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Frequency Domain Based Design of Iterative Learning Controllers for Monotonic Convergence

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Abstract: This paper presents a frequency domain based method to design iterative learning controllers (ILC) for monotonic convergence. This is an extension of a repetitive controller (RC) design that aims to achieve monotonic convergence of all frequency components of the tracking error from period to period. The monotonic convergence condition in the RC design requires a steady-state assumption that the ILC problem does not satisfy due to the transient at the beginning of every repetition. Additional fine-tuning of the ILC gains to ensure monotonic convergence is needed and two such techniques (iterative and non-iterative) are developed. Numerical examples are presented to illustrate the design method.

1. INTRODUCTION

Iterative learning control (ILC) is a body of control theory devoted to repeating processes. As an enabling technology ILC is capable of bringing tracking accuracy up to the same extremely high repeatability level of modern hardware. That is possible because ILC automatically compensates for all unknown deterministic sources of repeating errors. ILC is designed for a system that returns to the same initial condition before each new execution of the task, as in the case of a robot performing on each item that arrives one by one on an assembly line. A relative of ILC is repetitive control (RC) where the goal is to track a periodic trajectory without resetting between periods. Thus both ILC and RC are particularly suitable for ultra-precision repetitive manufacturing processes. Recent research in ILC and RC focuses on monotonic convergence and robustness as treated in the texts by Ahn, Moore, and Chen (2007), and Rogers, Galkowski, and Owens (2007). Earlier texts include Bien and Xu (1998), Moore (1993), and Rogers and Owens (1992).

Practical issues in ILC and RC designs are discussed in Longman (2000). Recent developments (Panomruttanarug and Longman, 2004; Longman, Xu, and Phan, 2007) produce repetitive controllers that try to achieve monotonic convergence of all frequency components of the tracking error from period to period. However, the frequency-domain monotonic condition on which the RC design is based requires steady-state assumption that is often violated in ILC because transient response is almost always present at the beginning of every repetition. Nevertheless, satisfying the same condition in ILC is still important because it implies monotonic convergence of all frequency components of the steady-state portions of the tracking error histories. This paper continues our previous line of work to addresses the necessary extension of the RC design to the ILC problem. The primary objective is to ensure that the resultant ILC design also guarantees monotonic convergence of the Euclidean norm of the entire tracking error histories from repetition to repetition in addition to monotonic convergence of all frequency components of the steady-state portions of those error histories. This paper focuses on the single-input single-output case as the multiple-input multiple-output case requires a different mathematical treatment. Robustification based on the probabilistic multiple-model design principle (Takanishi, Phan, and Longman, 2005) is recently treated in Lee, Phan, and Longman (2006) and Brown *et al.* (2007). Robustification of the ILC controllers developed in this paper will be treated in a later publication.

The paper begins with a brief description of the repetition-domain formulation of ILC (Phan and Longman, 1988) which establishes the necessary and sufficient condition for the stability of the learning process. A more restrictive condition for monotonic convergence of the Euclidean norm of the tracking error histories is then described, followed by an even more intuitive condition that describes how each frequency component of the steady-state tracking error history varies from repetition to repetition. This steady-state condition will then be used to produce a base-line ILC design that needs to be modified further to guarantee monotonic convergence of the entire tracking error history from repetition to repetition. Two such refinement methods (iterative and non-iterative) are developed. In the iterative method the base-line ILC gains are adjusted to satisfy the monotonic convergence condition. In the non-iterative method, the goal is to make the ILC dynamics that governs how the tracking error varies from repetition to repetition match the well-behaved RC dynamics that governs how the tracking error varies from period to period. Numerical results are used to illustrate the proposed ILC design methods.

2. REPETITION-DOMAIN FORMULATION

Consider an n-th order discrete-time system of the form

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + v_1(k) \\ y(k) &= Cx(k) + v_2(k) \end{aligned} \tag{1}$$

The vectors $x(k)$, $y(k)$ denote the system state and output, respectively. In ILC it is assumed that the initial state $x(0)$, the process and output disturbances $v_1(k)$ and $v_2(k)$ are unknown but they are the same from one repetition (or pass) to the next. Let $y^*(k)$, $k = 1,2,3,...,p$, denote the desired output to be tracked. For any repetition j, the relationship between the input and output time histories is

$$\underline{y}_j = P\underline{u}_j + \underline{w} \tag{2}$$

where

$$\underline{y}_j = \begin{bmatrix} y_j(1) \\ y_j(2) \\ \vdots \\ y_j(p) \end{bmatrix}, \ \underline{u}_j = \begin{bmatrix} u_j(0) \\ u_j(1) \\ \vdots \\ u_j(p-1) \end{bmatrix}, \ w_j = \begin{bmatrix} w_j(1) \\ w_j(2) \\ \vdots \\ w_j(p) \end{bmatrix}$$

$$P = \begin{bmatrix} CB & 0 & \cdots & 0 & 0 \\ CAB & CB & \ddots & \vdots & \vdots \\ CA^2B & CAB & \ddots & 0 & \vdots \\ \vdots & \ddots & \ddots & CB & 0 \\ CA^{p-1}B & \cdots & CA^2B & CAB & CB \end{bmatrix} \tag{3}$$

