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Boundary Predictive Control of Second-Order Linear Modulus-Constant Distributed Parameter Systems Based on Wavelets Transformation

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Abstract: Control system design of distributed parameter system is the difficulty of control theory. A new idea to control of distributed parameter system is to introduce the boundary control idea into predictive control of distributed parameter system based on wavelets transformation. Discrete-time boundary predictive control algorithm of second-order linear modulus-constant distributed parameter system based on orthogonal wavelets transformation is proposed in this paper. Second-order linear modulus-constant distributed parameter system in boundary control is approximated in Haar wavelets transformation. So the predictive control proposition of distributed parameter system has been transformed into the predictive control issue of lumped parameter system. The boundary predictive controller is designed for the input returning to the boundary predictive control rule of the original system. Simulation studies of the proposed algorithm, as well as the system robustness under uncertainty such as the parameters perturbations, and a disturb occurring to the system output are showed. The results have verified the control effectiveness of the proposed algorithm.

1. INTRODUCTION

Complexity of distributed parameter systems (DPS) that exist uncertainties and variety of disturbances cause the difficulty to get the analytic solution of the systems control. Model predictive control is an effective control algorithm for dealing with constrained control problems in process industries and is considered to be one of the most promising methods in control engineering. One type of orthogonal function base is built up and implemented to transform the distributed parameter systems into lumped parameter systems in the modeling of distributed parameter systems by XingSheng Gu. Integral operation matrix of orthogonal Haar wavelets is introduced in 1996 by JinSheng Gu and Weisun Jiang. Furthermore, product operation matrix of Haar wavelets is used in optimal approximating control of linear distributed parameter systems in 2001 by GuiGe Gao and XingSheng Gu. Output regulation for linear distributed parameter systems is discussed in 2000 by Christopher B.I.. Linear periodic robust output regulation by error feedback is briefly addressed by Chen Zhang and Andrea Serrani in 2005.

Model predictive control approaches are frequently adopted in predictive controller. Translating the model predictive control idea into the control of distributed parameter systems will make the online output predictive control feasible. Generally there are boundary constraints to distributed parameter systems. Explicating the boundary constraints into boundary control will deduce a new control strategy. A new discrete-time boundary predictive control algorithm of second-order linear modulus-constant distributed parameter systems based on Haar orthogonal wavelets transformation is proposed in this paper.

We have introduced the general idea of the proposed algorithm in this section. Operation matrixes of Haar wavelets are introduced in section 2. The discrete-time boundary predictive control algorithm of second-order linear modulus-constant distributed parameter systems based on Haar orthogonal wavelets transformation is deduced in detail in section 3. Second-order linear modulus-constant distributed parameter system is transformed into lumped parameter system by Haar wavelets transformation in section 4. Simulation studies of the proposed algorithm, as well as the system robustness under parameters perturbations, and disturb occurring to the system output are shown in section 5. The contribution of this paper is that the proposed algorithm is a new predictive control technique based on Haar wavelets transformation, boundary control idea is introduced to the control of second-order linear modulus-constant distributed parameter system, and this system is approximated into lumped parameter system by Haar wavelets transformation.

2. INTRODUCTION OF HAAR WAVELETS OPERATION MATRIX

Wavelets analysis is a new function approximating mathematical tool. Wavelets, which are orthogonal function, served as the base of function space so as to reach the function approximating in the specific space. The mathematical construction of Haar wavelets is quite simple, but it is known as the only orthogonal wavelets function that can be formulated of explicit.

2.1 Integral Operation Matrix

If satisfying the following formula, the $P_{m\times m}$ is called positive integral operation matrix.

$$\int_0^z \varphi(z)dz = P_{m\times m}\varphi(z) \tag{1}$$

The matrix $P_{m\times m}$ is constant. m is the resolution response of Haar wavelets $\varphi(z)$.

2.2 Product Operation Matrix

If satisfying the following formula:

$$\varphi_m(z)\varphi_m^T(z)\hat{f} = \tilde{f}_{m\times m}\varphi_m(z) \tag{2}$$

Here, $\hat{f}^T = [f_0, f_1, \cdots, f_{m-1}]$, the matrix $\tilde{f}_{m\times m}$ is called product operation matrix relating to the vector $\hat{f}$.

