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On Networked Control Architectures for MIMO Plants

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Abstract: In this work we focus on control of MIMO LTI plants and explore the potential benefits of replacing a traditional diagonal non-networked control architecture with a networked full MIMO one. Diagonal terms of the networked MIMO architecture employ transparent links, whereas the off-diagonal terms use communication channels which are subject to signal-to-noise ratio constraints. Within this setup, we show how to design LTI coding systems which optimize overall performance. Unsurprisingly, for high-quality channels the full MIMO architecture is preferable to decentralized architectures. However, our analysis reveals that the achievable performance in the networked situation may become arbitrarily poor, if the signal-to-noise ratio constraints in the communication links are sufficiently severe. In these cases, traditional decentralized controller structures are preferable. In the present work, we limit our analysis to the two-input two-output case and illustrate our results for networked control systems with bit-rate limited communication channels.

Keywords: Control over networks, control under communication constraints, decentralization.

1. INTRODUCTION

Practical control systems often use structurally constrained controllers such as diagonal or triangular ones (see, e.g., Salgado and Conley (2004); Skogestad and Postlethwaite (1996)). The reasons for this choice are manyfold and include ease of design, simplified tuning, and implementation related issues such as cabling or geographic plant distribution. Although it is well known that restricting the controller architecture may constrain the achievable performance (see, e.g., Goodwin et al. (2005); Kariwala (2007); Silva et al. (2007)), there exist situations where the implementation of centralized controllers is not feasible. For this reason, there has been ongoing interest in the design of decentralized control systems (see, e.g., Sandell et al. (1978); Hovd and Skogestad (1994); Campo and Morari (1994); Sourlas and Manousiousthakis (1995); Gündeş and Kabuli (2001); Rotkowitz and Lall (2006)). We note, however, that this is not an easy task. Even basic notions such as stability become non trivial in a decentralized framework (Wang and Davidson (1973)).

With the development of modern communication networks, low level networked control loops have become a reality (see, e.g., Antsaklis and Baillieul (2007); Hespanha et al. (2007)). The topic of networked control systems (NCS's) is a research area that has received significative attention during the last years. In fact, many successful controller design procedures that take communication constraints into account, have been proposed (see, e.g., Nair et al. (2007); Schenato et al. (2007) and the references in Hespanha et al. (2007)).

In the context of decentralized control systems, networks can play significative roles. Indeed, it is easy to envisage decentralized control architectures that, when enriched with additional communication links, may provide enhanced performance. This may (partially) overcome the limitations that arise as a consequence of the controller structure constraint. For example, Ishii and Francis (2002) showed that the set of plants that are stabilizable by decentralized architectures can be enlarged by means of appropriate communication resources usage. Also, Rawlings and Stewart (2007) have provided evidence that, by exploiting the possibility of transmitting large packets (as made feasible by modern networks such as Ethernet), one can recover centralized performance in an architecture where different agents communicate over a network. Other relevant results can be found in Yüksel and Başar (2007); Matveev and Savkin (2005); Jiang and Voulgaris (2007).

In this paper, we consider the control of MIMO LTI systems for which a decentralized controller has been successfully designed. We investigate the possible benefits of enriching the decentralized control structure with additional communication channels. These channels allow one to implement full MIMO (i.e., centralized) controllers, where the off-diagonal terms communicate through non ideal channels. Within this setup, we show, for a given MIMO controller, how to design LTI coding systems that optimize overall performance. Unsurprisingly, for high-quality channels the networked MIMO architecture outperforms the decentralized one. However, an interesting finding is that, in some situations, the networked architecture will perform better than the decentralized one only if the channels are *extremely* reliable. Our work builds upon the ideas presented in Goodwin et al. (2008).

The remainder of this paper is organized as follows. Section 2 presents the networked architecture of interest. Section 3 provides analysis guidelines. Section 4 shows how to synthesize optimal coding systems. Illustrative examples are provided in Section 5. Section 6 draws conclusions.

Notation: We use standard vector space notation for signals, i.e., x denotes $\{x(k)\}_{k\in\mathbb{N}_0}$. We also use z as both the argument of the z-transform and as the forward shift operator, where the meaning is clear from the context. Given any matrix, $(\cdot)^H$ and $(\cdot)^T$ denote conjugate transposition and transposition, respectively. Given any complex scalar, $|\cdot|$ denotes magnitude. I refers to the identity matrix and ε_i to the i-th element of the canonical basis in $\mathbb{R}^n$.

