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Dynamic Target Tracking Control for a Wheeled Mobile Robots Constrained by Limited Inputs

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Abstract: This paper focuses on the controller design for a wheeled mobile robot to track a moving target when its control inputs (linear and angular velocities) are limited. This issue is significant in practice, as the unpredicted target motions may cause the control actions beyond the robot's capability. In the paper, after the system model is formulated in a form suitable for the controller design, a Lyapunov function considering the limits of the robot's control efforts into consideration is proposed. A control law setting the robot's linear and angular velocities is then obtained. The conditions for asymptotic target tracking by the robot are also established. Simulation results are provided to verify the effectiveness of the approach.

Keywords: Mobile Robot; Control.

1. INTRODUCTION

It is well known that a wheeled mobile robot is a typical non-holonomic system which cannot be stabilized through a time unvarying linear feedback [1]. Many approaches were proposed to address this challenge. The most popular method is to treat a wheeled mobile robot as a special case of general non-honolomic chained systems, for which different control algorithms have been developed. These include smooth time varying or discontinuous controls [2][3][4][5][6][7], nonlinear feedback control[8][9][10], optimal control [11] and fuzzy control[12]. The control objective is to make the robot to achieve the desired posture required by the task, e.g., achieving a desired destination or tracking a target. Those methods are normally complex in structure and are computationally intensive.

In some researches, the robot state is described in polar coordinates, from which a system model suitable for the controller design is set up[13][14]. The polar coordinates contain all the information regarding the robot's posture relative to the target or the desired path. The controller design is mostly based on Lyapunov theory or potential field method. The shortcoming common to those approaches lies in the assumption that the target (or the desired path) is static and known in prior, and the robot can accept any control input. Such an assumption is too idealistic in practice. For example, the robot's actuators can only generate the torques within their limits. The trajectories of the target are normally unknown beforehand, and they may impose too demanding requirements on the control efforts for the robot to track the target. In [15], a potential field method is used to control an omni-wheeled mobile robot to track a moving target. The potential function is defined as a weighted sum of quadratic functions of the robot's position and velocity relative to the target. The robot and the target are simply modeled as point masses and the whole system is holonomic. By summing up the terms of different physical quantities (position and velocity) with different units, the physical meaning of the potential function disappears. The weight allocation to each term becomes difficult to be justified and may easily cause instability of the system. For example, consider the robot near the target when system potential is supposed to be low and the robot slows down from our common sensor, any error between the robot's velocity and that of the target will still keep the potential high and cause the robot continue to move with chattering or other forms of discontinuity. There is no consideration on the limit of the robot's control input in the approach. In [16][17], the traditional field method are extended for a differential wheel driven mobile robot to track a moving target. The limit on the control efforts are not considered. In [18], the limit on the robot's angular speed is considered in the controller design for a robot following a wall, a special case of target tracking by a robot. The limit on the robot's linear velocity and extending the approach to more general target tracking task are not considered.

In this paper, an approach is proposed to control the wheeled robot to track a moving target while its control inputs are limited. The closed loop stability, allowable linear velocity and angular velocity and their relations are studied. The controller guarantees that the robot reach the target's position and direction asymptotically under certain conditions.

The paper is organized as follows. The system model and problem formulation are described in Section 2. Controller design is given in Section 3. Simulation examples for verification of the proposed controller are provided in Section 4. Conclusion is given in Section 5.

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2. SYSTEM MODEL AND PROBLEM FORMULATION

The robot system to be studied is depicted in Figure 1. The following notations are used for the system description. XOY: world coordinate in the workspace; $o(x, y)$: center of gravity of the robot and its Cartesian coordinates; v: linear speed of the robot; ω: angular speed of the robot; ϕ: angle of v; $o_t(x_t, y_t)$: position of the target and its Cartesian coordinates; v_t: linear speed of the target; ϕ_t: directional angle of v_t;ρ: the distance between the robot and the target; θ: the angle of the vector from o_r to o_t.