3. TIME-DISCRETE BOUNDARY PREDICTIVE CONTROL ALGORITHM OF 2-ORDER DPS

Consider the following linear distributed parameter system:

$$x(k+1, z = \nabla_z x(k,z) \quad z \in \Omega \tag{3}$$

$$I.C.\ x(0,z) = x_0(z) \tag{4}$$

$$B.C.\ \Gamma_\xi x(k+1,\xi) = u_1(k) \quad \xi \in \partial\Omega \tag{5}$$

Here, k is discrete sampling time, z is the distributed output point of the system, $x(k,z)$ is the state of the system, $x(k+1,z)$ is the predicting state of the system, $u_1(k)$ is the boundary control input, $\nabla_z = a_2\frac{\partial^2}{\partial z^2} + a_1\frac{\partial}{\partial z} + a_0$ is the second-order linear partial differentiate operator, $x(0,z)$ is the initial condition of the system, Γ_ξ is the linear boundary partial differentiate operator, ξ is the boundary limit a or b of the system, Ω is the open set, that is $\Omega = (a,b)$, $\partial\Omega$ is the boundary of Ω, $\partial\Omega = a, b$.

When orthogonal Haar wavelets transformation is applied to the system model that is represented in partial differentiate equation, the approximated model will have the following form (refer to section 4 in this paper):

$$\begin{cases} \hat{x}_m(k+1) = \bar{A}(k)\hat{x}_m(k) + \bar{B}(k)\hat{u}(k) + v(k) \\ \hat{x}(0) = \hat{x}_0 \end{cases} \tag{6}$$

Here, $\hat{x}_m(k)$ is the state of the approximated system, i.e. $\hat{x}_m(k)$ is the spread coefficient vector of $x(k,z)$ in orthogonal wavelets transformation. $\bar{A}(k)$, $\bar{B}(k)$, and $v(k)$ are the parameter matrixes of approximated system(refer to (27),(28) and (29)in this paper), concerning with the partial differentiate operator ∇_z and system parameters a_2, a_1, a_0.

The output of the approximated system model in prediction is

$$y_m(k) = H[x_m(k,z_1), x_m(k,z_2), \cdots, x_m(k,z_\mu)]^T \tag{7}$$

The output of the system is

$$y(k) = H[x(k,z_1), x(k,z_2), \cdots, x(k,z_\mu)]^T \tag{8}$$

Here,$H \in R^{\gamma\times\mu}$ is observation matrix, $y(k) \in R^{\gamma\times 1}$ is multi-point output of the system at sampling time k, μ is the number of output points. γ is the resolution correspond to the Haar wavelets base.

Designating the anticipant output sequence of the system is $C(k) \in R^{\gamma\times 1}$, $k = 1, 2, \cdots$. The assignment of boundary predictive control system is to design the controller $u_1(k,z)$ so as to make the system output $y(k)$ matching $C(k)$ at terminal.

Designating the reference trajectory as follow:

$$\begin{cases} y_r(k+j) = \alpha^j y_r(k) + (I - \alpha^j)C(k) \\ y_r(k) = y(k) \end{cases} \tag{9}$$

Here, $\alpha = diag(\alpha_1, \alpha_2, \cdots, \alpha_\gamma)$, $0 < \alpha_i < 1$, α_i is called soften factor of reference trajectory. So the system output $y(k)$ will be asymptotically tracking of the smooth reference trajectory.

Designating the predictive horizon length is P, control horizon length is M $(M < P)$, that is

$$\hat{u}(k+M) = \hat{u}(k+M+1) = \cdots = \hat{u}(K+P) = 0 \tag{10}$$

From (6), The state prediction of the approximated system model is known as:

$$\hat{x}_m(k+j/k) = \psi(k+j,k)\hat{x}_m(k) + \sum_{l=k}^{k+j-1} \psi(k+j,l+1)[\bar{B}(l)\hat{u}(l) + v(l)] \tag{11}$$

Here $\psi(k,l)$ is the state-transfer matrix. j is the predicting step. $j = 0, 1, \cdots, P$. So the output prediction of the model can be expressed as:

$$y_m(k+j/k) = H\,[I_\mu \otimes \hat{x}_m^T(k+j/k)]\bar{\Phi}(z) \tag{12}$$

Here, $\bar{\phi}(z)$ is the orthogonal wavelets base with respect to the output point z.