The set of all $n\times m$ proper real rational transfer functions is denoted by $\mathcal{R}^{n\times m}$. The subset of $\mathcal{R}^{n\times m}$ containing all stable minimum phase and biproper transfer functions is denoted by $\mathcal{U}_\infty^{n\times m}$. Every $A(z)\in\mathcal{R}^{n\times m}$ with no poles on the unit circle belongs to $\mathcal{L}_2^{n\times m}$ (see, e.g., Morari and Zafiriou (1989)). If this is the case, then we define the 2−norm of $A(z)$ via

$$||A(z)||_2^2 \triangleq \operatorname{trace}\left\{\frac{1}{2\pi}\int_{-\pi}^{\pi} A(e^{j\omega})^H A(e^{j\omega})d\omega\right\}.$$

2. PARTLY NETWORKED MIMO CONTROL

As outlined in the Introduction, we will consider the control of a plant model, $G(z)\in\mathcal{R}^{n\times n}$, for which a decentralized controller is available. We are interested in exploring the possible benefits of replacing this control architecture by a networked full MIMO one. For simplicity, we consider $n = 2$ in the remainder of this paper. We assume that an admissible [1] full MIMO controller, say

$$C(z) = \begin{bmatrix} C_{11}(z) & C_{12}(z) \\ C_{21}(z) & C_{22}(z)\end{bmatrix} \in \mathcal{R}^{2\times 2},$$

has already been designed for $G(z)$. The diagonal terms of this controller are implemented without communication constraints, but the off-diagonal terms communicate using non transparent communication links. We will refer to this control architecture as a *partially networked* one.

We will focus on a situation where the non transparent communication links comprise a perfect reconstruction coder-decoder pair [2], and a fixed signal-to-noise ratio additive noise channel (see Fig. 1). In that figure, $F_i(z)\in\mathcal{R}^{1\times 1}$ is the i-th ($i\in\{1,2\}$) coder transfer function, v_i is the i-th channel input and w_i is the i-th channel output. These signals are related via

$$w_i = v_i + q_i,$$

where q_i is the i-th channel noise. Each noise sequence is considered white, having variance $0\le\sigma_i^2<\infty$ and power spectral density $\Sigma_i(e^{j\omega}) = \sigma_i^2$, $\forall\omega\in[-\pi,\pi]$. A key feature of our model is that each channel has a *fixed* signal-to-noise ratio. This means that σ_i^2 is not a given constant, but is proportional to the variance of the channel input (namely, proportional to $\sigma_{v_i}^2$). We define the associated i-th channel signal-to-noise ratio as

$$\gamma_i \triangleq \frac{\sigma_{v_i}^2}{\sigma_i^2}\in\mathbb{R}_0^+. \tag{1}$$

[1] i.e., an internally stabilizing controller that defines a well possed control loop (see,e.g., Zhou et al. (1996); Goodwin et al. (2001)).

[2] This guarantees that, save for the additive channel noise and the signal-to-noise ratio constraint, the communication links are transparent.

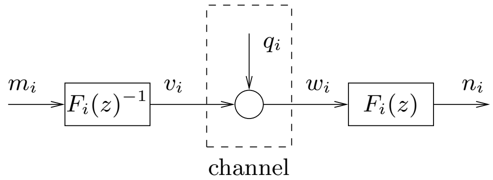

Fig. 1. i-th communication link.

We also assume that both channel noises are mutually uncorrelated.

The channel model described above has been used very successfully in the signal processing literature to model bit rate limited channels (see, e.g., Jayant and Noll (1984); Schreier and Temes (2004)). It has also been applied to the study of NCS architectures, as described in Xiao et al. (2003); Goodwin et al. (2008).

The NCS which results from employing the links described above to implement the off-diagonal terms of $C(z)$, can be visualized as in Fig. 2. In that figure, $u = [u_1\ u_2]^T$ is the plant input, $y = [y_1\ y_2]^T$ is the plant output, $r = [r_1\ r_2]^T$ is the reference sequence, and $e = [e_1\ e_2]^T$ denotes the tracking error, i.e.,

$$e \triangleq r - y.$$

In the remainder of this paper we will show how to choose $F_1(z)$ and $F_2(z)$ so as to minimize the variance of the tracking error. To that end, we will assume that the reference r is a zero mean wide sense stationary process, uncorrelated to q, having power spectral density $\Sigma_r(e^{j\omega}) \triangleq \Omega_r(e^{j\omega})\Omega_r(e^{j\omega})^H$, where $\Omega_r(z)\in\mathcal{R}^{2\times 2}$ is a stable spectral factor.

3. ANALYSIS

This section provides analysis guidelines for the NCS architecture described in Section 2. As a byproduct, we will show that, in some cases of interest, the advantages of a full MIMO controller design can be void due to the communication constraints that appear in the partially networked implementation.

