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Global exponential stability of delayed parabolic neural networks ⋆

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Abstract: The globally exponentially stable conditions for delayed parabolic neural networks with variable coefficients are considered in this paper. We first derive the globally exponentially stable condition for delayed parabolic neural networks with variable coefficients based on delay differential inequality combining with Young inequality. Compared with the method of Lyapunov functionals as in most previous studies, our method is simpler and more effective for stability analysis.

1. INTRODUCTION

Neural networks have many applications in pattern recognition, image processing, association, etc. Some of these applications require that the equilibrium points of the designed network be stable (Zhang *et al.*, 2005). Therefore, it is vital to study the stability of neural networks. In biological and artificial neural networks, time delays often arise in the processing of information storage and transmission. In recent years, the stability of delayed neural networks (DNN) have been investigated by many researchers (Joy M. 1999; Liao and Wang 1999; Arik 2000; Cao 2001; Cao and Wang 2003; Cui and Lou 2006; Lou and Cui 2006).

On the other hand, parabolic (and hyperbolic) evolution equations describe processes that are evolving in time. For such an equation the initial state of the system is part of the auxiliary data for a well-posed problem. We also notice that parabolic equations play a special role in the mathematical modelling of polymerization-type chemical reaction phenomena (coagulation and fragmentation of clusters), in atmosphere physics, biology, and immunology. Recently, the existence, stability and oscillation of such systems have been widely studied (Brzychczy 2002; Leiva and Sequera 2003; Li and Cui 2001; Minchev and Yoshida 2003; Wang and Teo 2005).

Furthermore, real neural networks are more likely to be time-varying evolving networks, namely, the topology is changing with the time. In this paper, we further extend the parabolic models to describe the varying topology neural networks. Using the Green's formula and boundary condition, we can easily deal with the parabolic terms. To the best of our knowledge, this is the first time to introduce and study delayed parabolic neural networks with variable coefficients. The main purposes of this paper are firstly to present the model of delayed parabolic neural networks with variable coefficients; and secondly to discuss the stability of delayed parabolic neural networks by using delay differential inequality combining with Young inequality. One criterion is given to guarantee the global exponential stability for delayed parabolic neural networks with variable coefficients.

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2. SYSTEM DESCRIPTION AND PRELIMINARY

In this paper, we obtain some conditions for delayed parabolic neural networks with variable coefficients of the form

$$\begin{aligned}\frac{\partial u_i(x,t)}{\partial t} &= a_i(t)\Delta u_i(x,t) + \sum_{k=1}^{s} b_{ik}(t)\Delta u_i(x,t-\rho_k(t)) \\ &\quad - c_i(t)u_i(x,t) + \sum_{j=1}^{n} w_{ij}(t) f_j(u_j(x,t)) \\ &\quad + \sum_{j=1}^{n} h_{ij}(t) f_j(u_j(x,t-\tau_j(t))) + I_i, \\ &\quad (x,t)\in\Omega\times[0,\infty), \qquad (1)\end{aligned}$$

for $i = 1, 2, \cdots, n$, where Ω is a bounded domain in $\mathbf{R}^n$ with a piecewise smooth boundary $\partial\Omega$, Δ is the Laplacian in $\mathbf{R}^n$ and $\Delta u(x,t) = \sum_{r=1}^{n} \frac{\partial^2 u(x,t)}{\partial x_r^2}$. $u_i(t,x)$ is the state of the ith unit at time t. $f_i(\cdot)$ denote the signal functions of the ith neurons at time t and in space x. $c_i(t) > 0$ represents the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnected from the network and external inputs at time t. $w_{ij}(t), h_{ij}(t)$ stand for the weights of neuron interconnections. I_i denote the external inputs on the ith neurons. $\tau_j(t)$ are time-varying delays of the neural network satisfying $0 \le \tau_j(t) \le \sigma$ (σ is a constant).

