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# REAL-TIME SLIDING-MODE ADAPTIVE CONTROL OF A MOBILE PLATFORM WHEELED MOBILE ROBOT

**A. Filipescu*[1], AL. Stancu*, S.Filipescu**, G. Stamatescu*****

*Department of Automation and Industrial Informatics
Adrian.Filipescu@ugal.ro, Alexndru.Stancu@ugal.ro
**Student in Electrical Engineering, Filipescus@galati.astral.ro
University "Dunarea de Jos" of Galati
Domneasca, 47, Galati, 800008, Romania
***Student in Automatic Control, gorrocs@gmail.com,
Polytechnic University of Bucharest
Splaiul Independentei, 313, Bucharest, 060042, Romania

Abstract: This paper presents parameter identification and discrete-time adaptive sliding-mode controller applied to control the mobile platform Pioneer 3-DX wheeled mobile robot (WMR) with 5-DOF manipulator. The dynamic model for mobile robot with two driving wheels, time–varying mass and moment of inertia have been used in sliding-mode control. Two closed-loop, on-line parameter estimators have been used in order to achieve robustness against parameter uncertainties (robot mass and moment of inertia). Two sliding-mode adaptive controllers have been designed corresponding to angular and linear motion. This paper presents closed-loop, circular trajectory tracking real-time control for the mobile robot Pioneer 3-DX. *Copyright © 2008 IFAC*

Keywords: Discrete-time Pioneer 3-DX model, sliding-mode adaptive control, on-line parameter estimation.

## 1. WHY DISCRETE-TIME SLIDING-MODE ADAPTIVE CONTROL? [2]

Different approaches have been proposed in the literature for output tracking of one pair of active wheels mobile robots (WMR), (Canudas de Wit and Sordalen, 1997, Canudas de Wit, Siciliano and Valavanis, 1998). The problem of controlling non-holonomic systems when there are model uncertainties has been widely addressed. However, relatively few results have been presented about the robustness of WMR control concerning model uncertainties and external disturbances. The structural (parameter) and/or un-structural uncertainties in the model of the MIMO non-linear systems, and the difficulties in parameter identification require the design of the controller so that the closed loop robustness is achieved.

[1]Corresponding author;

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It is well known that the robustness to structural, un-structural uncertainties and external disturbances of the WMR closed loop can be achieved with a variable structure controller, (Aghilar, and all. 1997; Filipescu, and all 2005; Yu and Xu 2002). Maintaining the system on a sliding surface reduces the influence of the uncertainties in the closed loop and quickly leads to an equilibrium point.

The main advantage of the discrete-time sliding mode control is the direct and easy real-time implementation. Since the sliding mode control is originally from continuous time, it is more difficult to choose a synthesis in discrete-time. The discrete-time sliding mode control, (Yu and Xu 2002, Leo and Orlando 1998), is quite different of performing the control design in the continuous-time domain. Discrete-time sliding-mode controller design is usually based on an approximate sliding-mode system evolution due to the non unique attractiveness condition and approximate evolution on sliding surface, (Furuta 1990; Yu and Xu, 2002). . The robust trajectory tracking problem has been addressed in Yang and Kim, 1999, using a continuous-time sliding-mode control. The performing control design, using the kinematical model of the vehicle, does not explicitly take into account parameters variation (robot mass and moment of inertia) and external disturbances (frictions and viscous forces), (Fierro and Lewis, 1997). The controller design using the WMR dynamical model, where uncertainties in the robot physical parameters can be explicitly taken into account, tends to interest the actual research on this field.

In this paper, the trajectory tracking problem for Pioneer 3-DX, WMR with two driving wheels, in the presence of uncertainties (time-varying mass and moment of inertia), has been solved by discrete-time sliding-mode controllers based on the discrete-time WMR dynamical model. Two closed loop, on-line parameter estimators have been used against parameter uncertainties.

The paper is structured as follows: Section 2 presents the dynamical model of the Pioneer 3-DX mobile robot. It includes the discrete-time state space model, its uncertainties, non-holonomic constraint and the output tracking errors of Pioneer 3-DX. Section 3 describes the on-line parameter estimators corresponding to angular and linear motion. The sliding adaptive controllers, associated with the angular and linear motion, are designed in Section 4 and 5. Pioneer 3-DX sliding-mode closed loop real-time results are presented in Section 6, while conclusions and remarks are presented in Section 7.