The entries in the $p \times p$ matrix P are the system Markov parameters. The Markov parameters can be identified from input-output data in the presence of repeating or periodic disturbances (Phan and Frueh, 1998; Phan *et al*., 2003). The vector $\underline{w}$ incorporates the effect of the unknown initial state and disturbances. Applying a backward difference operator $\delta_j z(k) = z_j(k) - z_{j-1}(k)$ to (2) yields an equation that describes how a change in the control input history affects the output history,

$$\delta_j \underline{y} = P\delta_j \underline{u} \tag{4}$$

Define the tracking error as $e(k) = y^*(k) - y(k)$, we have a similar equation that describes how a change in the control input history affects the output tracking error,

$$\delta_j \underline{e} = -P\delta_j \underline{u} \tag{5}$$

Notice that any unknown repeating terms are automatically eliminated by applying the backwards difference operator. Equation (4) or (5) forms the basis for the development of several ILC laws using modern space-space techniques (Phan and Longman, 1988; Phan, Longman, and Moore, 2000). For example, all linear ILC laws that rely on the tracking error of the previous repetition to modify the control to be used for the current repetition has the form

$$\delta_j \underline{u} = L\underline{e}_{j-1} \tag{6}$$

Stability of the learning process can be analyzed using the repetition-domain formulation as shown in the next section.

3. STABILITY AND MONOTONIC CONVERGENCE

Substituting (6) into (5) produces

$$\underline{e}_j = (I - PL)\underline{e}_{j-1} \tag{7}$$

The tracking error approaches zero as the number of repetitions approaches infinity if and only if the magnitudes of all eigenvalues of $I - PL$ are less than one, $i = 1,2,...,p$

$$|\lambda_i(I - PL)| < 1 \tag{8}$$

Although (8) is the true stability boundary, during learning the tracking error may badly diverge before converging zero, thus more practical conditions are derived (Panomruttanarug and Longman, 2006). The relationship between the Euclidean norm of the tracking error history from one repetition to the next can be shown to be

$$\|\underline{e}_j\| \le \sigma_{\max} \|\underline{e}_{j-1}\| \tag{9}$$

where $\sigma_{\max}$ denotes the maximum singular value of $I - PL$. Thus the condition for monotonic decay of the Euclidean norm of the tracking error from one repetition to the next is

$$\sigma_{\max}(I - PL) < 1 \tag{10}$$

A more revealing convergence condition is expressed in the frequency domain. Let $G(z)$ denote the discrete-time transfer function of the input-output model, $Y(z) = G(z)U(z)$, $G(z) = C(zI - A)^{-1}B$. The z-transfer function of the ILC law can be written as

$$\delta_j U(z) = L(z)E_{j-1}(z) \tag{11}$$

$$L(z) = \phi_q z^{2-q} + \cdots + \phi_3 z^{-1} + \phi_2 + \phi_1 z + \psi_2 z^2 + \cdots + \psi_\ell z^\ell \tag{12}$$

where $q = (p+1)/2$, $\ell = q$ for an odd p, or $q = p/2 + 1$, $\ell = q - 1$ for an even p. The repetition-domain error dynamics is governed by

$$E_j(z) = [1 - G(z)L(z)]E_{j-1}(z) \tag{13}$$

In the RC problem using a quasi-steady state argument, the approximate monotonic convergence of all frequency components of the tracking error is

$$|1 - G(z)L(z)| < 1, \ z = e^{j\omega\Delta t}, \ 0 \le \omega\Delta t \le \pi \tag{14}$$

The magnitude of $1 - G(e^{j\omega\Delta t})L(e^{j\omega\Delta t})$ can be plotted as $\omega\Delta t$ varies between 0 and π. In RC, to facilitate monotonic convergence we desire this plot to remain inside the unit half circle. In ILC, because transient response is present in each repetition, the condition (14) only applies to the steady-state portions of the trajectories. Nevertheless it is an important condition because of the monotonic convergence of all frequency components of the steady-state portions during learning that it implies. Therefore satisfaction of (14) is still necessary for good learning behavior.