We assume throughout this paper that

(H1) $a_i(t), b_{ik}(t) \in C([0,\infty);[0,\infty)), k = 1, 2, \cdots, s$;

(H2) $\rho_k(t) \in C([0,\infty);[0,\infty))$, $\lim_{t\to\infty}(t-\rho_k(t)) = \infty, k = 1, 2, \cdots, s$;

(H3) The neurons activation functions $f_i(\cdot)$ $(i = 1, 2, \ldots, n)$ are bounded and Lipchitz-continuous, that is, there exist constants $L_i > 0$ such that

$$|f_i(\xi_1) - f_i(\xi_2)| \le L_i|\xi_1 - \xi_2|$$

for all $\xi_1, \xi_2 \in \mathbf{R}$.

Consider the following boundary condition:

$$\frac{\partial u(x,t)}{\partial N} = 0, \quad (x,t) \in \partial\Omega \times [0,\infty), \tag{2}$$

where N is the unit exterior normal vector to $\partial\Omega$.

Suppose that the system (1) is supplemented with initial conditions of the form

$$u_i(s,x) = \phi_i(s,x), s \in [-\sigma, 0], i = 1,2,\cdots,n, \tag{3}$$

where $\phi_i(s,x)$ $(i = 1,2,\cdots,n)$ are continuous on $[-\sigma, 0] \times \Omega$ and system (1) has an equilibrium point $u^* = (u_1^*, u_2^*, \cdots, u_n^*)^T$. We denote

$$\|\phi - u^*\| = \sup_{-\sigma \le s \le 0} \left[\sum_{j=1}^{n} \int_{\Omega} |\phi_j(s,x) - u^*|^p dx\right]^{1/p}.$$

We say that an equilibrium point $u^* = (u_1^*, u_2^*, \cdots, u_n^*)^T$ is globally exponentially stable if there exist constants $\varepsilon > 0$ and $M \ge 1$ such that

$$\|u(t) - u^*\| \le M\|\phi - u^*\|e^{-\varepsilon t}. \tag{4}$$

We will use D^+ to denote the Dini derivative. For any continuous function $V : \mathbf{R} \to \mathbf{R}$, the Dini derivative of $V(t)$ is defined as

$$D^+V(t) = \lim_{h \to 0^+} \sup \frac{V(t+h) - V(t)}{h}. \tag{5}$$

In order to simplify the proofs and compare our results, we present some lemmas as follows.

Lemma 1 (Young inequality (Hardy *et al.*, 1952)). Assume that $a > 0$, $b > 0$, $p > 1$, $\frac{1}{p} + \frac{1}{q} = 1$, then the following inequality:

$$ab \le \frac{1}{p}a^p + \frac{1}{q}b^q \tag{6}$$

holds.

Lemma 2 (Halanay inequality (Gopalsamy 1992)). Let α and β be constants with $0 < \beta < \alpha$. Let $x(t)$ be a continuous nonnegative function on $t \ge t_0 - \tau$ satisfying inequality (6) for $t \ge t_0$

$$\dot{x}(t) \le -\alpha x(t) + \beta \widetilde{x}(t), \tag{7}$$

where $\widetilde{x}(t) = \sup_{t-\tau \le s \le t} \{x(s)\}$. Then

$$x(t) \le \widetilde{x}(t_0)e^{-r(t-t_0)}, \tag{8}$$

where r is a bound on the exponential convergence rate and is the unique positive solution of

$$r = \alpha - \beta e^{r\tau}.$$

3. MAIN RESULTS

Theorem 1. Suppose (H1)-(H3) hold. If there exist real constants ζ_{ij}, η_{ij} and positive constants $\lambda_i > 0$, $p \ge 1$, $i = 1,2,\cdots,n$ such that

$$\begin{aligned}\min_{1\le i\le n} \Bigg\{ & pc_i(t) - \sum_{j=1}^{n}\Big(\frac{\lambda_j}{\lambda_i}L_i|w_{ji}(t)|^{p(1-\zeta_{ji})} \\ & +(p-1)L_j|w_{ij}(t)|^{\frac{p\zeta_{ij}}{p-1}} + (p-1)L_j|h_{ij}(t)|^{\frac{p\eta_{ij}}{p-1}}\Big)\Bigg\} \\ > \max_{1\le i\le n} & \left\{\sum_{j=1}^{n}\frac{\lambda_j}{\lambda_i}L_i\frac{1}{p}|h_{ji}(t)|^{p(1-\eta_{ji})}\right\},\end{aligned} \tag{9}$$

then the equilibrium point u^* of system (1) is globally exponentially stable.