## 2. PIONEER 3-DX DICRETE-TIME DYNAMIC MODEL AND PARAMETER UNCERTAINTIES

*1) Assumption:* The WMR motion is assumed to be pure rolling, without of any slipping.

The figure 1 shows the mobile platform Pioneer 3-DX with Pioneer 5-DOF manipulator. The figure 2 shows the schematic of the WMR. The vehicle dynamics is fully described by a three-dimensional vector of generalized coordinates $q(t)$ defined by the coordinates $((x(t), y(t)))$ of the midpoint between the two driving wheels, and by the orientation angle $\Phi(t)$. The velocity constraint (non-holonomic constraint) of vehicle motion is $\dot{x}\sin\Phi - \dot{y}\cos\Phi = 0$.

Define by $\tau_r$ and $\tau_l$ the torques provided by DC motors to the right and left wheel, respectively. The vehicle is described by the following dynamical model where m, I, D, r are the robot mass, moment of inertia, distance between wheels, and wheels radius, respectively.

Fig. 1. Pioneer 3-DX wheeled mobile robot with 5-DOF robotic manipulator

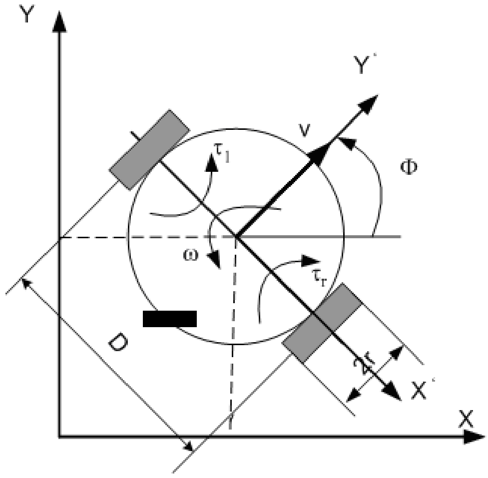


Fig. 2. WMR configuration variables for angular and linear motion.

$$
\begin{aligned}
m\ddot{x} &= -m\dot{y}\dot{\Phi} + \frac{\tau_r + \tau_l}{r}\cos\Phi \\
m\ddot{y} &= m\dot{x}\dot{\Phi} + \frac{\tau_r + \tau_l}{r}\sin\Phi \\
I\ddot{\Phi} &= \frac{D}{2r}(\tau_r - \tau_l)
\end{aligned}
\qquad (1)
$$

*1) Remark:* The real mass of the WMR is assumed to be time-varying with bounded uncertainty with known nominal mass. Due to the time-varying mass, the moment of inertia becomes time-depending with bounded uncertainty.

*2) Assumption:* Even if the moment of inertia is considered time-varying, the robot mass is assumed to be uniformly distributed all the time.

Let define two parameters corresponding to the angular and linear motion, such as: $\alpha(t) = D/(2I(t)r)$, $\pi(t) = 1/(m(t)r)$. The real values of the parameters are time-varying with upper bounded uncertainties

$$\alpha^{real}(t)=\alpha^{nom}-\Delta\alpha(t);\quad |\Delta\alpha|\le\Delta\alpha^{\max}$$
$$\pi^{real}(t)=\pi^{nom}-\Delta\pi(t);\quad |\Delta\pi|\le\Delta\pi^{\max} \tag{2}$$

Let $x\in R^6$ be the state vector, with the following components:

$$x_1=x,\quad x_2=y,\quad x_3=\Phi$$
$$x_4=\dot{x},\quad x_5=\dot{y},\quad x_6=\dot{\Phi} \tag{3}$$

Define the control input corresponding to angular, $u_A=\tau_r-\tau_l$ and linear motion, $u_P=\tau_r+\tau_l$, respectively. The state space model of WMR and the non-holonomic constraint have been discretized with the sampling period T, replacing the derivative by a finite difference and using a zero-order-hold for the control inputs, k being the k[th] time interval where the corresponding variable is evaluated ( $t=kT$ ). Let $e(k)\in R^6$ be, the vector of output errors: $e_i(k)=x_i(k)-x_i^{ref}(k)$, $x_i^{ref}(k)$; $i=1,\cdots,6$ is the trajectory to be tracked.