From Fig. 2 it follows that the tracking error e satisfies

$$e = S(z)r - S_d(z)F(z)q, \tag{2}$$

where

$$F(z) \triangleq \operatorname{diag}\{F_1(z), F_2(z)\}$$

and

$$S(z) \triangleq (I + G(z)C(z))^{-1}, \quad S_d(z) \triangleq S(z)G(z).$$

Given the model for q and r, (2) implies that the variance of the tracking error is given by

$$\sigma_e^2 = ||S(z)\Omega_r(z)||_2^2 + \sum_{i=1}^{2}\sigma_i^2\,||S_d(z)\varepsilon_i F_i(z)||_2^2, \tag{3}$$

where σ_i^2, $i\in\{1,2\}$, depends on v (recall (1)). From Fig. 2 one has that v_1 and v_2 satisfy

$$v_1 = F_1(z)^{-1}C_{12}(z)\varepsilon_2^T\left(S(z)r - S_d(z)F(z)q\right),$$
$$v_2 = F_2(z)^{-1}C_{21}(z)\varepsilon_1^T\left(S(z)r - S_d(z)F(z)q\right).$$

It thus follows that

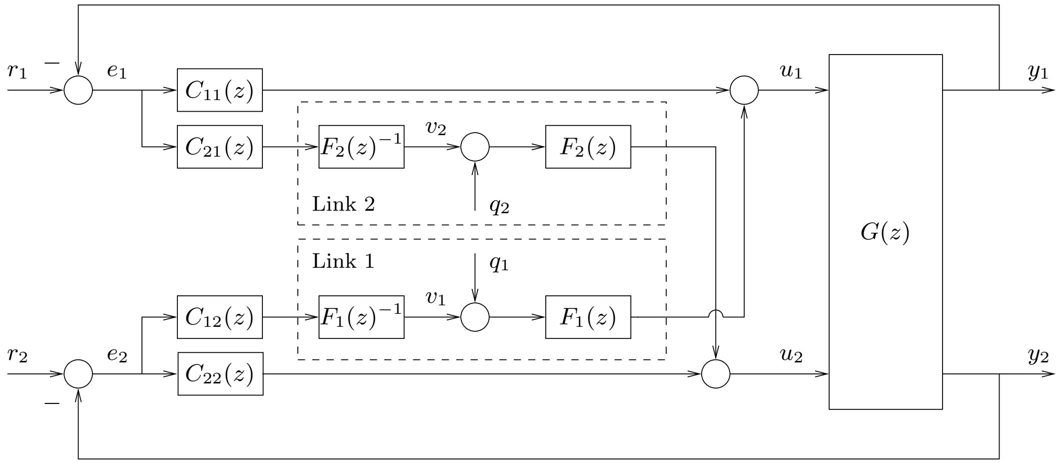

Fig. 2. Partly networked MIMO control architecture.

$$\frac{\sigma_{v_i}^2}{\sigma_i^2} = \frac{||A_i(z)\Omega_r(z)||_2^2}{\sigma_i^2} + \sum_{j=1}^{2} \frac{\sigma_j^2}{\sigma_i^2} ||A_i(z)G(z)F(z)\varepsilon_j||_2^2, \tag{4}$$

where

$$A_1(z) \triangleq F_1(z)^{-1}C_{12}(z)\varepsilon_2^T S(z),$$
$$A_2(z) \triangleq F_2(z)^{-1}C_{21}(z)\varepsilon_1^T S(z).$$

Equation (4) allows one to establish conditions that guarantee the mean square stability (MSS) of the NCS under study (i.e., conditions that guarantee that the variance matrices of the state vectors of all the involved systems are positive semi-definite and remain bounded as time goes to infinity):

Theorem 1. (MSS). The NCS described above is MSS if and only if both $F_1(z)$ and $F_2(z)$ belong to $\mathcal{U}_\infty^{1\times 1}$,

$$\gamma_1 > B_1 \triangleq \left|\left|C_{12}(z)\varepsilon_2^T S_d(z)\varepsilon_1\right|\right|_2^2, \tag{5}$$
$$\gamma_2 > B_2 \triangleq \left|\left|C_{21}(z)\varepsilon_2^T S_d(z)\varepsilon_2\right|\right|_2^2, \tag{6}$$

and

$$(\gamma_1 - B_1)(\gamma_2 - B_2) > ||A_1(z)G(z)F(z)\varepsilon_2||_2^2 \, ||A_2(z)G(z)F(z)\varepsilon_1||_2^2. \tag{7}$$

Proof. Since $\Omega_r(z)$ is stable, we have that MSS is equivalent to having an internally stable feedback system (in the standard sense) and $0 \le \sigma_i^2 < \infty$, $i \in \{1,2\}$ (see, e.g., Söderström (1994)). Since $C(z)$ is assumed to be an admissible controller for $G(z)$, internal stability is equivalent to $F_1(z), F_2(z) \in \mathcal{U}_\infty^{1\times 1}$ (see, e.g., Goodwin et al. (2001)). It remains to prove that (5)-(7) are equivalent to $0 \le \sigma_i^2 < \infty$, $i \in \{1,2\}$.