Proof. Integrating (1) with respect to x over the domain Ω, we have

$$\begin{aligned}&\frac{\partial}{\partial t}\Big[\int_{\Omega} u_i(x,t)dx\Big] \\ &= a_i(t)\int_{\Omega}\Delta u_i(x,t)dx + \sum_{k=1}^{s} b_{ik}(t)\int_{\Omega}\Delta u_i(x,t-\rho_k(t))dx \\ &\quad -c_i(t)\int_{\Omega} u_i(x,t)dx + \sum_{j=1}^{n} w_{ij}(t)\int_{\Omega} f_j(u_j(x,t))dx \\ &\quad +\sum_{j=1}^{n} h_{ij}(t)\int_{\Omega} f_j(u_j(x,t-\tau_j(t)))dx + I_i,\end{aligned} \tag{10}$$

From Green's formula and boundary condition (2), it follows that

$$\int_{\Omega}\Delta u_i(x,t)dx = \int_{\partial\Omega}\frac{\partial u_i(x,t)}{\partial N}dS = 0, \quad t \ge 0 \tag{11}$$

and

$$\begin{aligned}&\int_{\Omega}\Delta u_i(x,t-\rho_k(t))dx = \int_{\partial\Omega}\frac{\partial u_i(x,t-\rho_k(t))}{\partial N}dS = 0, \\ &t \ge 0, k = 1,2,\cdots,s,\end{aligned} \tag{12}$$

where dS is the surface element on $\partial\Omega$.
Combining (10)-(12), we have

$$\begin{aligned}&\frac{\partial}{\partial t}\Big[\int_{\Omega} u_i(x,t)dx\Big] \\ &= -c_i(t)\int_{\Omega} u_i(x,t)dx + \sum_{j=1}^{n} w_{ij}(t)\int_{\Omega} f_j(u_j(x,t))dx \\ &\quad +\sum_{j=1}^{n} h_{ij}(t)\int_{\Omega} f_j(u_j(x,t-\tau_j(t)))dx + I_i.\end{aligned} \tag{13}$$

Let $v_i(t) = \int_{\Omega}(u_i(x,t) - u_i^*)dx$, it follows from (13) that

$$\begin{aligned}\frac{dv_i(t)}{dt} &= -c_i(t)v_i(t) + \sum_{j=1}^{n} w_{ij}(t)g_j(v_j(t)) \\ &\quad +\sum_{j=1}^{n} h_{ij}(t)g_j(v_j(t-\tau_j(t))).\end{aligned} \tag{14}$$

Now we define a function

$$V(t) = \sum_{i=1}^{n}\lambda_i|v_i(t)|^p. \tag{15}$$

Calculating and estimating the upper right derivative D^+V of V along the solution of (14) as follows:

$$\begin{aligned}
D^+V(t) &= \sum_{i=1}^{n} \lambda_i p |v_i(t)|^{p-1} \mathrm{sign}(v_i(t)) \dot{v}_i(t) \\
&= \sum_{i=1}^{n} \lambda_i p |v_i(t)|^{p-1} \mathrm{sign}(v_i(t)) \Big[- c_i(t) v_i(t) \\
&\quad + \sum_{j=1}^{n} w_{ij}(t) g_j(v_j(t)) \\
&\quad + \sum_{j=1}^{n} h_{ij}(t) g_j(v_j(t-\tau_j(t))) \Big] \\
&\le \sum_{i=1}^{n} \lambda_i p \Big[- c_i(t) |v_i(t)|^p \\
&\quad + \sum_{j=1}^{n} |w_{ij}(t)| L_j |v_i(t)|^{p-1} |v_j(t)| \\
&\quad + \sum_{j=1}^{n} |h_{ij}(t)| L_j |v_i(t)|^{p-1} |v_j(t-\tau_j(t))| \Big] \\
&= \sum_{i=1}^{n} \lambda_i p \Big[- c_i(t) |v_i(t)|^p \\
&\quad + \sum_{j=1}^{n} L_j \Big(|w_{ij}(t)|^{1-\zeta_{ij}} |v_j(t)| \Big) \\
&\quad \times \Big(|w_{ij}(t)|^{\frac{\zeta_{ij}}{p-1}} |v_i(t)| \Big)^{p-1} \\
&\quad + \sum_{j=1}^{n} L_j \Big(|h_{ij}(t)|^{1-\eta_{ij}} |v_j(t-\tau_j(t))| \Big) \\
&\quad \times \Big(|h_{ij}(t)|^{\frac{\eta_{ij}}{p-1}} |v_i(t)| \Big)^{p-1} \Big].
\end{aligned} \tag{16}$$

Let $a = |w_{ij}(t)|^{1-\zeta_{ij}} |v_j(t)|$, $b = \Big(|w_{ij}(t)|^{\frac{\zeta_{ij}}{p-1}} |v_i(t)| \Big)^{p-1}$, by Lemma 1, we have

$$\begin{aligned}
&\Big(|w_{ij}(t)|^{1-\zeta_{ij}} |v_j(t)| \Big) \Big(|w_{ij}(t)|^{\frac{\zeta_{ij}}{p-1}} |v_i(t)| \Big)^{p-1} \\
&\le \frac{1}{p} |w_{ij}(t)|^{p(1-\zeta_{ij})} |v_j(t)|^p \\
&\quad + \frac{p-1}{p} |w_{ij}(t)|^{\frac{p\zeta_{ij}}{p-1}} |v_i(t)|^p.
\end{aligned} \tag{17}$$

Similarly, let $a = |h_{ij}(t)|^{1-\eta_{ij}} |v_j(t-\tau_j(t))|$, $b = \Big(|h_{ij}(t)|^{\frac{\eta_{ij}}{p-1}} |v_i(t)| \Big)^{p-1}$, by Lemma 1, we get

$$\begin{aligned}
&\Big(|h_{ij}(t)|^{1-\eta_{ij}} |v_j(t-\tau_j(t))| \Big) \Big(|h_{ij}(t)|^{\frac{\eta_{ij}}{p-1}} |v_i(t)| \Big)^{p-1} \\
&\le \frac{1}{p} |h_{ij}(t)|^{p(1-\eta_{ij})} |v_j(t-\tau_j(t))|^p \\
&\quad + \frac{p-1}{p} |h_{ij}(t)|^{\frac{p\eta_{ij}}{p-1}} |v_i(t)|^p.
\end{aligned} \tag{18}$$