From (11), When $j = 1$:

$$\hat{x}_m(k+1/k) = \psi(k+1,k)\hat{x}_m(k) + \psi(k+1,k+1)\bar{B}(k)\hat{u}(k) + \psi(k+1,k+1)v(k)$$

When $j = 2$:

$$\hat{x}_m(k+2/k) = \psi(k+2,k)\hat{x}_m(k) + \psi(k+2,k+1)\bar{B}(k)\hat{u}(k) + \psi(k+2,k+1)v(k) + \psi(k+2,k+2)\bar{B}(k+1)\hat{u}(k+1) + \psi(k+2,k+2)v(k+1)$$

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When $j = M$:

$$\begin{aligned}
&\hat{x}_m(k+M/k) = \psi(k+M,k)\hat{x}_m(k)\\
&+\psi(k+M,k+1)\bar{B}(k)\hat{u}(k) + \psi(k+M,k+1)v(k)\\
&\qquad +\psi(k+M,k+2)\bar{B}(k+1)\hat{u}(k+1)\\
&\qquad +\psi(k+M,k+2)v(k+1)\\
&+\cdots+\psi(k+M,k+M)\bar{B}(k+M-1)\hat{u}(k+M-1)\\
&\qquad +\psi(k+M,k+M)v(k+M-1)\\
&\qquad \cdots\cdots
\end{aligned}$$

When $j = P$:

$$\begin{aligned}
&\hat{x}_m(k+P/k) = \psi(k+P,k)\hat{x}_m(k)\\
&+\psi(k+P,k+1)\bar{B}(k)\hat{u}(k) + \psi(k+P,k+1)v(k)\\
&\qquad +\psi(k+P,k+2)\bar{B}(k+1)\hat{u}(k+1)\\
&\qquad +\psi(k+P,k+2)v(k+1)\\
&+\cdots+\psi(k+P,k+M)\bar{B}(k+M-1)\hat{u}(k+M-1)\\
&\qquad +\psi(k+P,k+M)v(k+M-1)\\
&\qquad +\psi(k+P,k+M+1)v(k+M)\\
&\qquad +\cdots+\psi(k+P,k+P)v(k+P-1)
\end{aligned}$$

By letting:

$$\begin{aligned}
F_1(k+1) &= [\psi(k+1,k+1)\bar{B}(k), 0, \cdots, 0]\\
F_2(k+2) &= [\psi(k+2,k+1)\bar{B}(k),\\
&\qquad \psi(k+2,k+2)\bar{B}(k+1), 0, \cdots, 0]\\
&\cdots\cdots\\
F_M(k+M) &= [\psi(k+M,k+1)\bar{B}(k),\\
&\qquad \psi(k+M,k+2)\bar{B}(k+1), \cdots,\\
&\qquad \psi(k+M,k+M)\bar{B}(k+M-1), 0, \cdots, 0]\\
&\cdots\cdots\\
F_P(k+P) &= [\psi(k+P,k+1)\bar{B}(k),\\
&\qquad \psi(k+P,k+2)\bar{B}(k+1), \cdots,\\
&\qquad \psi(k+P,k+M)\bar{B}(k+M-1), 0, \cdots, 0]\\
u_p(k) &= [\hat{u}(k), \hat{u}(k+1), \cdots,\\
&\qquad \hat{u}(k+M-1), 0, \cdots, 0]^T
\end{aligned}$$

Rewriting the above in the form of matrix:

$$\begin{bmatrix} \hat{x}_m(k+1/k)\\ \hat{x}_m(k+2/k)\\ \cdots\\ \hat{x}_m(k+M/k)\\ \cdots\\ \hat{x}_m(k+P/k) \end{bmatrix} = \begin{bmatrix} \psi(k+1,k)\\ \psi(k+2,k)(k)\\ \cdots\\ \psi(k+M,k)\\ \cdots\\ \psi(k+P,k) \end{bmatrix} \hat{x}_m(k)$$