$$\begin{aligned}
x_1(k+1)&=x_1(k)+Tx_4(k)\\
x_2(k+1)&=x_2(k)+Tx_5(k)\\
x_3(k+1)&=x_3(k)+Tx_6(k)\\
x_4(k+1)&=x_4(k)-Tx_5(k)x_6(k)+T\pi(k)\cos(x_3(k))u_P(k)\\
x_5(k+1)&=x_5(k)+Tx_4(k)x_6(k)+T\pi(k)\sin(x_3(k))u_P(k)\\
x_6(k+1)&=x_6(k)+T\alpha(k)u_A(k)
\end{aligned} \tag{4}$$

$$x_4(k)\sin(x_3(k))-x_5(k)\cos(x_3(k))=0 \tag{5}$$

## 3. PARAMETER UPDATING CORRESPONDING TO ANGULAR AND LINEAR MOTION

For each robot motion, angular and linear, respectively, an on-line parameter estimator and a sliding controller have been introduced. Due to the time-varying of the Pioneer 3-DX mass, the control input parameters $\alpha(t)$ and $\pi(t)$ are updated on-line in order to be used in the corresponding sliding mode control input. The robustness against mass uncertainty will be assured. The maximum bounds of control input parameters corresponding to angular and linear motion will be used in the attractiveness condition of appropriate sliding surface. As will be shown in the following sections, the attractiveness condition of the corresponding sliding surface is satisfied only in a certain interval. Outside of this interval, the control input is computed by on-line parameter estimates. Moreover, in discrete-time, the sliding condition with some approximation is satisfied. When the system is inside of the sliding sector or in the neighbourhood of sliding surface, the parameter updating law can provide convergent estimates. Let $S_A(k)$ and $S_P(k)$ be two sliding surfaces corresponding to the control input for angular and linear motion, respectively. As parameter updating law, the recursive least squares method is used. The control input for angular motion has two terms: the first, denoted compensation component $u_A^{comp}(k)$, has to compensate the rotational dynamics; the second, denoted sliding mode component, $u_A^{sm}(k)$, corresponds to system evolution inside the sliding surface neighbourhood. The whole control input for angular motion is

$$u_A(k)=u_A^{comp}(k)+u_A^{sm}(k) \tag{6}$$

The calculus and the steps for obtaining both components of the angular motion control input are presented in Section 4. Expressing the estimated value for angular motion control input parameter, $\hat{\alpha}(k)=\alpha^{nom}-\Delta\hat{\alpha}(k)$, the next sequence, corresponding to recursive least squares method, (Ljung, 1999; Stoica and Ahgren, 2002), can be used to provide an estimation of the uncertainty scalar term $\Delta\alpha(k)$ at the k[th] step

$$L_{\Delta\alpha}(k)=\frac{P_{\Delta\alpha}(k-1)u_A(k-1)}{1+[u_A(k-1)]^2P_{\Delta\alpha}(k-1)} \tag{7}$$

$$P_{\Delta\alpha}(k)=P_{\Delta\alpha}(k-1)-L_{\Delta\alpha}(k)u_A(k-1)P_{\Delta\alpha}(k-1) \tag{8}$$

$$\Delta\hat{\alpha}(k)=\Delta\hat{\alpha}(k-1)+L_{\Delta\alpha}\begin{bmatrix}\Delta\hat{\alpha}(k-1)u_A(k-1)+\\ \alpha^{nom}u_A^{sm}(k-1)-S_A(k)/T^2\end{bmatrix} \tag{9}$$

*2) Remark:* Since for each robot motion only one parameter is estimated, the gain $L_{\Delta\alpha}(k)$ and the covariance $P_{\Delta\alpha}(k)$ are scalars.

The control input for linear motion, $u_P(k)$, has only the sliding-mode component, $u_P(k)=u_P^{sm}(k)$. A similar updating law is used for the corresponding parameter, $\hat{\pi}(k)=\pi^{nom}-\Delta\hat{\pi}(k)$.

$$L_{\Delta\pi}(k)=\frac{P_{\Delta\pi}(k-1)u_P(k-1)}{1+[u_P(k-1)]^2P_{\Delta\pi}(k-1)} \tag{10}$$

$$P_{\Delta\pi}(k)=P_{\Delta\pi}(k-1)-L_{\Delta\pi}(k)u_\pi(k-1)P_{\Delta\pi}(k-1) \tag{11}$$

$$\Delta\hat{\pi}(k)=\Delta\hat{\pi}(k-1)+L_{\Delta\pi}(k)\begin{bmatrix}T\Delta\hat{\pi}(k-1)u_P(k-1)\\ +\pi^{nom}u_P(k-1)\\ +\tilde{S}_P(k)-S_P(k)\end{bmatrix} \tag{12}$$

where $L_{\Delta\pi}(k)$, $P_{\Delta\pi}(k)$ have the same meaning as above, and $\tilde{S}_P(k)$ will be defined later.