Substituting (17) and (18) into (16), we obtain

$$\begin{aligned}
D^+V(t) &\le \sum_{i=1}^{n} \lambda_i p \Big[- c_i(t) |v_i(t)|^p \\
&\quad + \sum_{j=1}^{n} L_j \frac{1}{p} |w_{ij}(t)|^{p(1-\zeta_{ij})} |v_j(t)|^p \\
&\quad + \sum_{j=1}^{n} L_j \frac{p-1}{p} |w_{ij}(t)|^{\frac{p\zeta_{ij}}{p-1}} |v_i(t)|^p \\
&\quad + \sum_{j=1}^{n} L_j \frac{p-1}{p} |h_{ij}(t)|^{\frac{p\eta_{ij}}{p-1}} |v_i(t)|^p \\
&\quad + \sum_{j=1}^{n} L_j \frac{1}{p} |h_{ij}(t)|^{p(1-\eta_{ij})} |v_j(t-\tau_j(t))|^p \Big] \\
&= \sum_{i=1}^{n} \lambda_i \Big[- p c_i(t) + \sum_{j=1}^{n} \frac{\lambda_j}{\lambda_i} L_i |w_{ji}(t)|^{p(1-\zeta_{ji})} \\
&\quad + \sum_{j=1}^{n} (p-1) L_j |w_{ij}(t)|^{\frac{p\zeta_{ij}}{p-1}} \\
&\quad + \sum_{j=1}^{n} (p-1) L_j |h_{ij}(t)|^{\frac{p\eta_{ij}}{p-1}} \Big] |v_i(t)|^p \\
&\quad + \sum_{i=1}^{n} \lambda_i \Big[\sum_{j=1}^{n} \frac{\lambda_j}{\lambda_i} L_i \frac{1}{p} |h_{ji}(t)|^{p(1-\eta_{ji})} \Big] \\
&\quad \times |v_i(t-\tau_i(t))|^p \\
&\le - \min_{1 \le i \le n} \Big\{ p c_i(t) - \sum_{j=1}^{n} \Big(\frac{\lambda_j}{\lambda_i} L_i |w_{ji}(t)|^{p(1-\zeta_{ji})} \\
&\quad + (p-1) L_j |w_{ij}(t)|^{\frac{p\zeta_{ij}}{p-1}} \\
&\quad + (p-1) L_j |h_{ij}(t)|^{\frac{p\eta_{ij}}{p-1}} \Big) \Big\} V(t) \\
&\quad + \max_{1 \le i \le n} \Big\{ \sum_{j=1}^{n} \frac{\lambda_j}{\lambda_i} L_i \frac{1}{p} |h_{ji}(t)|^{p(1-\eta_{ji})} \Big\} \widetilde{V}(t).
\end{aligned} \tag{19}$$

Applying Lemma 2, then it follows (9) and (15) that

$$\lambda_{min} \int_{\Omega} |u(x,t) - u^*|^p dx \le V(t) \le \widetilde{V}(t_0) e^{-\varepsilon(t-t_0)}. \tag{20}$$

So, we have

$$\int_{\Omega} |u(x,t) - u^*|^p dx \le \frac{\lambda_{max}^{1/p}}{\lambda_{min}^{1/p}} e^{-\frac{\tau}{p} t} \|\phi(x,t) - u^*\|^p. \tag{21}$$

Therefore, the proof is completed.

Corollary 1. Suppose (H1)-(H3) hold. If there exist constants $\lambda_i > 0$ such that

$$\begin{aligned}
&\min_{1 \le i \le n} \Big\{ c_i(t) - \sum_{j=1}^{n} \Big(\frac{\lambda_j}{\lambda_i} L_i |w_{ji}(t)| \Big) \Big\} \\
&> \max_{1 \le i \le n} \Big\{ \sum_{j=1}^{n} \frac{\lambda_j}{\lambda_i} L_i |h_{ji}(t)| \Big\},
\end{aligned} \tag{22}$$

then the equilibrium point u^* of system (1) is globally exponentially stable.

Proof. Taking $p = 1$, $\zeta_{ij} = \eta_{ij} = 0$ in theorem 1 above, then we can easily obtain Corollary 1.

Corollary 2. Suppose (H1)-(H3) hold. If there exist constants $\lambda_i > 0$ such that

$$\min_{1\le i\le n}\left\{2c_i(t) - \sum_{j=1}^{n}\Big(\frac{\lambda_j}{\lambda_i}L_i|w_{ji}(t)| + L_j(|w_{ij}(t)| + |h_{ij}(t)|)\Big)\right\} > \max_{1\le i\le n}\left\{\sum_{j=1}^{n}\frac{\lambda_j}{\lambda_i}L_i\frac{1}{p}|h_{ji}(t)|^{p(1-\eta_{ji})}\right\}, \qquad (23)$$

then the equilibrium point u^* of system (1) is globally exponentially stable.

Proof. It is easy to check that the inequality (8) is satisfied by taking $p = 2$, $\zeta_{ij} = \eta_{ij} = 0.5$ and hence the theorem 1 implies Corollary 2.