$$+ \begin{bmatrix} F_1(k+1)\\ F_2(k+2)\\ \cdots\\ F_M(k+M)\\ \cdots\\ F_P(k+P) \end{bmatrix} u_p(k) + \begin{bmatrix} \sum_{l=k}^{k}\psi(k+1,l+1)v(l)\\ \sum_{l=k}^{k+1}\psi(k+2,l+1)v(l)\\ \cdots\\ \sum_{l=k}^{k+M-1}\psi(k+M,l+1)v(l)\\ \cdots\\ \sum_{l=k}^{k+P-1}\psi(k+P,l+1)v(l) \end{bmatrix}$$

$$= \breve{\psi}\hat{x}_m(k) + Fu_p(k) + V \tag{13}$$

Here, $\breve{\psi}$, F,V are matrixes showed in the above equation respectively. F is the matrix concerning with $\bar{A}(k)$, $\bar{B}(k)$.

So the output prediction of the model can be written as:

$$y_m(k+j/k) = H \begin{bmatrix} \bar{\Phi}^T(z_1)\hat{x}_m(k+j/k)\\ \bar{\Phi}^T(z_2)\hat{x}_m(k+j/k)\\ \cdots\\ \bar{\Phi}^T(z_\mu)\hat{x}_m(k+j/k) \end{bmatrix}$$

$$\begin{aligned}
&= H \begin{bmatrix} \bar{\Phi}^T(z_1)\\ \bar{\Phi}^T(z_2)\\ \cdots\\ \bar{\Phi}^T(z_\mu) \end{bmatrix} \hat{x}_m(k+j/k)\\
&= H\Phi_\mu \hat{x}_m(k+j/k)
\end{aligned} \tag{14}$$

Here $\bar{\Phi}^T(z_i)$ is the wavelets base relating to the output point z_i. $i = 1, 2, \cdots, \mu$. Φ_μ is a matrix. $\Phi_\mu = [\bar{\Phi}^T(z_1), \bar{\Phi}^T(z_2), \cdots, \bar{\Phi}^T(z_\mu)]^T$.

Subsequently the output prediction of the model can be expressed in matrix as:

$$\begin{aligned}
Y_m &= H\Phi_\mu[\breve{\psi}\hat{x}_m(k) + Fu_p(k) + V]\\
&= H\Phi_\mu\breve{\psi}\hat{x}_m(k) + H\Phi_\mu Fu_p(k) + H\Phi_\mu V
\end{aligned} \tag{15}$$

According to the feedback revising principle in model predictive control, the output prediction equation of the system is as follow:

$$\begin{aligned}
Y_p &= Y_m + \tilde{I}[y(k) - y_m(k)]\\
&= H\Phi_\mu[\breve{\psi}\hat{x}_m(k) + Fu_p(k) + V]\\
&\qquad + \tilde{I}[y(k) - y_m(k)]\\
&= H\Phi_\mu[\breve{\psi}\hat{x}_m(k) + Fu_p(k) + V]\\
&\qquad + \tilde{I}[y(k) - H\Phi_\mu\hat{x}_m(k)]
\end{aligned} \tag{16}$$

Since model is only an approximation of the real process, it is extremely important for model predictive control to be robust to model uncertainty. There are uncertainties such as prediction errors, parameter perturbations, and variety of disturbances in distributed parameter systems. To conquer the affections of uncertainties, a terminal constrained tuning functionβC is adopted into the objective function as follow:

$$J = \|Y_P - Y_r + \beta C\|_Q^2 + \|u_p(k)\|_R^2 \quad (17)$$

Here, Q_j are the block-diagonal weighted matrixes with warp, $j = 1, 2, \cdots, P$, R_j are the block-diagonal weighted matrixes with control, $j = 0, 1, \cdots, M-1$. Y_r is the reference trajectory matrix.

Minimizing the objective function on a prediction horizon, the assignment of system predictive control transforms into the proposition of rolling optimization as follow:

$$\begin{cases} \min\limits_{u(k)} J = \|Y_P - Y_r + \beta C\|_Q^2 + \|u_p(k)\|_R^2 \\ \text{s.t. } Y_p = H\Phi_\mu[\breve{\psi}\hat{x}_m(k) + Fu_p(k) + V] \\ \qquad + \tilde{I}[y(k) - H\Phi_\mu \hat{x}_m(k)] \\ k = 0, 1, 2, \cdots \end{cases} \quad (18)$$