Because the z-transform is based on steady-state response thinking, satisfying (14), although is important, does not guarantee stability of the learning process. Another reason for this is that for ILC it is not possible to involve all the gains of $L(z)$ at every time step in computing the control input. The learning matrix L based on $L(z)$ can be at most,

$$L=\begin{bmatrix} \phi_1 & \psi_2 & \cdots & \psi_\ell & 0 & \cdots & 0 \\ \phi_2 & \phi_1 & \psi_2 & \cdots & \psi_\ell & \ddots & \vdots \\ \vdots & \phi_2 & \phi_1 & \psi_2 & \cdots & \ddots & 0 \\ \phi_q & \cdots & \phi_2 & \phi_1 & \psi_2 & \cdots & \psi_\ell \\ 0 & \phi_q & \cdots & \phi_2 & \ddots & \cdots & \vdots \\ \vdots & \ddots & \ddots & \cdots & \ddots & \phi_1 & \psi_2 \\ 0 & \cdots & 0 & \phi_q & \cdots & \phi_2 & \phi_1 \end{bmatrix} \tag{15}$$

Notice that the gains are truncated at the beginning and end of portions of the each trajectory. Thus for any designed learning controller gain matrix L, it is important to check that (10) is satisfied. It should be noted that satisfaction of (10) automatically implies satisfaction of (8) because the largest magnitude of any eigenvalue of a matrix is always less than the largest singular value of that matrix.

4. THE ILL-CONDITIONED P MATRIX

Another practical problem that deserves attention in any ILC design is the fact that the matrix P in (3) is ill-conditioned. Issues associated with the ill-conditioned P are studied in Li and Longman (2007). Theoretically P is full rank as long as $CB \neq 0$, but in practice it is badly ill-conditioned, hence the exact inverse solution to produce zero tracking error for the entire p-step trajectory becomes necessarily large. This is clearly not desirable, or necessary in practice because the situation can be avoided by not asking for zero tracking error at all p time steps (e.g., via the use of control input weighting or basis functions in Frueh and Phan, 2000), or by asking for zero error at fewer than p time steps in the desired trajectory. In the following we take the latter option.

Consider the case where one specifies zero tracking error from $e(2)$ to $e(p)$ but leaving out $e(1)$. Then,

$$\delta_j \underline{e}_S = -P_S \delta_j \underline{u} \tag{16}$$

where $\underline{e}_S$ contains the tracking error from $e(2)$ to $e(p)$, and P_S is P without its first row. The mathematics generalizes easily when the tracking error is not specified to be zero at more than one time step. The corresponding $p_S \times 1$ vector $\underline{e}_S$ does not contain the tracking errors at those time steps, and the $p_S \times p$ matrix P_S does not contain the rows associated with those errors. The ILC law then has the form

$$\delta_j \underline{u} = L_S \underline{e}_{S\,j-1} \tag{17}$$

Let n_S be the number of nearly zero singular values of P. In general, if the number of tracking errors not specified to be zero is at least n_S, P_S will become well-conditioned. The corresponding condition for monotonic convergence of the Euclidean norm of $\underline{e}_S$ is

$$\sigma_{\max}(I_S - P_S L_S) < 1 \tag{18}$$

The identity matrix I_S has dimensions $(p_S - 1) \times (p_S - 1)$.

5. ILC DESIGN FOR MONOTONIC CONVERGENCE

From the above discussion, our basis for designing L_S in this paper is based on (14) and (18). Let $L(z)$ be written as $L(z) = M(z)\Phi$ where the ILC gains are collected in Φ and $M(z)$ is a vector of the positive and negative powers of z,

$$\begin{aligned} \Phi &= \begin{bmatrix} \phi_q & \cdots & \phi_2 & \phi_1 & \psi_2 & \cdots & \psi_\ell \end{bmatrix}^T \\ M(z) &= \begin{bmatrix} z^{2-q} & \cdots & z^{-1} & 1 & z & \cdots & z^\ell \end{bmatrix} \end{aligned} \tag{19}$$

As discussed in the previous section, the approximate monotonic condition for all frequency components of the tracking error in the steady state is that the plot of the magnitude of $1 - G(z)L(z)$ remains inside the unit half circle. We seek a learning controller that minimizes the shape of this plot by a cost function with $z_i = e^{j\omega_i \Delta t}$,

$$J = \sum_{i=0}^{N-1} W_i [1 - G(z_i)M(z_i)\Phi][1 - G(z_i)M(z_i)\Phi]^* + \Phi^T R \Phi \tag{20}$$

In (20), the * denotes the complex conjugate operation, W_i a frequency-dependent scalar weighting factor, R the weighting factor for the control gain magnitude, and N the number of points that define this half unit circle plot.