Equating (4) to γ_i, and using the definition of $A_i(z)$, one obtains that

$$\gamma_i - B_i = \frac{||A_i(z)\Omega_r(z)||_2^2}{\sigma_1^2} + \sum_{\substack{j=1\\ j\neq i}}^{2} \frac{\sigma_j^2 \, ||A_i(z)G(z)F(z)\varepsilon_j||_2^2}{\sigma_i^2}. \tag{8}$$

Solving (8) for σ_i^2, it follows that

$$\sigma_i^2 = \Delta^{-1}\left(\left(\gamma_j - ||A_j(z)G(z)F(z)\varepsilon_j||_2^2\right)||A_i(z)\Omega_r(z)||_2^2 + ||A_j(z)\Omega_r(z)||_2^2 \, ||A_i(z)G(z)F(z)\varepsilon_j||_2^2\right), \tag{9}$$

where $j \in \{1,2\}$, $j \neq i$, and

$$\Delta = \left(\gamma_1 - \left|\left|C_{12}(z)\varepsilon_2^T S_d(z)\varepsilon_1\right|\right|_2^2\right) \times \left(\gamma_2 - \left|\left|C_{21}(z)\varepsilon_2^T S_d(z)\varepsilon_2\right|\right|_2^2\right) - ||A_1(z)G(z)F(z)\varepsilon_2||_2^2 \, ||A_2(z)G(z)F(z)\varepsilon_1||_2^2. \tag{10}$$

Using the definition of $A_i(z)$, the result follows directly from (8)-(10). ■

It is illustrative to note that, as long as $F_1(z), F_2(z) \in \mathcal{U}_\infty^{1\times 1}$, condition (5) (resp. (6)) is necessary and sufficient for MSS when $q_2 = 0$, i.e., when $\gamma_2 \to \infty$ (resp. when $\gamma_1 \to \infty$). This means that conditions (5)-(6) arise when the channels do not interact. On the other hand, for finite γ_i's both channels interact through the plant and controller and hence, (5)-(6) are no longer sufficient for MSS, and (7) is required. Indeed, if both $C(z)$ and $G(z)$ are anti-diagonal (and hence the channels do not interact), then (7) is trivially satisfied if (5)-(6) hold.

An immediate consequence of Theorem 1 is the following:

Corollary 2. (Arbitrary poor performance). Consider the NCS described above with $F_1(z), F_2(z) \in \mathcal{U}_\infty^{1\times 1}$. Assume that (5)-(7) are satisfied and define

$$\mathcal{S} \triangleq \left\{(\gamma_1,\gamma_2) \in \mathbb{R}^2 : \ \gamma_1 \text{ and } \gamma_2 \text{ achieve equality in (7)}\right\}.$$

Then,

$$\lim_{(\gamma_1,\gamma_2)\to(\bar{\gamma}_1,\bar{\gamma}_2)} \sigma_e^2 = \infty$$

for any $C(z)$, and any choice for $F_1(z)$ and $F_2(z)$.

Proof. Immediate from the definition of $\mathcal{S}$ and expressions (3) and (9). ■

Although very simple, Corollary 2 has an interesting implication: For any given full MIMO controller, and no matter how the coding system is chosen, there exist sufficiently poor channels which render the performance of the resulting partially networked closed loop arbitrary bad. In these cases, *any* stabilizing decentralized controller (that makes no use of the non-transparent channels) will provide better performance. As will become clear in Section 5, the channel signal-to-noise ratios do not need to be artificially low for the MIMO controller to perform poorly. Indeed, depending on plant and controller features, the left hand side of (7) may be large, thus requiring high signal-to-noise ratios to be satisfied.

4. OPTIMAL CODER DESIGN

In this section we show how to design optimal coders $F_1(z)$ and $F_2(z)$ under a mild simplifying assumption.