Solving the above rolling optimization proposition, we have:

$$u^*(k) = -[R + F^T\Phi_\mu^T H^T Q H \Phi_\mu F]^{-1} F^T \Phi_\mu^T H^T Q$$

$$[H\Phi_\mu(\breve{\psi}\hat{x}_m(k) + V) + \tilde{I}[y(k) - H\Phi_\mu \hat{x}_m(k)] - Y_r + \beta C]$$

By letting

$$K = [R + F^T\Phi_\mu^T H^T Q H \Phi_\mu F]^{-1} F^T \Phi_\mu^T H^T Q$$

Then

$$u^*(k) = -K[H\Phi_\mu \breve{\psi}\hat{x}_m(k) + H\Phi_\mu V + \tilde{I}[y(k) - H\Phi_\mu \hat{x}_m(k)] - Y_r + \beta C] \quad (19)$$

It turns out M steps control inputs as follow:

$$u^*(k) = [\hat{u}^*(k), \hat{u}^*(k+1), \cdots, \hat{u}^*(k+M-1)]^T \quad (20)$$

Although more than one input is computed, the controller implements only the first control input $\hat{u}^*(k)$. This method known as receding horizon control. At the next sampling time, system measurements are used to update the optimization problem, and the optimization computation is repeated in this rolling optimization strategy. In this way, the control at every sampling time becomes a closed loop approach, though open-loop optimal control is used within a moving horizon. It enhances the robust stability of the control system. The first control input $\hat{u}^*(k)$ return to the boundary predictive control of the original system $u_1^*(k, z_i) \approx \hat{u}^{*T}(k)\bar{\Phi}(z_i)$ at every sampling time in online optimization computation. Tuning the coefficient β finely can reach the robustness output in dealing with the uncertainties.

4. WAVELETS APPROXIMATION OF LINEAR SECOND-ORDER MODULUS-CONSTANT DPS

Considering the linear second-order modulus-constant distributed parameter system:

$$x(k+1, z) = a_2 \frac{\partial^2 x(k,z)}{\partial z^2} + a_1 \frac{\partial x(k,z)}{\partial z} + a_0 x(k,z) \quad (21)$$

$$I.C. \quad x(0, z) = f(z) \quad (22)$$

$$B.C. \quad \begin{cases} x(k, 0) = u_1(k) \\ \dfrac{\partial x(k,z)}{\partial z}|_{z=0} = g_2(k) \end{cases} \quad (23)$$

Taking the Haar orthogonal wavelets transformation to all the variables and coefficients of the above system:

$$\begin{cases} x(k,z) \approx \hat{x}^T(k)\Phi(z) \\ x(k+1,z) \approx \hat{x}^T(k+1)\Phi(z) \\ x(k,0) = u_1(k) \approx \hat{u}_1^T(k)\Phi(z) \\ a_0(z) \approx \hat{a}_0^T\Phi(z) \\ a_1(z) \approx \hat{a}_1^T\Phi(z) \\ a_2(z) \approx \hat{a}_2^T\Phi(z) \\ x(0,z) = f(z) \approx \hat{f}^T\Phi(z) \\ a_1(0) = a_{10} \approx \hat{a}_{10}^T\Phi(z) \\ a_2(0) = a_{20} \approx \hat{a}_{20}^T\Phi(z) \\ \dfrac{\partial a_1}{\partial z} = a_{11}(z) \approx \hat{a}_{11}^T\Phi(z) \\ \dfrac{\partial a_2}{\partial z} = a_{21}(z) \approx \hat{a}_{21}^T\Phi(z) \\ \dfrac{\partial^2 a_2}{\partial z^2} = a_{22} \approx \hat{a}_{22}^T\Phi(z) \\ \dfrac{\partial a_2}{\partial z}|_{z=0} = a_{210}(z) \approx \hat{a}_{210}^T\Phi(z) \\ \hat{e} = [1, 0, \cdots, 0] \in R^{n\times 1} \\ \dfrac{\partial x(k,z)}{\partial z}|_{z=0} = g_2(k) \approx \hat{g}_2^T(k)\Phi(z) = g_2(k)\hat{e}^T\Phi(z) \end{cases}$$

Here, the variable k and z in $x(k, z)$ are isolated separate into $\hat{x}^T(k)$ and $\Phi(z)$. This property makes the computation easier. $\Phi(z)$ is similar to $\sin\omega(t)$ in Fourier transformation but orthogonal, $\hat{x}^T(k)$ is the wavelets coefficient vector similar to Fourier coefficient vector.