*3) Remark:* For both parameter updating laws, (9) and (12), the expression in brackets is valid when the system evolves in the neighborhood of the corresponding sliding surface.

## 4. SLIDING-MODE ADAPTIVE CONTROL OF ANGULAR MOTION

The following stable sliding surface has been chosen, in order to design the control input for angular motion

$$S_A(k)=A(k+1)-\mu A(k)=0 \tag{13}$$

where

$$A(k)=x_3(k)-arctg\left(\frac{x_5^{ref}(k)-\delta_2 e_2(k-1)}{x_4^{ref}(k)-\delta_1 e_1(k-1)}\right) \tag{14}$$

with: $\mu\in(-1\quad 1)$, $\delta_1,\delta_2\in\left(0\quad \frac{1}{T}\right)$. Parameter $\mu$ and the linear errors, $e_1, e_2$, defines the dynamics of the sliding surface. The interval set of $\delta_1$ and $\delta_2$ assures the stability of linear errors.

If the non-holonomic constraint corresponding to the reference trajectory is

$$x_3^{ref}(k)=arctg\left(x_5^{ref}(k)/x_4^{ref}(k)\right) \tag{15}$$

then the angular error $e_3(k)$ tends to zero when $e_1(k)$, $e_2(k)$ tend to zero.

*4) Remark:* The sliding surface defined in (13) has been chosen in such a manner that whenever a sliding mode is achieved on it and $e_1(k), e_2(k)$ ->0, the orientation angle $\Phi$ tends to its reference value.

For computing the control input, the following attractiveness condition, (Furuta, 1990; Yu and Xu, 2002), has been used:

$$S_A(k)\Delta S_A(k+1)<-\frac{1}{2}\Delta S_A^2(k+1) \tag{16}$$

where

$$\Delta S_A(k+1)=S_A(k+1)-S_A(k) \tag{17}$$

An approximate sliding-mode evolution can be assured on the surface (13). If the compensation component of the control input is defined by the expression

$$u_A^{comp}(k)=\left(T^2\alpha^{nom}\right)^{-1}\left[\begin{array}{l} arctg\left(\dfrac{x_5^{ref}(k+2)-\delta_2 e_2(k+1)}{x_4^{ref}(k+2)-\delta_1 e_1(k+1)}\right)\\ +x_3(k+1)-Tx_6(k)-\mu A(k+1)\end{array}\right] \tag{18}$$

then, after replacing (6), (13) and (14) in (17), one obtains:

$$\Delta S_A(k+1)=T^2\left(\alpha^{nom}-\Delta\alpha(k)\right)u_A^{sm}(k) + \Delta\alpha(k)u_A^{comp}(k)-S_A(k) \tag{19}$$

With (19), (16) becomes

$$T^2\left[\alpha^{nom}-\Delta\alpha(k)\right]^2\left[u_A^{sm}(k)\right]^2 + 2T^2\left[\alpha^{nom}-\Delta\alpha(k)\right]\Delta\alpha(k)\left|u_A^{sm}(k)\right|\left|u_A^{comp}(k)\right| + T^2\left[\Delta\alpha(k)\right]^2\left[u_A^{comp}(k)\right]^2-\left[S_A^2(k)\right]^2<0 \tag{20}$$

Introducing the upper bound of the angular motion parameter uncertainty, the above second degree inequality can be written in the compact form

$$T^2\left[\begin{array}{l}\left(\alpha^{nom}-\Delta\alpha^{\max}\right)\left|u_A^{sm}(k)\right|\\ +\Delta\alpha^{\max}\left|u_A^{comp}(k)\right|\end{array}\right]^2-\left[S_A^2(k)\right]^2<0 \tag{21}$$

If $u_A^{sm}(k)>0$ and $\left|S_A(k)\right|/T^2>\Delta\alpha^{\max}\left|u_A^{comp}(k)\right|$, then the sliding-mode component of the control input becomes

$$u_A^{sm}(k)<\left|S_A(k)/T^2\right|-\frac{\Delta\alpha^{\max}\left|u_A^{comp}(k)\right|}{\alpha^{nom}-\Delta\alpha^{\max}} \tag{22}$$