Taking the derivative of J with respect to the gain vector Φ and setting the result to zero will yield the desired solution:

$$\Phi = \mathsf{A}^{-1}\mathsf{B} \tag{21}$$

where $0 < \omega_i \Delta t < \pi$, and

$$\begin{aligned} \mathsf{A} &= \sum_{i=0}^{N-1} W_i \left[\mathrm{Re}(Q(z_i)) + \mathrm{Re}(Q(z_i))^T \right] + 2R \\ \mathsf{B} &= \sum_{i=0}^{N-1} W_i \left[\mathrm{Re}(S^H(z_i)) + \mathrm{Re}(S(z_i))^T \right], \ z_i = e^{j\omega_i \Delta t} \\ Q(z_i) &= S^H(z_i)S(z_i), \quad S(z_i) = G(z_i)M(z_i) \end{aligned} \tag{22}$$

In (22) Re(.) denotes the real part of the quantity in the parentheses, the T denotes the regular (real) transpose, and the H denotes the complex conjugate transpose.

To use the gains derived in (21), we form the L matrix from Φ as in (15). The ILC law is given in (17) where the candidate L_S is L with the appropriate column(s) deleted. For example, if zero tracking error at first time step is not specified, then L_S is formed by deleting the first column of L, and P_S by deleting the first row of P. We need to check if this candidate L_S satisfies (18). If it does not then L_S needs to be further fine-tuned. This paper presents two methods for doing so: an iterative fine-tuning method (Section 6), and a non-iterative method (Section 7).

6. AN ITERATIVE FINE-TUNING METHOD

The condition in (14) is derived under steady-state assumption whereas the true monotonic condition for the Euclidean norm of the tracking error histories in ILC is given in (18). It is possible that the candidate L_S based on (21) might violate (18) in that some of the singular values of $I_S - P_S L_S$ are slightly larger than one. This section we describe a procedure to fine-tune L_S to satisfy (18). Let σ_r denote a singular value of $I_S - P_S L_S$. Then $\psi_r = \sigma_r^2$, $r = 1, 2, ..., p_S$, is an eigenvalue of $H = (I_S - P_S L_S)^T (I_S - P_S L_S)$. For each $\psi_r = \sigma_r^2$,

$$\left[\psi_r\right]_{k+1} \approx \left[\psi_r\right]_k + \left(\frac{\partial \psi_r}{\partial \ell_{11}}\right)_k \left(\ell_{11}^{(k+1)} - \ell_{11}^{(k)}\right) + \left(\frac{\partial \psi_r}{\partial \ell_{12}}\right)_k \left(\ell_{12}^{(k+1)} - \ell_{12}^{(k)}\right) + \cdots + \left(\frac{\partial \psi_r}{\partial \ell_{pp_S}}\right)_k \left(\ell_{pp_S}^{(k+1)} - \ell_{pp_S}^{(k)}\right) \quad (23)$$

The subscripts k, $k+1$, or the superscripts (k), $(k+1)$ denote the iteration (not repetition) numbers. Writing (23) for all r and grouping the resultant equations produces

$$\underline{\psi}_{k+1} = \underline{\psi}_k + S_k\ \delta_{k+1}\underline{L}_S \quad (24)$$

where

$$\underline{\psi}_{k+1} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_{p_S} \end{bmatrix}_{k+1}, \underline{\psi}_k = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_{p_S} \end{bmatrix}_k, \ \delta_{k+1}\underline{L}_S = \begin{bmatrix} \ell_{11}^{(k+1)} - \ell_{11}^{(k)} \\ \ell_{12}^{(k+1)} - \ell_{12}^{(k)} \\ \vdots \\ \ell_{pp_S}^{(k+1)} - \ell_{pp_S}^{(k)} \end{bmatrix} \quad (25)$$

$$S_k = \begin{bmatrix} \left(\frac{\partial \psi_1}{\partial \ell_{11}}\right)_k & \left(\frac{\partial \psi_1}{\partial \ell_{12}}\right)_k & \cdots & \left(\frac{\partial \psi_1}{\partial \ell_{pp_S}}\right)_k \\ \vdots & \vdots & \cdots & \vdots \\ \left(\frac{\partial \psi_p}{\partial \ell_{11}}\right)_k & \left(\frac{\partial \psi_p}{\partial \ell_{12}}\right)_k & \cdots & \left(\frac{\partial \psi_p}{\partial \ell_{pp_S}}\right)_k \end{bmatrix} \quad (26)$$