Taking the integration to both sides of the original system for variable z from 0 to z at the same time:

$$\int_0^z\int_0^z x(k+1,z)dzdz = \int_0^z\int_0^z a_2 \frac{\partial^2 x(k,z)}{\partial z^2}dzdz + \int_0^z\int_0^z a_1 \frac{\partial x(k,z)}{\partial z}dzdz + \int_0^z\int_0^z a_0 x(k,z)dzdz \quad (24)$$

Taking the Haar orthogonal wavelets transformation, the left of the equation (24) becomes:

$$\int_0^z\int_0^z x(k+1,z)dzdz = \hat{x}^T(k+1)P\int_0^z \Phi(z)dz = \hat{x}^T(k+1)P^2\Phi(z)$$

The first item of the right of (24) will be as follow:

$$\int_0^z\int_0^z a_2 \frac{\partial^2 x(k,z)}{\partial z^2}dzdz = \int_0^z\int_0^z a_2 \frac{\partial}{\partial z}\left(\frac{\partial x(k,z)}{\partial z}\right)dzdz$$

$$= \int_0^z \left(a_2 \frac{\partial x(k,z)}{\partial z}\Big|_0^z - \int_0^z \frac{\partial x(k,z)}{\partial z}\frac{\partial a_2}{\partial z}dz\right)dz$$

$$= a_2 x(k,z) - a_2(0)x(k,0) - \int_0^z x(k,z)\frac{\partial a_2}{\partial z}dz$$

$$-\int_0^z a_2(0)\frac{\partial x(k,z)}{\partial z}|_{z=0}dz - \int_0^z \frac{\partial a_2}{\partial z}x(k,z)dz$$

$$+\int_0^z \frac{\partial a_2}{\partial z}|_{z=0}x(k,0)dz + \int_0^z\int_0^z \frac{\partial^2 a_2}{\partial z^2}x(k,z)dzdz$$

$$= \hat{x}^T(k)\tilde{a}_2\Phi(z) - \hat{u}_1^T(k)\tilde{a}_{20}\Phi(z) - \hat{x}^T(k)\tilde{a}_{21}P\Phi(z)$$
$$-g_2(k)\hat{e}^T\tilde{a}_{20}P\Phi(z) - \hat{x}^T(k)\tilde{a}_{21}P\Phi(z)$$
$$+\hat{u}_1^T(k)\tilde{a}_{210}P\Phi(z) + \hat{x}^T(k)\tilde{a}_{22}P^2\Phi(z)$$

The second item of the right of (24) will be as follow:

$$\int_0^z\int_0^z a_1\frac{\partial x(k,z)}{\partial z}dzdz$$
$$= \int_0^z (a_1 x(k,z)\mid_0^z - \int_0^z x(k,z)\frac{\partial a_1}{\partial z}dz)dz$$
$$= \int_0^z (\hat{x}^T(k)\Phi(z)\Phi^T(z)\hat{a}_1 - \hat{u}_1^T(k)\Phi(z)\Phi^T(z)\hat{a}_{10}$$
$$-\int_0^z (\hat{x}^T(k)\Phi(z)\Phi^T(z)\hat{a}_{11}dz)dz$$
$$= \hat{x}^T(k)\tilde{a}_1 P\Phi(z) - \hat{u}_1^T(k)\tilde{a}_{10}P\Phi(z) - \hat{x}^T(k)\tilde{a}_{11}P^2\Phi(z)$$

The third item of the right of (24) will be as follows:

$$\int_0^z\int_0^z a_0 x(k,z)dzdz = \int_0^z\int_0^z \hat{x}^T(k)\Phi(z)\Phi^T(z)\hat{a}_0 dzdz$$
$$= \int_0^z \hat{x}^T(k)\tilde{a}_0 P\Phi(z)dz = \hat{x}^T(k)\tilde{a}_0 P^2\Phi(z)$$