The robot's kinematics is described by:

$$\dot{x} = v\cos\phi$$
$$\dot{y} = v\sin\phi$$
$$\dot{\phi} = \omega$$
$$\|v\| \le v_{max} \tag{1}$$
$$\|\omega\| \le \omega_{max} \tag{2}$$

where $v_{max} > 0$ and $\omega_{max} > 0$ in the last two equations are the robot's maximum linear and angular speeds respectively.

The speeds of the left and the right wheels: ω_l and ω_r are related to the robot's speeds by:

$$v = \frac{(\omega_r + \omega_l)L}{2}, \quad \omega = \frac{(\omega_r - \omega_l)R}{L}$$

If the maximum wheel speed is ω_{wmax}, we have

$$v_{max} = L\omega_{wmax}, \quad v_{max} = \frac{2R}{L}\omega_{wmax}$$

and

$$v_{max} = \frac{L^2}{2R}\omega_{max}$$

Our task is to find u and ω to make the robot track the target, while they are limited by equations (1) and (2). For the controller design, the robot's posture and velocity relative to the target should be studied first.

The positions of the robot and the target are related by

$$\rho\cos\theta = x_t - x$$
$$\rho\sin\theta = y_t - y$$

To describe the angular relation between the robot and the target, the following variables are defined:

$$\alpha = \theta - \phi$$
$$\beta = \theta - \phi_t$$

Their derivatives with respect to time t are:

$$\dot{\rho}\cos\theta - \rho\dot{\theta}\sin\theta = v_t\cos\phi_t - v\cos\phi \tag{3}$$
$$\dot{\rho}\sin\theta + \rho\dot{\theta}\cos\theta = v_t\sin\phi_t - v\sin\phi \tag{4}$$
$$\dot{\alpha} = \dot{\theta} - \omega, \quad \dot{\beta} = \dot{\theta} - \dot{\phi}_t \tag{5}$$

From equations (3) to (5), we have,

$$\dot{\rho} = v_t\cos\beta - v\cos\alpha \tag{6}$$
$$\dot{\alpha} = v\frac{\sin\alpha}{\rho} - v_t\frac{\sin\beta}{\rho} - \omega, \quad \rho \ne 0 \tag{7}$$
$$\dot{\beta} = v\frac{\sin\alpha}{\rho} - v_t\frac{\sin\beta}{\rho} - \dot{\phi}_t, \quad \rho \ne 0 \tag{8}$$

This is the kinematic model of the system represented by new state variables ρ, α and β.

The original target tracking problem becomes the regulation of the system described by equations (6) to (8) by choosing proper u and ω.

3. CONTROLLER DESIGN

For the controller design, the following candidate Lyapunov function is defined:

$$V = \frac{1}{2}(\rho^2 + \alpha^2 + \beta^2)$$

Its derivative with respect to time t is

$$\dot{V} = \rho\dot{\rho} + \alpha\dot{\alpha} + \beta\dot{\beta} \tag{9}$$

From equations (6), (8) and (9), we have

$$\dot{V} = v_t(\rho\cos\beta - \frac{\beta+\alpha}{\rho}\sin\beta) - v(\rho\cos\alpha - \frac{\beta+\alpha}{\rho}\sin\alpha)$$
$$-\alpha\omega - \beta\dot{\phi}_t \tag{10}$$

By defining

$$\gamma \triangleq \arctan\frac{\beta+\alpha}{\rho^2}$$
$$\eta \triangleq \sqrt{\rho^2 + \frac{(\beta+\alpha)^2}{\rho^2}} = \frac{\rho}{\cos\gamma}$$

equation (10) is simplified:

$$\dot{V} = \eta(v_t\cos(\beta+\gamma) - v\cos(\alpha+\gamma)) - \alpha\omega - \beta\dot{\phi}_t \tag{11}$$

Letting

$$v = \cos(\alpha+\gamma)(\lambda_1(\eta)\eta + v_t(\cos(\beta+\gamma)) \tag{12}$$
$$\omega = \lambda_2(\alpha)\alpha - \dot{\phi}_t\frac{\beta}{\alpha} + \frac{\eta\sin(\alpha+\gamma)\tan(\alpha+\gamma)}{\alpha}v \tag{13}$$

and substituting them into equation (11), we have

$$\dot{V} = -\lambda_1(\eta)\eta^2 - \lambda_2(\alpha)\alpha^2$$

where $\lambda_1(\eta) > 0$ and $\lambda_2(\alpha)$ are the functions to be decided to make sure the control constraints in equations (1) and (2) are met. Obviously they will also affect the rate of the convergence of the tracking errors.