When $u_A^{sm}(k)<0$, the inequality (21) is satisfied for

$$u_A^{sm}(k)>-\frac{\left|S_A(k)/T^2\right|-\Delta\alpha^{\max}\left|u_A^{comp}(k)\right|}{\alpha^{nom}-\Delta\alpha^{\max}} \tag{23}$$

*5) Remark:* Both expressions of the sliding-mode component, (22) and (23), can be written compactly

$$u_A^{sm}(k)=\frac{\rho_A\left|\frac{S_A(k)}{T^2}\right|-\Delta\alpha^{\max}\left|u_A^{comp}(k)\right|}{\alpha^{nom}-\Delta\alpha^{\max}} \tag{24}$$

where $\rho_A\in(-1\quad 1)$.

When $\left|S_A(k)\right|/T^2\le\Delta\alpha^{\max}\left|u_A^{comp}(k)\right|$, the attractiveness condition (16) can not be satisfied. The sliding mode component of the control input still can be computed by using estimates of parameter $\Delta\alpha$. The recursive least square method used to compute $\Delta\hat{\alpha}$, given by (7), (8) and (9), is convergent only when the system evolves in the neighbourhood of the sliding surface. Therefore, an approximate sliding mode condition is satisfied $S_A(k+1)/T^2\approx 0$

$$\left[\alpha^{nom}-\Delta\hat{\alpha}(k)\right]u_A^{sm}(k)+\Delta\hat{\alpha}(k)u_A^{comp}(k)\approx 0 \tag{25}$$

This approximation is used to compute the control input for the angular motion

$$u_A^{sm}(k) = -\Delta\hat{\alpha}(k) u_A^{comp}(k) \Big/ \left(\alpha^{nom} - \Delta\hat{\alpha}(k)\right) \quad (26)$$

*6) Remark:* Using (24), the updating law (9) can be rewritten as

$$\Delta\hat{\alpha}(k) = \Delta\hat{\alpha}(k-1) + L_{\Delta\alpha}\begin{bmatrix} \left[\alpha^{nom} - \Delta\hat{\alpha}(k-1)\right] u_A^{sm}(k-1) \\ + \Delta\hat{\alpha}(k-1) u_A^{comp}(k-1) - S_A(k)/T^2 \end{bmatrix} \quad (27)$$

## 5. SLIDING-MODE ADAPTIVE CONTROL OF LINEAR MOTION

The following sliding surface is proposed

$$S_P(k) = \left([x_4(k)]^2 + [x_5(k)]^2\right)^{1/2} - \begin{pmatrix} \left[x_4^{ref}(k) - \delta_1 e_1(k-1)\right]^2 \\ + \left[x_5^{ref}(k) - \delta_2 e_1(k-1)\right]^2 \end{pmatrix}^{1/2} = 0 \quad (28)$$

Starting with the third equation of model (4), using a trigonometric equation and the non-holonomic constraint (5), the following equation holds

$$tg(Tx_6(k)) = \left(\frac{x_5(k+1)}{x_4(k+1)} - \frac{x_5(k)}{x_4(k)}\right) \Big/ \left(1 + \frac{x_5(k+1)x_5(k)}{x_4(k+1)x_4(k)}\right) \quad (29)$$

Introducing the expression of the state variables, from state model (4), and using the constraint (5), the above equation becomes

$$tg(Tx_6(k))\left(\left([x_4(k)]^2 + [x_5(k)]^2\right)^{1/2} - T\pi(k) u_P(k)\right) = Tx_6(k)\left([x_4(k)]^2 + [x_5(k)]^2\right)^{1/2} \quad (30)$$

Let define

$$\tilde{S}_P(k) = \left[\cos(Tx_6(k))\right]^{-1}\left([x_4(k)]^2 + [x_5(k)]^2\right)^{1/2} - \begin{pmatrix} \left[x_4^{ref}(k+1) - \delta_1 e_1(k)\right]^2 \\ + \left[x_5^{ref}(k+1) - \delta_2 e_2(k)\right]^2 \end{pmatrix}^{1/2} \quad (31)$$