Substituting the above four outcomes back into (24):

$$\hat{x}^T(k+1)P^2\Phi(z)$$
$$= \hat{x}^T(k)\tilde{a}_2\Phi(z) - \hat{u}_1^T(k)\tilde{a}_{20}\Phi(z) - \hat{x}^T(k)\tilde{a}_{21}P\Phi(z)$$
$$-g_2(k)\hat{e}^T\tilde{a}_{20}P\Phi(z) - \hat{x}^T(k)\tilde{a}_{21}P\Phi(z)$$
$$+\hat{u}_1^T(k)\tilde{a}_{210}P\Phi(z) + \hat{x}^T(k)\tilde{a}_{22}P^2\Phi(z)$$
$$+\hat{x}^T(k)\tilde{a}_1 P\Phi(z) - \hat{u}_1^T(k)\tilde{a}_{10}P\Phi(z)$$
$$-\hat{x}^T(k)\tilde{a}_{11}P^2\Phi(z) + \hat{x}^T(k)\tilde{a}_0 P^2\Phi(z)$$

Reducing the $\Phi(z)$, transposing the both sides of the equation and rearranging the items, we have:

$$(P^2)^T\hat{x}(k+1) = \tilde{a}_2^T\hat{x}(k) - \tilde{a}_{20}^T\hat{u}_1(k) - P^T\tilde{a}_{21}^T\hat{x}(k)$$
$$-P^T\tilde{a}_{20}^T g_2(k)\hat{e} - P^T\tilde{a}_{21}^T\hat{x}(k)$$
$$+P^T\tilde{a}_{210}^T\hat{u}_1(k) + (P^2)^T\tilde{a}_{22}^T\hat{x}(k)$$
$$+P^T\tilde{a}_1^T\hat{x}(k) - P^T\tilde{a}_{10}^T\hat{u}_1(k)$$
$$-(P^2)^T\tilde{a}_{11}\hat{x}(k) + (P^2)^T\tilde{a}_0^T\hat{x}(k)$$

Multiplying the $(P^2)^{-T}$ to the left to both sides and making arrangement, we have:

$$\hat{x}(k+1) = [(P^2)^{-T}\tilde{a}_2^T$$
$$-2P^{-T}\tilde{a}_{21}^T + \tilde{a}_{22}^T + P^{-T}\tilde{a}_1^T - \tilde{a}_{11}^T + \tilde{a}_0^T]\hat{x}(k)$$
$$+[P^{-T}\tilde{a}_{210}^T - P^{-T}\tilde{a}_{10}^T - (P^2)^{-T}\tilde{a}_{20}^T]\hat{u}_1(k)$$
$$-P^{-T}\tilde{a}_{20}^T g_2(k)\hat{e} \quad (25)$$

$$I.C.\ \hat{x}(0,z) = \hat{x}_0(z) = \hat{f}(z) \quad (26)$$

Refer to (6), we have:

$$\bar{A}(k) = (P^2)^{-T}\tilde{a}_2^T - 2P^{-T}\tilde{a}_{21}^T + \tilde{a}_{22}^T + P^{-T}\tilde{a}_1^T$$
$$-\tilde{a}_{11}^T + \tilde{a}_0^T \quad (27)$$

$$\bar{B}(k) = P^{-T}\tilde{a}_{210}^T - P^{-T}\tilde{a}_{10}^T - (P^2)^{-T}\tilde{a}_{20}^T \quad (28)$$

$$v(k) = -P^{-T}\tilde{a}_{20}^T g_2(k)\hat{e} \quad (29)$$

Here: $\bar{A}(k)$, $\bar{B}(k)$ are parameter matrixes of the approximated system. $\tilde{a}_0$,$\tilde{a}_1$,$\tilde{a}_{10}$,$\tilde{a}_{11}$,$\tilde{a}_2$,$\tilde{a}_{20}$,$\tilde{a}_{21}$,$\tilde{a}_{22}$,$\tilde{a}_{210}$ are Haar wavelets product operation matrixes corresponding to the parameters a_0,a_1,a_{10},a_{11},a_2,a_{20},a_{21},a_{22},a_{210} respectively. P is Haar wavelets positive integral operation matrix.