From (8) one can immediately conclude that, provided (5)-(7) are satisfied,

$$\sigma_i^2 \geq \frac{||A_i(z)\Omega_r(z)||_2^2}{\gamma_i - B_i} \geq 0.$$

As a consequence (recall (3)),

$$\sigma_e^2 \geq ||S(z)\Omega_r(z)||_2^2 + \sum_{i=1}^{2} \frac{||A_i(z)\Omega_r(z)||_2^2 \, ||S_d(z)\varepsilon_i F_i(z)||_2^2}{\gamma_i - B_i}. \quad (11)$$

If γ_1 and γ_2 are high enough, then the bound in (11) will be tight. If this is the case, then the coders that minimize σ_e^2 also minimize

$$J \triangleq \sum_{i=1}^{2} \frac{||A_i(z)\Omega_r(z)||_2^2 \, ||S_d(z)\varepsilon_i F_i(z)||_2^2}{\gamma_i - B_i}.$$

We will denote the coders that minimize J by $F_1^o(z)$ and $F_2^o(z)$, i.e.,

$$(F_1^o(z), F_2^o(z)) \triangleq \arg \min_{\substack{F_1(z)\in\mathcal{U}_\infty^{1\times 1} \\ F_2(z)\in\mathcal{U}_\infty^{1\times 1}}} J;$$

the corresponding (minimal) value of J will be denoted by J_o. Note that the constraints on $F_1(z)$ and $F_2(z)$ arise from MSS considerations (recall Theorem 1).

The next theorem characterizes $F_1^o(z)$, $F_2^o(z)$ and J_o.

Theorem 3. (Optimal Coders). The optimal coders $F_i^o(z)$, $i \in \{1, 2\}$, are such that

$$\left|F_i^o(e^{j\omega})\right|^4 = \alpha_i \frac{M_i(e^{j\omega})M_i(e^{j\omega})^H}{\left(S_d(e^{j\omega})\varepsilon_i\right)^H S_d(e^{j\omega})\varepsilon_i},$$

where

$$M_1(z) \triangleq C_{12}(z)\varepsilon_2^T S(z)\Omega_r(z),$$
$$M_2(z) \triangleq C_{21}(z)\varepsilon_1^T S(z)\Omega_r(z),$$

and α_i is an arbitrary positive constant. With this choice,

$$J_o = \sum_{i=1}^{2} \frac{\left(\int_{-\pi}^{\pi} D_i(e^{j\omega})d\omega\right)^2}{4\pi^2\left(\gamma_i - B_i\right)},$$

where

$$D_i(e^{j\omega}) \triangleq \sqrt{M_i(e^{j\omega})M_i(e^{j\omega})^H \left(S_d(e^{j\omega})\varepsilon_i\right)^H S_d(e^{j\omega})\varepsilon_i}\,.$$

Proof. The definition of the 2−norm allows one to conclude that, for every $X(z) \in \mathcal{L}_2^{n\times 1}$, the following identities hold:

$$||X(z)||_2^2 = \left|\left|X(z^{-1})^T\right|\right|_2^2 = \left|\left|\sqrt{X(z^{-1})^T X(z)}\right|\right|_2^2. \quad (12)$$

Equation (12) and the Cauchy Schwartz inequality imply that, for every $i \in \{1, 2\}$,

$$\begin{aligned} &4\pi^2 \, ||A_i(z)\Omega_r(z)||_2^2 \, ||S_d(z)\varepsilon_i F_i(z)||_2^2 \\ &= 4\pi^2 \left|\left|\sqrt{A_i(z)\Omega_r(z)\left(A_i(z^{-1})\Omega_r(z^{-1})\right)^T}\right|\right|_2^2 \times \\ &\left|\left|\sqrt{F_i(z^{-1})^T \left(S_d(z^{-1})\varepsilon_i\right)^T S_d(z)\varepsilon_i F_i(z)}\right|\right|_2^2 \\ &\geq \left(\int_{-\pi}^{\pi} \sqrt{M_i(e^{j\omega})M_i(e^{j\omega})^H \left(S_d(e^{j\omega})\varepsilon_i\right)^H S_d\varepsilon_i(e^{j\omega})}\, d\omega\right)^2, \end{aligned} \quad (13)$$

where we have used the definitions of $M_i(z)$ and $A_i(z)$. In (13), equality holds if and only if there exists $\alpha_i \in \mathbb{R}^+$ such that

$$\sqrt{A_i(e^{j\omega})\Omega_r(e^{j\omega})\left(A_i(e^{j\omega})\Omega_r(e^{j\omega})\right)^H} = \frac{1}{\sqrt{\alpha_i}}\sqrt{F_i(e^{j\omega})^H \left(S_d(e^{j\omega})\varepsilon_i\right)^H S_d(e^{j\omega})\varepsilon_i F_i(e^{j\omega})},$$

for every $\omega \in [-\pi, \pi]$. Since B_i does not depend on $F_1(z)$ or $F_2(z)$, both results are now immediate. ■

Theorem 3 shows how to synthesize coding systems that minimize the impact of the communication links on overall closed loop performance. An interesting feature of the proposed filters is that they do not depend on the channel signal-to-noise ratios. This allows one to conjecture that the optimal filters will perform well for a large class of communication channels. Moreover, our result opens the possibility of investigating optimal communication resource allocation schemes, as discussed in, e.g., Xiao et al. (2003); Quevedo et al. (2007); Ishii and Hara (2006).

