $F_{ext}(t)$ ,  $\hat{F}_{ext}^{(s)}$  is the estimated external force for each value of  $\delta v(s)$  and  $n_P$  is the number of parameters to be estimated ( $n_P = 4$ , in this case). The standard deviation,  $\sigma_s$ , is proportional to the variability of the estimated external force, thus the parameters are given by the region where  $\sigma_s$  is minimal.

Another statistical tool that can be used to select the correct parameters is the correlation coefficient,  $R_S^2$ , that is given by (Ravanbod-Shirazi and Besançon-Voda, 2003):

$$R_{S}^{2} = \frac{\sum_{t=1}^{n_{S}} \left[ \hat{F}_{ext}^{(S)}(t) - \overline{F}_{ext} \right]^{2}}{\sum_{t=1}^{n_{S}} \left[ F_{ext}(t) - \overline{F}_{ext} \right]^{2}}$$
(18)

where  $\overline{F}_{ext}$  is the time average of  $F_{ext}(t)$ . As the estimated external force approaches the actual one, the correlation coefficient,  $R_S^2$ , becomes closer to unity.

From the standard deviation and correlation coefficient (Fig. 4) plots, a natural choice of the parameters is made for the region around  $\delta v(s) \approx 0.01$  m/s, where the variability of the estimated force is minimal and the correlation between the measured and the estimated force is maximal. The same procedure was applied to the data generated with the valve gaskets strongly tightened, i.e., for a high friction situation and the parameters obtained are shown in "model 1" labeled columns of Table 1.

Except for the mass m, for which the nominal value was considered, the other model parameters (k,  $F_c$ ,  $F_s$  and  $F_v$ ) were alternatively estimated using force balance equations, as in Garcia (2007). This alternative was named "model 2" and it was used to compare with "model 1" results. The estimation

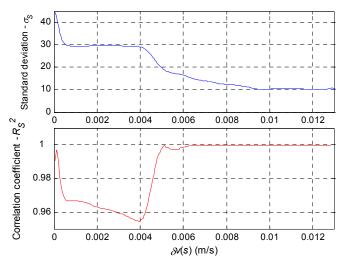


Fig. 4. Standard deviation and correlation coefficient between the estimated external force and the actual one for the low friction experiment.

Table 1. Estimated valve models parameters for low and high friction experiments.

Parameter	Low friction		High friction	
	model 1	model 2	model 1	model 2
m (kg)	1.542	1.6	1.75	1.6
k (N/m)	$2.05 \times 10^{5}$	$2.03 \times 10^{5}$	$2.05 \times 10^{5}$	$2.04 \times 10^{5}$
$F_v (N\cdot s/m)$	4865	1.15×10 <sup>4</sup>	$4.48 \times 10^4$	$2.15 \times 10^4$
$F_c(N)$	152.7	157.5	606	691.27
$F_s(N)$	152.7	157.5	606	691.27

of "model 2" parameters is discussed next.

The spring constants of the "model 2" are the ones calculated by (16). In addition, as the valve signature plot (Fig. 2.) did not exhibit the slip jump phenomenon ( $F_c \approx F_s$ ), another way to estimate the Coulomb friction coefficient is by means of the force balance equation (11) or (12).

The dataset generated with triangular excitation can also be used to provide an alternative estimate of the viscous friction coefficient. As  $\ddot{x}(t) = 0$  during slip periods, the force balance (1) can be rewritten as:

$$F_{\nu} = \frac{1}{\dot{x}(t)} \left\{ F_{ext}(t) - \operatorname{sgn}[\dot{x}(t)] \cdot F_{c} - k \cdot x(t) \right\}$$
 (19)

Then,  $F_v$  is given by the mean of (19) for each instant where the valve is sliding (Table 1).

As stated before in this section, the nominal spring constant k is 215530 N/m ( $\pm$ 5%). Therefore, the values presented in Table 1 for this parameter are very close to the expected one.