This shows that V is non-increasing. Given $V > 0$ as defined, V converges to a non-negative limit asymptotically. As a result, the state variables ρ, β and α are all bounded. This in turn makes $\dot{V}$ uniformly continuous. From Barblat's Lemma [19], $\dot{V} \to 0$, and accordingly, $\alpha \to 0$ and $\eta \to 0$.

From the definition of η, we have

$$\rho^2 + \frac{(\beta+\alpha)^2}{\rho^2} \to 0 \tag{14}$$

and $\rho \to 0$ and $\beta \to 0$ accordingly.

When $\alpha \to 0$, $\cos(\alpha+\gamma) \to \cos\gamma$. As defined in equation (11), $\cos\gamma > 0$. As such, equation (14) implies that

$$\rho^2 + \frac{(\beta+\alpha)^2}{\rho^2} \to 0$$

and $\rho \to 0$ and $\beta + \alpha \to 0$ accordingly. As $\alpha \to 0$, we can conclude that $\beta \to 0$.

The fact that $\beta \to 0$ can also be drawn from the basic formula $2ab \le a^2 + b^2$ (a and b are real numbers),

$$0 \le (\beta+\alpha)^2 \le \frac{1}{2}(\rho^2 + \frac{(\beta+\alpha)^2}{\rho^2})$$

As the both sides of $(\beta+\alpha)^2$ approximate zero, $\beta+\alpha \to 0$, or, $\beta \to 0$ given that $\alpha \to 0$.

To complete the controller design, the functions $\lambda_1(\eta) > 0$ and $\lambda_2(\alpha)$ are to be decided. To begin with, each control input can be decomposed into two parts, depending whether it is affected by $\lambda_1(\eta)$ or $\lambda_2(\alpha)$:

$$v = v_1 + v_2 \tag{15}$$
$$\omega = \omega_1 + \omega_2 \tag{16}$$
$$v_1 = \lambda_1(\eta)\cos(\alpha+\gamma)\eta$$
$$v_2 = v_t\cos(\alpha+\gamma)(\cos(\beta+\gamma))$$
$$\omega_1 = \lambda_2(\alpha)\alpha$$
$$\omega_2 = -\dot{\phi}_t\frac{\beta}{\alpha} + \frac{\eta\sin(\alpha+\gamma)\tan(\alpha+\gamma)}{\alpha}v$$

Obviously the terms v_1 and ω_1 can be varied with $\lambda_1(\eta)$ and $\lambda_2(\alpha)$ respectively. The terms of v_2 and ω_2 are contributed by the motion of the target.

Letting

$$\lambda_1(\eta) = \frac{k_1 v_{max}}{a_1 + |\eta|}, \quad 0 < k_1 \le 1 \tag{17}$$
$$\lambda_2(\alpha) = \frac{k_2 \omega_{max}}{a_2 + |\alpha|}, \quad 0 < k_2 \le 1 \tag{18}$$

where k_1 and k_2 are the constants in the interval (0, 1), and $a_1 > 0$ and $a_2 > 0$ are positive constants.

Substituting equations (17) and (18) into equations (15) and (16) respectively, we have

$$v = k_1 v_{max}\frac{\cos(\alpha+\gamma)\eta}{|\eta|} + v_2, \quad |\eta| \le k_1 v_{max} + |v2| \tag{19}$$
$$\omega = k_2\omega_{max}\frac{\alpha}{a_2 + |\alpha|} + \omega_2, \quad |\omega| \le k_2\omega_{max} + |\omega_2| \tag{20}$$

If the following conditions are met:

$$|v_2| < (1-k_1)v_{max} \quad \text{and} |\omega_2| < (1-k_2)\omega_{max} \tag{21}$$

it follows from equations (19) and (20) that

$$|v| \le v_{max} \quad \text{and} |\omega| \le \omega_{max}$$

In summary, under that conditions specified by equation (21), the limited control inputs derived in equations (12), (13), (17) and (18) can make the robot successfully track the moving target.