An iterative scheme to reduce the singular values ψ_r can be found by minimizing

$$T_{k+1} = \underline{\psi}_{k+1}^T Q\ \underline{\psi}_{k+1} + \delta_{k+1}^T \underline{L}_S R_L \delta_{k+1}\underline{L}_S \quad (27)$$

Taking the derivative of (27) with respect to $\delta_{k+1}\underline{L}_S$ and setting it zero yields the following rule to refine the elements of L_S,

$$\left(\underline{L}_S\right)_{k+1} = \left(\underline{L}_S\right)_k - \left[\left(S_k^T Q S_k + R_L\right)^{-1} S_k^T Q\right] \underline{\psi}_k \quad (28)$$

The elements of S_k in (26) can be shown to be

$$\left(\frac{\partial \psi_r}{\partial \ell_{ij}}\right)_k = \left(v_r^T\right)_k \left(\frac{\partial H}{\partial \ell_{ij}}\right)_k \left(v_r\right)_k \quad (29)$$

$$\left(\frac{\partial H}{\partial \ell_{ij}}\right)_k = -\left[I - P\left(L_S\right)_k\right]^T PI(i,j) - I(j,i)P^T\left[I - P\left(L_S\right)_k\right] \quad (30)$$

where $I(i,j)$ is a $p \times p_S$ matrix of 0's everywhere except a 1 at the (i,j) element.

7. A NON-ITERATIVE FINE-TUNING METHOD

We now develop another method to design L_S by making the ILC dynamics that governs how the tracking error varies from repetition to repetition match the RC dynamics that governs how the tracking error varies from period to period. To this end we need a mapping that relates how the tracking error varies from period to period for RC. The following is a generalization of a similar result in Panomruttanarug and Longman (2006). The RC counterpart of (13) is

$$z^p E(z) = \left[1 - G(z)F(z)\right]E(z) \quad (31)$$

$$G(z) = CBz^{-1} + CABz^{-2} + CA^2Bz^{-3} + \cdots + CA^{p-1}Bz^{-p} + \cdots \quad (32)$$

$$F(z) = \psi_\ell z^\ell + \cdots + \psi_2 z^2 + \phi_1 z + \phi_2 + \phi_3 z^{-1} + \cdots + \phi_q z^{2-q} \quad (33)$$

It can be shown that

$$G(z)F(z) = T_{1\ell}z^{\ell-1} + \cdots + T_{13}z^2 + T_{12}z + T_{11} + T_{21}z^{-1} + \cdots + T_{p1}z^{2-q-p} + \cdots \quad (34)$$

Converting (31) back into the time domain, and setting $e(k) = 0$ for $k = 0$ and all negative values of k produces

$$\left[e(p+1) \quad e(p+2) \ \cdots \ e(2p)\right]^T = T\left[e(1) \quad e(2) \ \cdots \ e(p)\right] \quad (35)$$

where T is a $p \times p$ Toeplitz matrix having its first row and first column as R_T and C_T, respectively

$$R_T = \left[1 - T_{11} \quad -T_{12} \quad -T_{13} \quad \cdots \quad -T_{1\ell} \quad 0 \quad \cdots \quad 0\right]$$
$$C_T = \left[1 - T_{11} \quad -T_{21} \quad -T_{31} \quad \cdots \quad -T_{p1}\right]^T \quad (36)$$

Suppose that in ILC, we choose to specify zero tracking error from $e(2)$ to $e(p)$, then in RC the corresponding mapping is

$$\left[e(p+2) \quad \cdots \quad e(2p)\right]^T = T_S\left[e(2) \quad \cdots \quad e(p)\right]^T + T_R e(1) \quad (37)$$

where T_S is T without its first row and first column. In general the $p_S \times p_S$ matrix T_S is T without the rows and columns associated with the tracking errors not specified to be zero. To match the convergence dynamics of ILC to that of RC, we need $I_S - P_S L_S = T_S$. Such a solution for L_S is

$$L_S = P_S^+\left(I_S - T_S\right) \quad (38)$$

where the + denotes the pseudo-inverse. Because T is formed from an RC design that facilitates monotonic convergence of all frequency components of the tracking error, one might expect that $\sigma_i(T_S) < 1$ when the steady-state condition holds. If this is not the case, adjustments to T_S can be made as follows. Let the singular value decomposition of T_S be $T_S = U_S \Sigma_S V_S^T$. Let Σ_S^* be a modified Σ_S where any singular values that are larger than one can be set to be marginally less than one, and a new $T_S^* = U_S \Sigma_S^* V_S^T$ can be used in place of T_S to compute L_S from (38).