Remark 1. If the parameters $\lambda_i, p, \zeta_{ij}, \eta_{ij}$ are properly chosen, we can easily obtain a series of corollaries.

Remark 2. In many papers, the delay function $\tau_j(t)$ is needed to be differentiable, for example (Joy M. 1999; Cao and Wang 2003;). However, the restriction is neglected. Moreover, we introduce the parabolic models to describe the delayed neural networks. Thus, the conditions given in this paper are less restrictive, general and conservative.

4. NUMERICAL EXAMPLES

In this section, we will give two numerical examples to show the validity of our results.

Example 1. Consider a delayed parabolic neural network with variable coefficients

$$\frac{\partial u_i(x,t)}{\partial t} = a_i(t)\Delta u_i(x,t) + \sum_{k=1}^{s} b_{ik}(t)\Delta u_i(x, t-\rho_k(t)) - c_i(t)u_i(x,t) + \sum_{j=1}^{n} w_{ij}(t)f_j(u_j(x,t)) + \sum_{j=1}^{n} h_{ij}(t)f_j(u_j(x, t-\tau_j(t))) + I_i,$$
$$(x,t) \in (0,\pi)\times[0,\infty), \qquad (24)$$

with boundary condition
$$\frac{\partial u_i(0,t)}{\partial x} = \frac{\partial u_i(\pi,t)}{\partial x} = 0, t \ge 0, i = 1,2.$$

The system parameters are chosen as follows:

$a_1(t) = a_2(t) = 1$,
$b_{ik}(t) = e^t$ $(i = 1,2;\ k = 1,2)$,
$\rho_1(t) = \frac{\pi}{2}$, $\rho_2(t) = \frac{3\pi}{2}$,

$c_1(t) = c_2(t) = 1$, $I_1 = 1$, $I_2 = 2$,

$$W = (w_{ij})_{n\times n} = \begin{bmatrix} 0.1 & -0.1 \\ 0.1 & 0.1 \end{bmatrix},$$

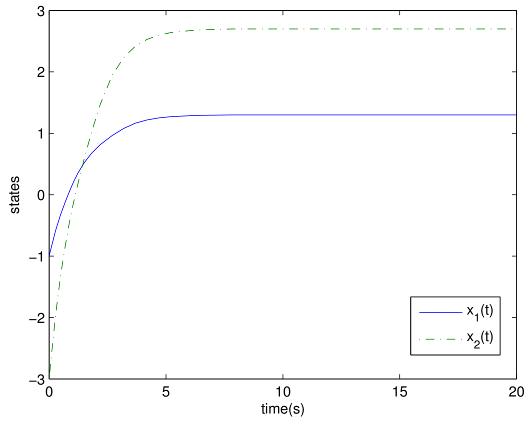

Fig.1. Numeric simulation for the exponential stability of the system (26)

$$H = (h_{ij})_{n\times n} = \begin{bmatrix} 0.4 & -0.1 \\ 0.1 & 0.4 \end{bmatrix}.$$

The activation function is PWL: $f_i(y) = \frac{1}{2}(|y+1| - |y-1|)$. Clearly, f_i satisfies the assumption (H3) with $L_1 = L_2 = 1$. Furthermore, let $\lambda_1 = \lambda_2 = 1$, then one can easily check that

$$\min_{1\le i\le n}\left\{c_i(t) - \sum_{j=1}^{n}\Big(\frac{\lambda_j}{\lambda_i}L_i|w_{ji}(t)|\Big)\right\} = 0.8 > \max_{1\le i\le n}\left\{\sum_{j=1}^{n}\frac{\lambda_j}{\lambda_i}L_i|h_{ji}(t)|\right\} = 0.5. \qquad (25)$$

Therefore, by Corollary 1, the equilibrium point of (24) is globally exponentially stable. By a simple computation, we can easily seen that the matrix $C + C^T = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ is not negative semidefinite. Thus the condition in (Li *et al.*, 2003) does not hold. For this example, our result is less restrictive than that given in (Li *et al.*, 2003).