Remarks:

In practice, a modification to the controller is needed in either of the following cases:

Case 1: The angle tracking error α is close to zero. As α is the divisor in the some terms of the controller, this will cause the robot's angular velocity to reach an undesirable magnitude with chatterings. In this case, a minimum threshold of α should be set in the calculation.

Case 2: One or all the conditions in equation (21) are not met, and the magnitude of the robot's angular or linear velocity will exceed its maximum magnitude. When this happens, the input should be set at the level of its maximum magnitude.

Though, in theory the modification made in the above cases will compromise the performance of the controlled system, they are necessary in practice. Their affects on the stability of are also limited by the short period of their occurrence.

4. SIMULATION

In the simulation example, a wheeled robot is controlled to track a moving target whose trajectory is a circle defined by

$$x_t = 3 - 50\cos(0.01t), \quad y_t = 47 + 50\sin(0.01t), \tag{22}$$

By default, ISO units are used in all the measurements in the simulation, e.g, second for time, meter for distance and meter / second for speed. The angles are normalized between $-\pi$ and π. The center of circle is at (3, 47) and its radius is 50.

The target's speed and the angle are derived from equation (22), $v_t = 0.5$ and $phi_t = \frac{\pi}{2} - 0.01t$. At $t = 0$, the robot's posture are $x = 0$, $y = 0$ and $\phi = \pi$.

Through some calculations, we get the initial state of the system on which the controller is designed: $\rho = 47\sqrt{2} = 66.468$, $\alpha = -\frac{\pi}{4}$ and $\beta = \frac{\pi}{4}$.

The following controller parameters are selected: $\omega_{max} = 10$, $v_{max} = 4$,$k_1 = 0.15$, $k_2 = 0.10$, $a_1 = a_2 = 0.25$. we fix the control parameters $\lambda_1(\eta) = 0.25$ and $\lambda_2(\alpha) = 0.25$. The system responses are shown in Figures 2 to 4. The control inputs are plotted in Figures 5 and 6 respectively. The paths of the robot (dashed line) and the target (solid line) are plotted in Figure 7. It can be seen that the robot catches up with the target and alight itself with the target.

It can be observed that all the systems states ρ, α and β approximate to zero within 140 seconds. It is a good result considering the big range of the target's movements and the distance between the robot and the target at the beginning. At the initial stage, the robot's angular velocity exceeds its maximum level. There are overshoots and chatterings in ω while it approximate zero quickly. At the mean time, α exhibits frequent chattering near zero. Thanks to the moderation of the controller gains and the limit on the magnitude of α, this only happens in a short time and the control inputs are within the reasonable range afterwards.

The paths of the robot and the target are plotted in Figure 7. Beginning from point B_0, the robot moves along the path $B_0B_1 \ldots$ and catches up and align with the target at the point C. The target moves from point A_0 and travel along the circle. The directions of the robot and the target are indicated by arrows.

5. CONCLUSION

The control of a wheeled mobile robot to track a moving target with limited control inputs is studied in the paper. Based on a proper system modeling, a Lyapunov based controller design approach is used to determine the robot's angular and linear velocities to achieve the asymptotic convergence of the tracking errors. Considering the limits on the control inputs, a condition for a asymptotic stability of the closed loop system is also established. Simulation results are provided to verify the effectiveness of the proposed approach.

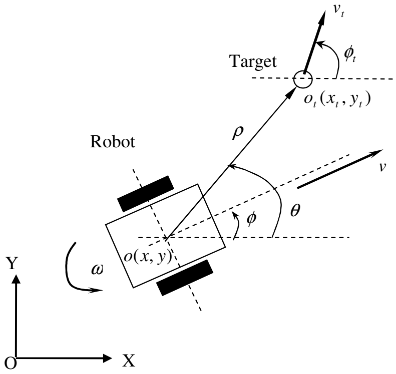

Fig. 1. A mobile robot chasing a target

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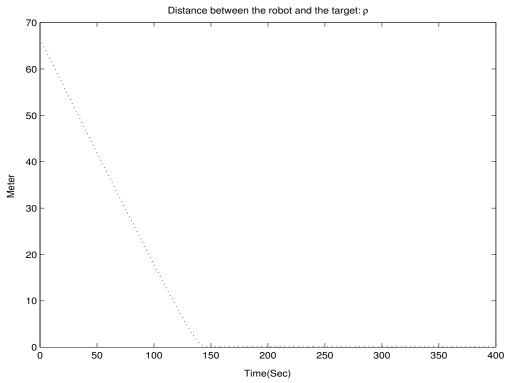