5. DESIGN STUDIES

This section illustrates the results in this paper with two examples that employ bit rate limited channels. We assume that each channel is able to transmit b_i bits per sampling interval, and that a b_i-bit uniform quantizer is employed to quantize v_i prior to transmission. In this context, it is well known that one can use the model described in Section 2 to model quantization noise, with a channel signal-to-noise ratio given by

$$\gamma_i = k_i^{-1}\left(2^{b_i} - 1\right)^2. \quad (14)$$

(see, e.g., Jayant and Noll (1984); Goodwin et al. (2008); Xiao et al. (2003)). In (14), k_i is a constant that depends on the assumed statistics of v_i and on the quantizer parameters. We choose a quantizer dynamic range equal to $\alpha\sigma_{v_i}$,[3] and assume that v_i is Gaussian, that overflow is rare, and that q_i is uniformly distributed. Under these conditions, $k_i = \alpha^2/3$ (see Jayant and Noll (1984) for the details).

For simplicity, we consider channels with equal bit rates, i.e., $b_1 = b_2 = b$,[4] and take the sampling interval as $1[s]$ (in both channels). In the sequel, we will consider

[3] Usually, $\alpha = 4$ (Jayant and Noll (1984)).

[4] One can envisage alternative bit allocation schemes: For example, given a total budget of b_T bits, one can choose b_1 and b_2 so as to minimize J, while respecting $b_T = b_1 + b_2$ (we refer the reader to Jayant and Noll (1984) for details).

fractional bit rates to illustrate our theoretical analysis. We also include simulation results that use actual b-bit uniform quantizers in both channels and, of course, only integer bit rates are then considered.

Example 4. We first consider the following simple plant model:

$$G_1(z) = \begin{bmatrix} \frac{0.6}{(z-0.8)} & \frac{0.4}{(z-0.8)} \\ \frac{1}{(z-0.5)} & \frac{1}{(z-0.5)} \end{bmatrix}.$$

For this plant we synthesize the decentralized controller

$$C_{d1}(z) = \begin{bmatrix} \frac{1.3333(z-0.8)}{(z+0.8)(z-1)} & 0 \\ 0 & \frac{0.8(z-0.5)}{(z+0.8)(z-1)} \end{bmatrix},$$

and the full MIMO controller

$$C_1(z) = \begin{bmatrix} \frac{5(z-0.8)}{z-1} & \frac{-2(z-0.5)}{z-1} \\ \frac{-5(z-0.8)}{z-1} & \frac{3(z-0.5)}{z-1} \end{bmatrix}.$$

We also assume that the reference description is given by

$$\Omega_r(z) = \frac{0.0049627(z+0.9934)}{(z^2-1.97z+0.9802)} I,$$

and that $\alpha = 4$.

Fig. 3 shows the tracking error variance in the partially networked MIMO architecture as a function of the per-channel bit-rate b in several situations: "Analytical no coding" refers to the performance predicted by (3) and (9) when no coding is employed; "Empirical no coding" refers to simulated[5] performance when no coding is considered; "Analytical opt. coding" and "Empirical opt. coding" refer to analytical and simulated performance when the coders suggested by Theorem 3 are employed. For comparison purposes, Fig. 3 also shows the non networked full MIMO performance ("Ideal full MIMO") and the performance achieved when using $C_{d1}(z)$ ("Decentralized"). The results allow one to conclude that, in this case, the benefits of coding are significative. Indeed, for $b = 4$ and no coding, the performance achieved by the partially networked MIMO controller is more than 5 times worse that the ideal full MIMO performance. On the other hand, if optimal coding is employed, then the performance deterioration is only 3.5%. We also note in passing that the match between the predictions of our model and the results of non idealized simulations is remarkably good.

As expected, non-networked full MIMO performance is recovered as $b \to \infty$, irrespective of the coders used. Moreover, it is apparent that the non coded networked full MIMO architecture should be preferred to the decentralized one for $b > 3.36$. On the other hand, the optimally coded networked MIMO architecture provides significant improvement in performance for $b > 3.20$, when compared to the decentralized architecture. It is also interesting to note that, if $b \to 3.07$, then the performance becomes arbitrary poor no matter what the coding is. This is consistent with Corollary 2 as straightforward calculations reveal. □

We next examine a second example, where the plant is much harder to control.

[5] The simulations use actual b-bit uniform quantizers in both channels. For each b the results correspond to the average of 200 simulations (each one 10^5 samples long and using a different reference realization).