The sliding motion on the surface (27) concerns the reduced order system of the robotic model, without of 3rd and 6th equation. The same attractiveness condition, as in [6], has been considered for computing the linear motion control input

$$S_P(k)\Delta S_P(k+1) < -\frac{1}{2}\Delta S_P^2(k+1) \quad (32)$$

$$\Delta S_P(k+1) = S_P(k+1) - S_P(k) \quad (33)$$

An approximate sliding mode evolution on the surface (27) can be assured. Consequently of sliding-mode evolution on (13), the state $x_3(k)$ tends to hold the following expressions

$$\cos(Tx_3(k)) = \left(x_4^{ref}(k) - \delta_1 e_1(k-1)\right)\left(\left[x_4^{ref}(k) - \delta_1 e_1(k-1)\right]^2 + \left[x_5^{ref}(k) - \delta_2 e_1(k-1)\right]^2\right)^{-1/2} \quad (34)$$

$$\sin(Tx_3(k)) = \left(x_5^{ref}(k) - \delta_1 e_1(k-1)\right)\left(\left[x_4^{ref}(k) - \delta_1 e_1(k-1)\right]^2 + \left[x_5^{ref}(k) - \delta_2 e_1(k-1)\right]^2\right)^{-1/2} \quad (35)$$

Using (28), the following expression can be obtained

$$[x_4(k+1)]^2 + [x_5(k+1)]^2 = \left[\cos(Tx_6(k))\right]^{-2}\left[\left([x_4(k)]^2 + [x_5(k)]^2\right)^{1/2} - T\left(\pi^{nom} - \Delta\pi(k)\right)u_P(k)\right]^2 \quad (36)$$

With (35) and (29), (25) and (32) become

$$S_P(k+1) = \tilde{S}_P(k) - T\left|\cos(Tx_6(k))\right|^{-1}\left(\pi^{nom} - \Delta\pi(k)\right)u_P(k) \quad (37)$$

$$\Delta S_P(k+1) = \tilde{S}_P(k) - S_P(k) = T\left|\cos(Tx_6(k))\right|^{-1}\left(\pi^{nom} - \Delta\pi(k)\right)u_P(k) \quad (38)$$

Using (36), (37) and upper bound of linear motion uncertainty, from (2), the second degree inequality can be written

$$\left[\frac{\left(\pi^{nom} - \Delta\pi^{\max}\right)T}{\left|\cos(Tx_6(k))\right|}\left|u_P(k)\right| + \left|\tilde{S}_P(k)\right|\right]^2 - [S_P(k)]^2 < 0 \quad (39)$$

If $u_P^{sm}(k) > 0$, and $\left|S_P(k)\right| > \left|\tilde{S}_P(k)\right|$, then the sliding control input for linear motion is:

$$u_P(k) = \rho_P \frac{S_P(k) - \left|\tilde{S}_P(k)\right|}{T\left[\left|\cos(Tx_6(k))\right|\right]^{-1}\left(\pi^{nom} - \Delta\pi^{\max}\right)} \quad (40)$$

where $\rho_P \in (0 \quad 1)$.

When $\left|S_P(k)\right| \le \left|\tilde{S}_P(k)\right|$, the attractiveness condition (31) can not be satisfied. The control input still can be computed using on-line estimates for $\Delta\pi$.

*7) Remark*: The recursive least square method used to compute $\Delta\hat{\pi}$, given by (10), (11) and (12), is convergent only when the system evolves in the neighborhood of the sliding surface. Therefore, the approximate sliding mode condition is satisfied, $S_P(k+1) \approx 0$, i.e.

$$T\left[\left|\cos(Tx_6(k))\right|\right]^{-1}\left(\pi^{nom} - \Delta\hat{\pi}(k)\right)u_P(k) + \tilde{S}_P(k) \approx 0 \quad (41)$$

Considering the above, the control input can be expressed as:

$$u_P(k) = -\tilde{S}_P(k)\big/T\left[\left|\cos(Tx_6(k))\right|\right]^{-1}\left(\pi^{nom} - \Delta\hat{\pi}(k)\right) \tag{42}$$

*8) Remark:* As a result of (40), (12) can be rewritten as

$$\begin{aligned}\Delta\hat{\pi}(k) &= \Delta\hat{\pi}(k-1) \\ &+ L_{\Delta\pi}(k)\begin{bmatrix} T\left[\left|\cos(Tx_6(k-1))\right|\right]^{-1}\left[\pi^{nom} - \Delta\hat{\pi}(k-1)\right] \\ u_P(k-1) + \tilde{S}_P(k-1) - S_P(k) \end{bmatrix}\end{aligned} \tag{43}$$