8. NUMERICAL EXAMPLES

The examples are based on the experimental model of a link of a 7-degree-of-freedom robot (Elci *et al.*, 2002). For the illustration we use both a 3rd-order model $G_1(s) = G_a G_b$ and a 5th-order model $G_2(s) = G_a G_b G_c$, discretized via a zero-order-hold with $dt = 0.02s$,

$$G_a(s) = \frac{\alpha}{s+\alpha}, G_b(s) = \frac{\omega_1^2}{s^2 + 2\zeta_1\omega_1 s + \omega_1^2}, \ G_c(s) = \frac{\omega_2^2}{s^2 + 2\zeta_2\omega_2 s + \omega_2^2}$$

where $\alpha = 8.8$, $\omega_1 = 37\,rad/s$, $\omega_2 = 113\,rad/s$, $\zeta_1 = \zeta_2 = 0.1$ which is less than the actual value of 0.5 to make the systems more challenging for ILC. The 51-time step desired rise-dwell trajectory (Fig. 1) is short relative to the dynamics of the unit pulse responses of the two models (Fig. 2). The first set of examples is for the 3rd-order model $G_1(s)$. The ILC gains (Fig. 3) are designed from (21) with $W_i = 1$, and $R = I$. The 51×51 matrix P in (3) has one singular value at 2.30×10^{-18}. The matrix P_S is formed by deleting the first row of *P*, and L_S by deleting the first column of *L*. The 50×50 matrix $I_S - P_S L_S$ has one singular value larger than 1 at 1.32. Applying the iterative fine-tuning method given in (28) with Q and R_L as identity matrices of appropriate size reduces the largest singular value to be less than 1 after 7 iterations shown in Fig. 4 for a number of singular values. The monotonic convergence of the tracking error is shown in Fig. 5. Figure 6 reveals that the iteration mainly modifies the upper left and lower right corners of the original L_S. To illustrate the non-iterative method, the 51×51 Toeplitz T is formed from (36), and the 50×50 T_S extracted. Both *T* and T_S are found to have all singular values less than 1. A new ILC gain L_S is computed from (38), and used in the simulation. With the gain derived by the non-iterative method, the monotonic convergence of the tracking error is also shown in Fig. 5 for comparison. Figure 7 reveals how the non-iterative method modifies the original L_S. The second set of examples is for the 5th-order model $G_2(s)$. The 51×51 matrix P now has 2 singular values at 2.25×10^{-13} and 3.33×10^{-19}. Then P_S is formed by deleting the first two rows of *P*, and L_S by deleting the first two columns of L built from (21). The 49×49 matrix $I_S - P_S L_S$ has one singular value larger than 1 at 1.05. The 51×51 Toeplitz T is formed from (36), then the 49×49 T_S is extracted. All the singular values of *T* and T_S are less than 1, hence an ILC gain matrix L_S can be designed directly from (38). Monotonic convergence is indeed observed in Fig. 8. Figure 9 shows how the non-iterative method modifies the original ILC gain matrix L_S for the 5th-order model.

9. REFERENCES

Ahn, H.-S., Moore, K.L., and Chen, Y.Q. (2007). Iterative Learning Control: Robustness and Monotonic Convergence for Interval Systems, Springer-Verlag, London.

Bien, Z. and Xu, J.-X., Editors, (1998) *Iterative Learning Control: Analysis, Design, Integration, and Applications*, Kluwer Academic Publishers, Boston.

Brown, H., Phan, M.Q., Lee, S.C., and Longman, R.W. (2007). Robustified Repetitive Controllers with Monotonic Convergence for Mulitple-Input Multiple-Output Systems. In: *Proceeding of AAS/AIAA Astro-dynamics Specialist Conference*, Mackinac Island, MI.

Elci, H., Longman, R.W., Phan, M.Q., Juang, J.-N., and Ugoletti, R. (2002). Simple Learning Control Made Practical by Zero-Phase Filtering: Applications to Robotics. *IEEE Transactions on Circuits and Systems–I: Fundamental Theory and Applications,* **49**(6), 753-767.

Frueh, J.A. and Phan, M.Q. (2000). Linear Quadratic Optimal Learning Control (LQL). *International Journal of Control,* **73**(10), 832-839.

Lee, S.C., Phan, M.Q., and Longman, R.W. (2006). Multiple-Model Design of Robustified Repetitive Controllers with Optimized Convergence Rate. In: *Proceedings of the AIAA Guidance, Navigation, and Control Conference and Exhibit,* Keystone, CO.

Li, Y. and Longman, R.W. (2007). Addressing Problems of Instability in Intersample Error in Iterative Learning Control. In: *Proceedings of the AAS/AIAA Astrodynamics Specialist Conference*, Mackinac Island, MI.