Example 2. If we do not consider the parabolic term, the system (24) reduce to a normal delayed neural network as follows

$$\frac{du_i(t)}{dt} = -c_i(t)u_i(t) + \sum_{j=1}^{n} w_{ij}(t)f_j(u_j(t)) + \sum_{j=1}^{n} h_{ij}(t)f_j(u_j(t-\tau_j(t))) + I_i, \qquad (26)$$

We choose the same parameters as (24) and let $\tau(t) = 0.5$. Then, by Corollary 1, the system (26) is exponentially stable. Fig. 1 indicates that $[x_1(t), x_2(t)]^{\mathrm{T}}$ converge to $[1.3000, 2.7000]^{\mathrm{T}}$ with the initial values $[-1.0, -3.0]^{\mathrm{T}}$.

5. CONCLUSIONS

The global exponential stability of parabolic neural networks with variable coefficients and time-varying delays has been studied. Some stability criteria, which are independent of the delay parameter, have been derived by employing delay differential inequality and Young inequality. The conditions given in this paper are less restrictive, general and conservative. Moreover, the approaches presented in this paper can be applied to some other neural networks, such as neural networks with reaction-diffusion terms, robust neural networks, and Cohen-Grossberg neural networks.

REFERENCES

Arik S. (2000). Stability analysis of delayed neural networks. *IEEE Trans. Circ. Syst. I*, **47**, 1089-1092.

Brzychczy S. (2002). Existence of Solutions of Nonlinear Infinite Systems of Parabolic Differential-Functional Equations. *Mathematical and Computer Modelling*, **36**, 435-443.

Cao J. (2001). Global stability conditions for delayed CNNS. *IEEE Trans. Circuits Syst. I*, **48**, 1330-1333.

Cao J. and Wang J. (2003). Global asymptotic stability of a general class of recurrent neural networks with time-varying delays. *IEEE Trans. Circuits Syst. I*, **50**, 34-44.

Cui B.T. and Lou X.Y. (2006). Global asymptotic stability of BAM neural networks with distributed delays and reaction-diffusion terms. *Chaos, Solitons and Fractals*, **27**, 1347-1354.

Gopalsamy K. (1992). *Stability and Oscillations in Delay Differential Equations of Population Dynamics*, Kluwer Academic Publishers, Dordrecht.

Hardy G.H., Littlewood J.E. and Polya G. (1952). *Inequalities*, Cambridge University Press, London.

Joy M. (1999). On the global convergence of a class of functional differential equations with applications in neural network theory. *J. Math. Anal. Appl.*, **232**, 61-81.

Leiva H. and Sequera I. (2003). Existence and stability of bounded solutions for a system of parabolic equations. *J. Math. Anal. Appl.*, **279**, 495-507.

Li W.N and Cui B.T. (2001). Necessary and sufficient conditions for oscillation of neutral delay parabolic differential equations. *Applied Mathematics and Computation*, **121**, 147-153.

Li X.M., Huang L.H. and Zhu H. (2003). Global stability of cellular neural networks with constant and variable delays. *Nonlinear Anal.*, **53**, 319-333.

Liao T.L. and Wang F.C. (1999). Global stability condition for cellular neural networks with delay. *Electron. Lett.*, **35**, 1347-1359.

Lou X.Y. and Cui B.T. (2006). New LMI conditions for delay-dependent asymptotic stability of delayed Hopfield neural networks. *Neurocomputing*, **69**, 2374-2378.

Minchev E. and Yoshida N. (2003). Oscillation of solutions of nonlinear parabolic equations via comparison method. *Applied Mathematics and Computation*, **134**, 561-566.

Wang P.G. and Teo K.L. (2005). Oscillation of solutions of parabolic differential equations of neutral type. *J. Math. Anal. Appl.*, **311**, 615-625.

Zhang H.B., Li C.G. and Liao X.F. (2005). A note on the robust stability of neural networks with time delay. *Chaos, Solitons and Fractals*, **25**, 357-360.