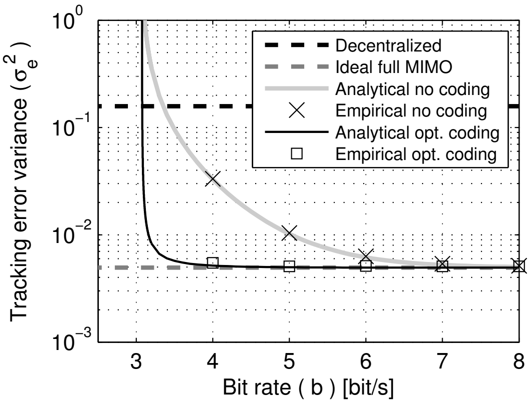

Fig. 3. Tracking error variance as a function of the per-channel bit rate (Example 4).

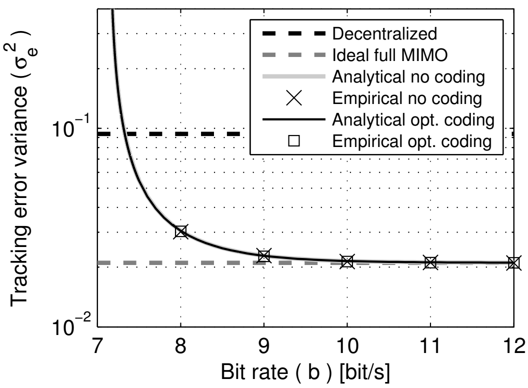

Fig. 4. Tracking error variance as a function of the per-channel bit rate (Example 5).

Example 5. (Distillation process). We consider a classical 2×2 model of a distillation process, as described in Skogestad and Postlethwaite (1996). Assuming properly scaled variables and a zero order hold at the plant input, it is possible to derive the following discrete time model:

$$G_2(z) = \frac{1}{z-0.9868} \begin{bmatrix} 1.1629 & -1.1444 \\ 1.4331 & -1.4516 \end{bmatrix}.$$

Although simple in appearance, this plant is extremely difficult to control. This is due to the fact that it is almost singular (Skogestad and Postlethwaite (1996)).

For the full MIMO controller we use

$$C_2(z) = \frac{z-0.9868}{z-1} \begin{bmatrix} 30.1564 & -23.7729 \\ 29.7712 & -24.1582 \end{bmatrix},$$

and for the decentralized controller we use

$$C_{d2}(z) = \begin{bmatrix} \frac{0.261(z-0.734)}{(z-1)} & 0 \\ 0 & \frac{-0.375(z-0.6979)}{(z-1)} \end{bmatrix}.$$

We note that $C_{d2}(z)$ is a discretized version of a carefully tuned MIMO PI controller for $G_2(z)$, as proposed in Skogestad and Postlethwaite (1996). The reference description is assumed to be given by $\Omega_r(z) = 0.1(z-0.9)^{-1} I$, and we take $\alpha = 5$.[6]

[6] For this plant, saturation may compromise loop stability even when transparent channels are employed. To avoid this, we extended the quantizer dynamic range so that quantizer overflow (i.e., saturation) probability becomes completely negligible.

Fig. 4 shows the partially networked MIMO tracking error variance as a function of the per-channel bit-rate in the same cases as those in Figure 3. The same qualitative behavior as in Example 4 is observed. However, the performance gains arising from coding are negligible in this case. This is due to the fact that the optimal coders have an almost flat frequency response.

An interesting feature of this example is that the performance achieved by the networked full MIMO controller may become significatively worse than the decentralized performance for relatively high bit rates (namely, for $b \to 7.15$). Again, this is consistent with Corollary 2 and can be easily visualized when looking at (7). In this case, (7) leads to $(\gamma_1 - 1.1607 \cdot 10^3)(\gamma_2 - 1.1607 \cdot 10^3) > 1.4274 \cdot 10^6$, i.e., only relative high signal-to-noise ratios (bit rates) are allowable. □

6. CONCLUSIONS

This paper has studied a partially networked MIMO control architecture aimed at improving the performance of standard decentralized control loops. In this context, and assuming a given MIMO controller design, we have shown how to design LTI coding systems that optimize overall closed loop performance. An interesting byproduct of our analysis is that, in some cases, the additional channels need to be of high quality if the partially networked architecture is to outperform a given decentralized design. This opens the question of how to actually design partially networked MIMO controllers. This is the subject of future study, together with extensions to the $n \times n$ case.

REFERENCES

P. Antsaklis and J. Baillieul. Special issue on technology of networked control systems. *Proc. IEEE*, 95(1):5–8, January 2007.

P. Campo and M. Morari. Achievable closed-loop properties of systems under decentralized control: Conditions involving the steady-state gain. *IEEE Trans. on Automatic Control*, 39(5):932–943, May 1994.

G.C. Goodwin, S.F. Graebe, and M.E. Salgado. *Control System Design.* Prentice Hall, New Jersey, 2001.