Fig. 2. Distance tracking error: ρ

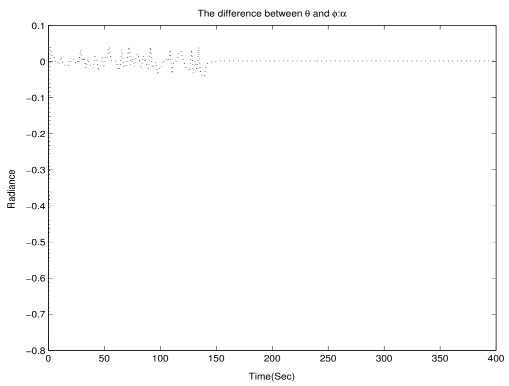

Fig. 3. Angle tracking error: α

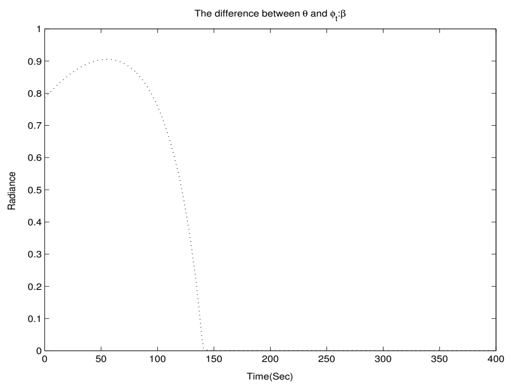

Fig. 4. Angle tracking error: β

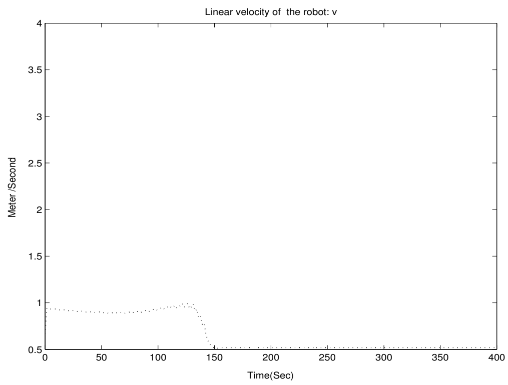

Fig. 5. Robot's linear velocity

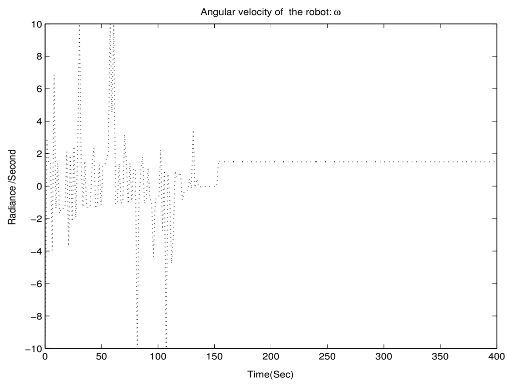

Fig. 6. Robot's angular velocity

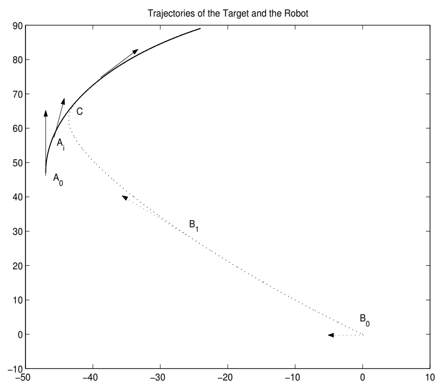

Fig. 7. The target's path (solid line) and the robot's path (dotted line)