Longman, R.W., Xu, K., and Phan, M.Q. (2007). Design of Repetitive Controllers in the Frequency Domain for Multi-Input Multi-Output Systems. In: *Proceedings of the AAS/AIAA Astrodynamics Specialist Conference*, Mackinac Island, MI.

Longman, R.W. (2000). Iterative Learning and Repetitive Control for Engineering Practice. *International Journal of Control*, **73**(10), 930-954.

Moore, K. (1993). *Iterative Learning Control for Deterministic Systems*, Advances in Industrial Control, Springer-Verlag, London.

Panomruttanarug B. and Longman, R.W. (2006). Converting Repetitive Control into Stable Learning Control by Iterative Adjustment of Initial State. *Advances in the Astronautical Sciences*, **124**, 667-686.

Panomruttanarug B. and Longman, R.W. (2004). Repetitive Controller Design Using Optimization in the Frequency Domain. In: *Proceedings of the AIAA/AAS Astrodynamics Specialist Conference and Exhibit,* Providence, RI.

Phan, M.Q., Longman, R.W., and Moore, K.L. (2000). A Unified Formulation of Linear Iterative Learning Control. *Advances in the Astronautical Sciences*, **105**, 93-111.

Phan, M.Q., Longman, R.W., Lee, S.C, and Lee, J.W. (2003). System Identification from Multiple Trial Data Corrupted by Non-Repeating Periodic Disturbances. *International Journal of Applied Mathematics and Computer Science*, **13**(2), 185-192.

Phan, M.Q. and Frueh, J.A. (1998). System Identification and Learning Control. *Iterative Learning Control: Analysis, Design, Integration, and Applications*, Z. Bien and J. Xu, (Editors), Kluwer Academic Publishing, Norwell, MA.

Phan, M.Q. and Longman, R.W. (1988). A Mathematical Theory of Learning Control for Linear Discrete Multivariable Systems. In: *Proceedings of the AIAA/AAS Astrodynamics Conference*, Minneapolis, MN.

Rogers, E., Galkowski, K., and Owens, D.H. (2007). Control Systems Theory and Applications for Linear Repetitive Processes, Springer (2007).

Rogers, E. and Owens, D.H. (1992). *Stability Analysis for Linear Repetitive Processes*, Springer (2007).

Takanishi, K., Phan, M.Q., and Longman, R.W. (2005). Multiple-Model Probabilistic Design of Robust Iterative Learning Controllers. In: *Transactions of the North American Manufacturing Research Institution of SME*, **33**, 533-540.

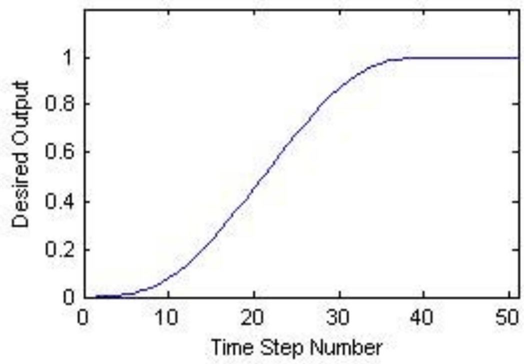

Fig. 1: Desired 51-step trajectory to be tracked.

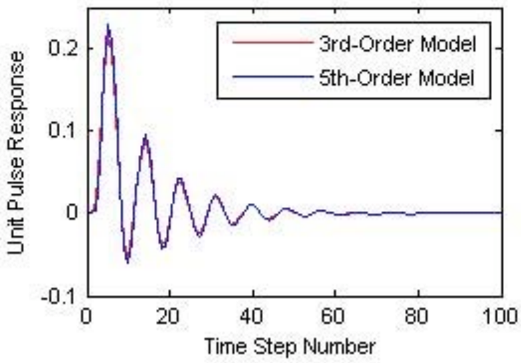

Fig. 2: Unit pulse responses of the 3rd- and 5th-order models.

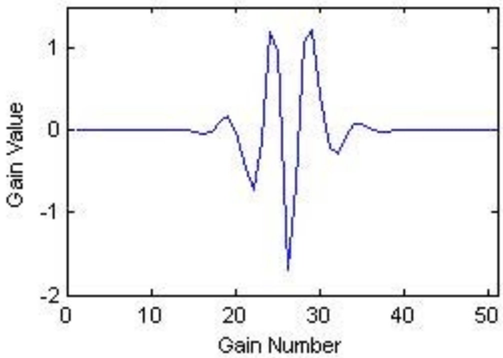

Fig. 3: Original ILC gains $\phi_{26},...,\phi_1,\psi_2,...,\psi_{26}$.