G.C. Goodwin, M.E. Salgado, and E.I. Silva. Time-domain performance limitations arising from decentralized architectures and their relationship to the RGA. *Int. J. Contr.*, 78(13):1045–1062, September 2005.

G.C. Goodwin, D.E. Quevedo, and E.I. Silva. Architectures and coder design for networked control systems. *Automatica*, 44(1):248–257, 2008.

A.N. Gündeş and M.G. Kabuli. Reliable decentralized integral action controller design. *IEEE Trans. on Automatic Control*, 46(2):296 – 301, 2001.

J.P. Hespanha, P. Naghshtabrizi, and Y. Xu. A survey of recent results in networked control systems. *Proc. IEEE*, 95(1):138–162, January 2007.

M. Hovd and S. Skogestad. Sequential design of decentralized controllers. *Automatica*, 30:1601–1607, 1994.

H. Ishii and B.A. Francis. Stabilization with control networks. *Automatica*, 38:1745 – 1751, 2002.

H. Ishii and S. Hara. A subband coding approach to networked control. In *Proc. 17th Int. Symp. Mathematical Theory of Networks and Systems*, Kyoto, Japan, 2006.

N. Jayant and P. Noll. *Digital Coding of Waveforms. Principles and Approaches to Speech and Video.* Prentice Hall, 1984.

S. Jiang and P.G. Voulgaris. Cooperative control over link limited abd packet dropping networks. In *Proc. of the European Control Conf.*, 2007.

V. Kariwala. Fundamental limitation on achievable decentralized performance. *To appear in Automatica*, 2007.

A.S. Matveev and A.V. Savkin. Decentralized stabilization of linear systems via limited capacity communication networks. In *Proc. of the 44th IEEE CDC and the ECC 2005*, pages 1155–1161, Sevilla, Spain, 2005.

M. Morari and E. Zafiriou. *Robust process control.* Prentice Hall, 1989.

G. Nair, F. Fagnani, S. Zampieri, and R. Evans. Feedback control under data rate constraints: An overview. *Proc. IEEE*, 95(1):108–137, January 2007.

D.E. Quevedo, E.I. Silva, and G. C.Goodwin. Performance limits in multi-channel networked control system architectures. In *3rd IFAC SSSC (to appear)*, 2007.

J.B. Rawlings and B.T. Stewart. Coordinating multiple optimization based controllers: New opportunities and challenges. In *Proc. of the 8th Int. IFAC DYCOPS*, Cancún, México, June 2007.

M. Rotkowitz and S. Lall. A characterization of convex problems in decentralized control. *IEEE Trans. on Automatic Control*, 51(2):274–286, 2006.

M.E. Salgado and A. Conley. MIMO interaction measure and controller structure selection. *Int. J. Contr.*, 77(4): 367–383, March 2004.

N. Sandell, P. Varaiya, M. Athans, and M. Safonov. Survey of decentralized control methods for large scale systems. *IEEE Trans. on Automatic Control*, 23(2):108–128, 1978.

L. Schenato, B. Sinopoli, M. Franceschetti, K. Poolla, and S. S. Sastry. Foundations of control and estimation over lossy networks. *Proc. IEEE*, 95(1), January 2007.

R. Schreier and G.C. Temes. *Understanding Delta Sigma Data Converters.* Wiley-IEEE Press, 2004.

E.I. Silva, D.A. Oyarzún, and M.E. Salgado. On structurally constrained $\mathcal{H}_2$ performance bounds for stable MIMO plant models. *IET Control Theory & Applications*, 1(4):1033–1045, 2007.

S. Skogestad and I. Postlethwaite. *Multivariable Feedback Control: Analysis and Design.* Wiley, New York, 1996.

T. Söderström. *Discrete-time stochastic systems.* Prentice Hall, 1994.

D. Sourlas and V. Manousiousthakis. Best achievable decentralized performance. *IEEE Trans. on Automatic Control*, 40(11):1858–1871, November 1995.

S.H. Wang and E.J. Davidson. On the stabilization of decentralized control systems. *IEEE Trans. on Automatic Control*, 18(5):473–478, October 1973.

L. Xiao, M. Johansson, H. Hindi, S. Boyd, and A. Goldsmith. Joint optimization of communication rates and linear systems. *IEEE Trans. on Automatic Control*, 48 (1):148–153, January 2003.

S. Yüksel and T. Başar. Communication constraints for decentralized stabilizability with time-invariant policies. *IEEE Trans. on Automatic Control*, 52(6):1060–1066, 2007.

K. Zhou, J.C. Doyle, and K. Glover. *Robust and optimal control.* Prentice Hall, Englewood Cliffs, N.J., 1996.