When the system evolves in sliding-mode on the surface (27)

$$x_4(k) = x_4^{ref}(k) - \delta_1 e_1(k-1) \tag{44}$$

$$x_5(k) = x_5^{ref}(k) - \delta_2 e_1(k-1) \tag{45}$$

Therefore, the dynamics of the output tracking error associated to the reduced order system can be expressed as:

$$e_1(k+1) = e_1(k) - \delta_1 T e_1(k-1) \tag{46}$$

$$e_2(k+1) = e_2(k) - \delta_2 T e_2(k-1) \tag{47}$$

For $\delta_1, \delta_2 \in \left(0 \quad \frac{1}{T}\right)$, the above dynamics errors are stable.

## 6. PIONEER PLATFORM REAL-TIME SLIDING-MODE CLOSED LOOP CONTROL

For testing the proposed discrete-time sliding-mode adaptive controller Pioneer 3-DX with on board PC and wireless adapter has been used in circular trajectory tracking. The basic Pioneer platform contains all of the components of an intelligent mobile robot for sensing and navigation in real world environment , including battery power, drive motors and wheels, linear encoders, and range-finding ultrasonic sonar transducers. The mobile platform is client-server architecture. The client is an embedded PC with Ethernet connection. The robot client software runs on the onboard PC and wireless Ethernet is used to monitor and control PC operations. The rugged P3-DX is 44cm x 38cm x 22cm aluminum body with 16.5cm diameter drive wheels. The two motors use 38.3:1 gear ratios and include 500-tick encoders. This differential drive platform is highly holonomic and can rotate in place moving both wheels, or it can swing around a stationery wheel in a circle of 32cm radius. A rear caster balances the robot. The following parameters of model (3) are used: m=10kg, D=50cm, I=0.0624 kgm$^2$, T=0.3s. The moment of inertia has been computed assuming the mass uniformly distributed.

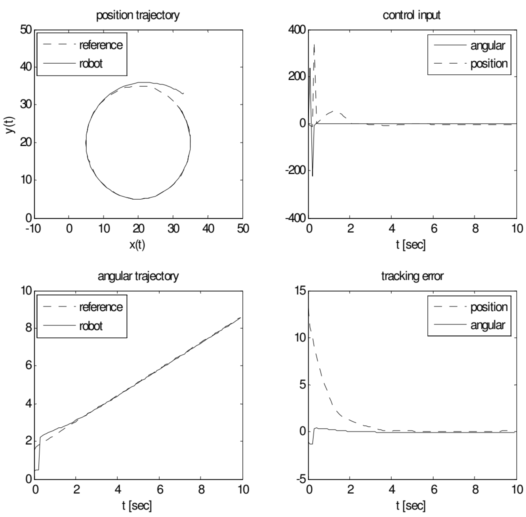


Fig.3. WMR closed loop response for circular reference and initial conditions x1(0)=33cm; x2(0)=33cm; x3(0)=$\pi$/7; x4(0)=-0.5 m/s; x5(0)=0.2 m/s; x6(0)=0.1 rad/s.

A linear time-varying mass additionally to the nominal one has been considered. Specifically, the robotic time-varying mass has been increased linearly from 12kg to 16kg. The circle trajectory tracking, shown in figures 3, was obtained for $\Delta\alpha^{\max} = 0.4$, $\Delta\pi^{\max} = 0.033$. The following values have been chosen for the constants: $\mu = 0.001$, $\rho_P = \rho_A = 0.99$, $\delta_1 = \delta_2 = 3.33$, $P_{\Delta\alpha}(0) = P_{\Delta\pi}(0) = 10$.

## 7. CONCLUSIONS

Discrete-time, sliding-mode adaptive controllers and parameter estimators for trajectory tracking applied to control angular and linear motion of Pioneer 3-DX mobile robot, have been presented in this paper. The controllers have been designed by considering time-varying mass and moment of inertia and a dynamical state space model. Even if only the robotic mass and moment of inertia have been considered, as parameter uncertainties, the proposed controllers assure closed loop robustness to a wide range of parameter and model uncertainties and external disturbances. Two sliding-mode adaptive controllers have been designed, for angular and linear motion, respectively. The robustness is guaranteed by sliding-mode controllers and by on-line parameter estimators. The on-line updated parameters, assure an approximate sliding-mode evolution, even if the attractiveness condition is not satisfied and contribute to an increased robustness.

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