Another aspect that should be highlighted is that the Coulomb friction coefficient estimation of both models in the low friction experiment was similar. On the other hand, in the high friction situation they were somewhat different. Moreover, the results summarized in Table 1, showed that the estimations of  $F_{\nu}$  were quite different for any of the friction situations, what probably happens because the effect of the term  $F_{\nu} \cdot \dot{x}(t)$  is insignificant in the valve behavior. The influence of each term in the force balance (4), for the high friction situation, when a sinusoidal input is used, is presented in Fig. 5. Note that the predominant term is  $k \cdot x(t)$ , followed by  $F_{c\cdot}$ , whereas the other two terms are negligible. In the low

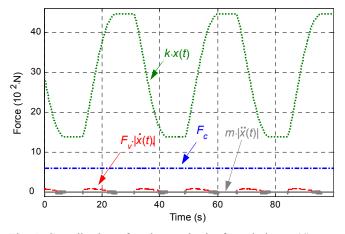


Fig. 5. Contribution of each term in the force balance (4).

friction case, the spring component is even more prominent.

The validation procedure for such models, for the low and high friction experiment, is shown in the following section.

## 5. VALVE MODEL VALIDATION

Firstly, in order to validate the friction models, their response for a sinusoidal excitation with 35.5 s period and varying from 63.95 to 160.44 kPa (13.65% to 72%) were analyzed. This signal was chosen because in slip periods, the stem velocity and acceleration are not null, consequently, all of the model coefficients should affect the valve force balance.

The comparison of the actual stem position with the simulated ones by both models of the low friction valve is depicted in Fig. 6, which showed a good resemblance between the real and the simulated stem positions. Although the simulated

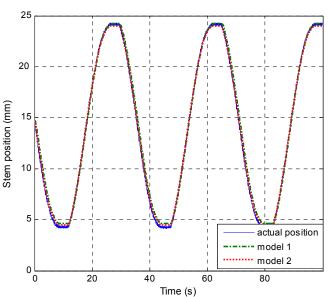


Fig. 6. Model validation of the low friction experiment for a sinusoidal excitation.

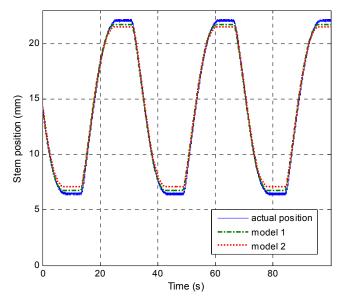


Fig. 7. Model validation of the high friction experiment for a sinusoidal excitation.

lations of the high friction models were close to the valve behavior, such results were slightly worse than the ones obtained when the friction level was lower. In addition, as illustrated in Fig. 7, it is important to note that "model 2" presented greater deviations from the real valve response.

Another test that can be used to verify if the models are able to reproduce the dead zone or to evaluate the valve dynamic response time is proposed in ISA (2000). This test is based on steps of 0.1, 0.2, 0.5, 1, 2, 5, 10, 20 and 50% in the input signal, from the actuator pressure in 50% of the input range and the stem position in steady-state. In Fig. 8 it is presented the results of the test performed with such excitation for the low friction condition.

As for the sinusoidal excitation, the "model 1" for the low friction experiment presented a simulated stem position that matches exactly the actual one. However, the force balance

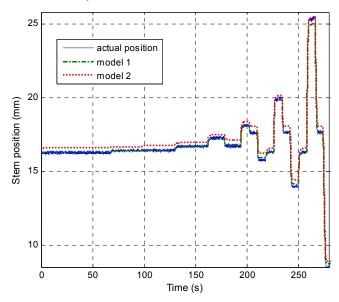


Fig. 8. Analysis of the low friction model responses for an increasing amplitude step sequence excitation. (ISA, 2000).