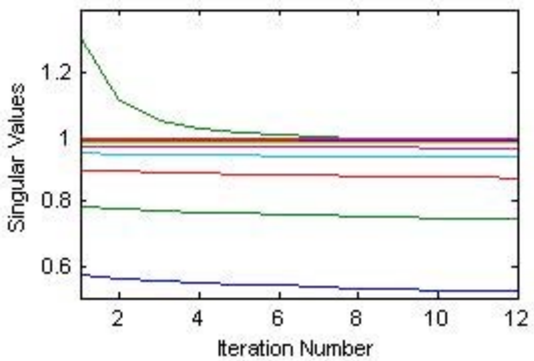

Fig. 4: Convergence of singular values.

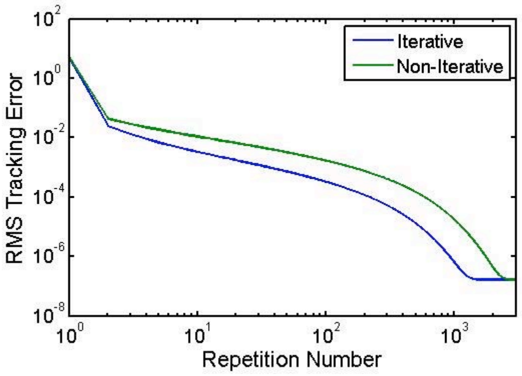

Fig. 5: Monotonic convergence of tracking error.

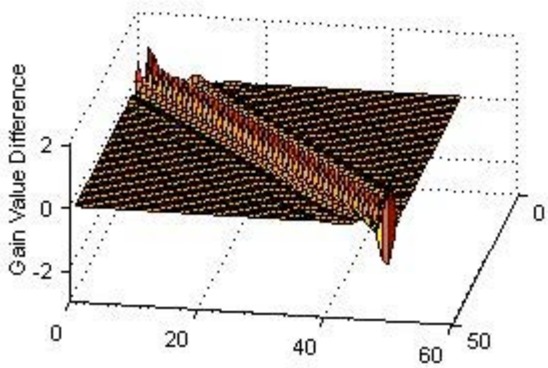

Fig. 6: Difference between original and iterative ILC gain matrices for 3rd-order model.

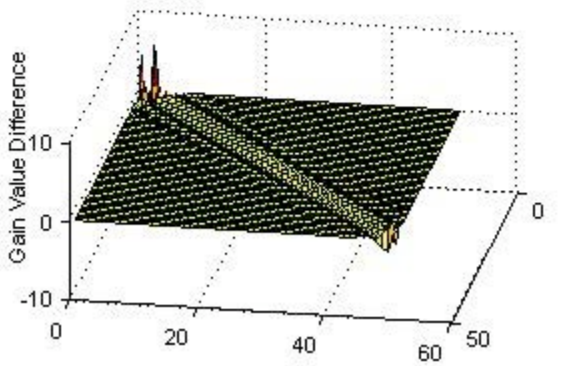

Fig. 7: Difference between original and non-iterative ILC gain matrices for 3rd-order model.

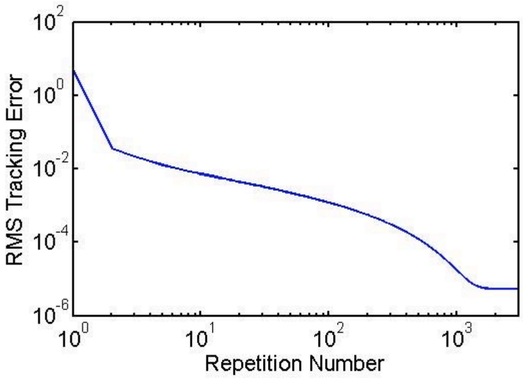

Fig. 8: Monotonic convergence of tracking error.

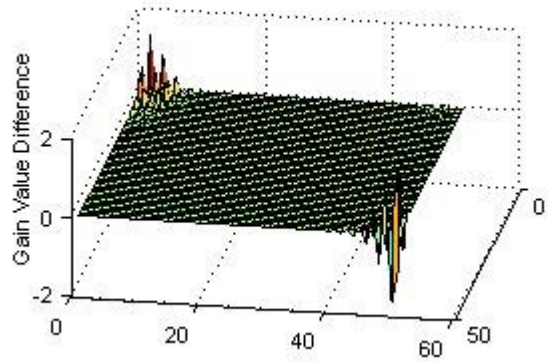

Fig. 9: Difference between original and non-iterative ILC gain matrices for 5th-order model.