5. SIMULATION STUDY OF BOUNDARY PREDICTIVE CONTROL ALGORITHM OF LINEAR SECOND-ORDER MODULUS-CONSTANT DPS

5.1 Algorithm Simulation

For the second-order linear modulus-constant DPS showed in (21), (22), (23), founding the parameters as follow: $f(z) = z(z-1)$, $g_2(k) = 0$, $a_0 = 0.82$, $a_1 = -0.00022$, $a_2 = 0.000011$. Taking the Haar wavelets base that $m = 16$, the boundary predictive horizon length $P = 10$ and the control horizon length $M = 5$, simulation result is showed in fig.1. It means that the proposed algorithm is feasible and has good performances.

5.2 Simulation When System Parameters Perturbations

Since the complexity of distributed parameter systems, the systems parameters may have perturbations in real process resulting in the system model deviating. The capability of keeping the system work proper under the conditions of system model deviations represents the robustness of the control system.

Founding the parameters perturbations: $a_0 = 0.83$, that is $\Delta a_0 = 0.01$; $a_1 = -0.00025$, that is $\Delta a_1 = -0.00003$; $a_2 = 0.000012$, that is $\Delta a_2 = 0.000001$; maintaining $f(z) = z(z-1)$, $g_2(k) = 0$ unchanged. Taking the Haar wavelets base that m=16, the boundary predictive horizon length P=10 and the control horizon length M=5, simulation result is showed in fig.2. It proves the robust effectiveness of the proposed algorithm.

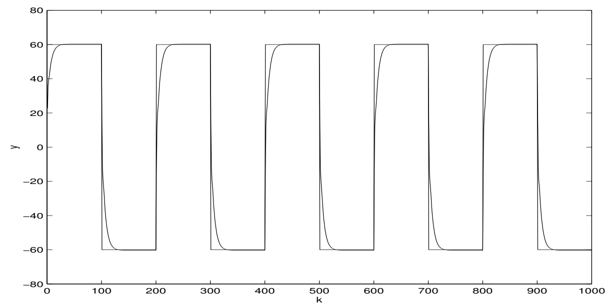

Fig. 1. Output curve as C=60 Q=100 R=I α=0.8 z=0.6

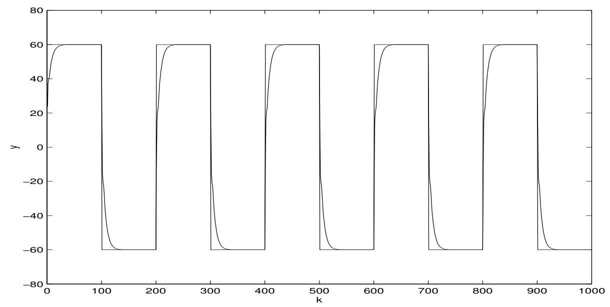

Fig. 2. Output curve when parameters perturbations as C=60 Q=100 R=I α=0.8 z=0.6

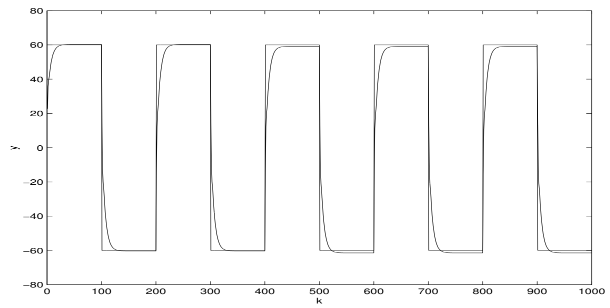

Fig. 3. Output curve when disturb occurs as C=60 Q=100 R=I α=0.8 z=0.6

5.3 Simulation When Disturb Occurs to the Output

There is variety of disturbances in distributed parameter systems generally. These disturbances strike the proper work of the system. Founding the system parameters: $f(z) = z(z-1)$, $g_2(k) = 0$, $a_0 = 0.82$, $a_1 = -0.00022$, $a_2 = 0.000011$, Suppose a disturb step-signal $y_d(k) = 1$ occurs to the system output when k=400 and on. Taking the Haar wavelets base that m=16, the boundary predictive horizon length P=10 and the control horizon length M=5, simulation result is showed in fig.3. It indicates that system output has a deviation from $C(k)$.

6. CONCLUSION

The algorithm extends the study and potential application about predictive control, wavelets and distributed parameter system. It is considered that our next work is applying the algorithm to non-linear distributed parameter system and putting the algorithm into application in the future.

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