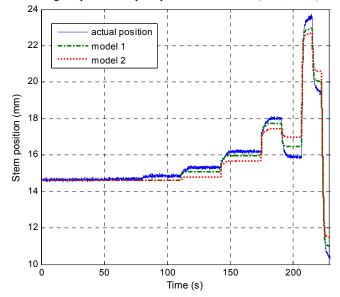


Fig. 9. Model validation of the high friction experiment for an increasing amplitude step sequence excitation.

based model exhibited a poorer performance, especially for describing the static behavior of the valve. Probably it happens because the first model approximated better  $F_c$  and k, as these parameters determine the steady state behavior.

Finally, the high friction valve identified models are validated for increasing amplitude step sequence excitation. As depicted in Fig. 9, both models outputs are able to describe the real valve stem position. Nevertheless, as well as noted when the models were simulated with a sinusoidal input, "model 1" resulted in closer results than the other one. This denotes that the Coulomb friction coefficient was better estimated by (7) than by the force balance at the points  $P_{IH}$  or  $P_{2H}$  in Fig. 2. In addition, as the actual stem position varies between a wider range, a lower value of  $F_c$  would increase the model fit, i.e., the real valve is submitted to less friction than the suggested by the models.

In a general way, the model that was estimated through force balance equations presented higher deviations from the real valve behavior. This could be explained by the fact that in such method, the parameters are obtained graphically, which is less accurate than a regression based method like the one used to identify "model 1".

# 6. CONCLUSIONS

The validation tests, performed with different input signals from the ones used during the estimation procedure, showed that both identified models represented quite well the static and dynamic behavior of the control valve, for high and low friction conditions. However, it is necessary to stress that the moving parts mass estimations suffered a strong influence of the measurement noise, considering that the acceleration, used to estimate m, was calculated through a discrete derivation algorithm, which tends to amplify such noise. The authors have tested some numeric derivation algorithms (backward difference approximation, wavelets and spline based methods, among others), nevertheless the results obtained showed small differences. Based on this problem, the authors are now starting to test an accelerometer installed in the valve stem, in order not to estimate but to directly measure the acceleration.

In a similar way, the estimated viscous friction coefficient was quite different in both models. But this divergence is not significant before the influence of the Coulomb friction coefficient or the spring constant, since the stem velocity is very small. For instance, when one is worried about friction compensation, it is necessary to focus in the model parameters that actually affect the valve behavior, that is, the Coulomb friction coefficient and the spring constant.

As a continuation of this work, the authors intend to remove the Teflon gaskets and to substitute them by carbon ones, with the purpose of evaluating the identification algorithm when the slip jump phenomenon is present.

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## Appendix A. NUMERIC DERIVATION ALGORITHM

The construction of regression vector  $\varphi(t)$ , defined in (5), requires the computation of the velocity and the acceleration of the valve stem. The derivations of x(t) are computed approximating each data window of length d (the length must be odd) among x(t) samples by a polynomial of degree n (for second derivative estimation,  $n \ge 2$ ):

$$h(t) = a_0 + a_1 t + \dots + a_{n-1} t^{n-1} + a_n t^n$$
(20)

then the derivations on instant  $t_i$  are given by:

$$\left. \hat{\dot{x}}(t_i) = \frac{dh_i(t)}{dt} \right|_{t=ti} = a_1 + \dots + (n-1) \cdot a_{n-1} t_i^{n-2} + n \cdot a_n t_i^{n-1}$$

$$\begin{aligned} \hat{x}(t_i) &= \frac{dh_i(t)}{dt} \bigg|_{t=ti} = a_1 + \dots + (n-1) \cdot a_{n-1} t_i^{n-2} + n \cdot a_n t_i^{n-1} \\ \hat{x}(t_i) &= \frac{d^2 h_i(t)}{dt^2} \bigg|_{t=ti} = a_2 + \dots + (n-2) \cdot (n-1) \cdot a_{n-1} t_i^{n-3} \\ &+ (n-1) \cdot n \cdot a_n t_i^{n-2} \end{aligned}$$

where  $h_i(t)$  is the polynomial of order n that best fitted the data window  $[x(t_i - (d-1)/2), x(t_i + (d-1)/2)]$ . This derivation algorithm was applied in this work with windows of length d = 31 that were fitted by polynomials of order n = 3.