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Stable IHMPC for Unstable Systems

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Abstract: An alternative method to formulate the stable Model Predictive Control (MPC) optimization problem, which allows controlling unstable systems with a large domain of attraction, is presented in this work. Usually, stability is guaranteed by means of an appropriate selection of a terminal cost, a terminal constraint, and a local unconstrained controller for predictions beyond the control horizon. This is the case, for instance, of the infinite horizon MPC (IHMPC) with a null local controller, and the dual MPC with a local Linear Quadratic Regulator (LQR). In the last case, the MPC formulation also allows a local optimality. However, its domain of attraction is limited (small, in most of the cases) and depends on the size of the terminal set and the length of the control horizon. Here we propose the inclusion of an appropriate set of slacked terminal constraints into the optimization problem as a way to enlarge the domain of attraction of the MPC that uses the null local controller. In addition, this slack allows a simple offset-free operation in the proximities of the input saturation. Despite the proposed controller does not achieve local optimality, simulations show that its performance is similar to the one obtained with the dual MPC that uses a LQR local controller.

1. INTRODUCTION

MPC is a control strategy that computes the current control by solving, at each time step, a finite-horizon open-loop optimization problem and applying to the system the first element of the optimal sequence so obtained. The strategy incorporates constraints in both, the states and input of the system in a relatively simple way. In addition, a complete framework for analyzing stability, robustness, optimality, etc. has been developed in the last decade (see Mayne et al. 2000, for a survey). Usually, stability is assured by means of an appropriate selection of three components: a terminal cost which is an associated Lyapunov function, a terminal constraint that forces the terminal states to belong to a positively invariant set for the system, and a local unconstrained controller for predictions beyond the control horizon (the strategy that uses such a terminal controller is usually named dual control). For stable systems, a simple choice for the local controller is the null controller $K(x) \equiv 0$ (Rawlings and Muske, 1993), which produces a bounded terminal cost. In the regulator case, such a control strategy produces an unlimited terminal region. However, for unstable system the optimization problem needs to incorporate a terminal constraint that zeroes the unstable modes – since they cannot be steered to the origin by the proposed local null controller –, reducing in this way the original terminal set. Another choice for both, stable and unstable systems consists in using a Linear Quadratic Regulator (LQR) as a local controller (Scokaert and Mayne, 1998). In this case, the controller presents a local optimality, i.e., inside the terminal set the control action obtained by means of the MPC optimization is that of the LQR. However, since the local LQR control is unconstrained, the terminal set, and then, the domain of attraction, could be rather reduced.

The domain of attraction of the MPC controller is the set of states that can be steered to the terminal region in a number of time steps equal or smaller than the control horizon. The size of the domain of attraction depends on the size of the terminal region and the length of the control horizon. The simplest procedure to enlarge the domain of attraction is to increase the control horizon. However, this implies a larger number of decision variables and, consequently, an increase in the computational effort.

Several papers have described how to enlarge the domain of attraction: De Doná et al. (2002), Limon et al. (2005) and Magni et al. (2001). In the first two cases, the authors used a saturated local control law; in Magni et al. (2001), the enlargement of the domain of attraction (for non-linear systems) is obtained by considering a prediction horizon larger than the control horizon. On the other hand, Limon et al. (2005) proposed a contractive terminal set given by a sequence of reachable sets. This approach allows the enlargement of the domain of attraction while maintaining the local optimality of the standard MPC controller.

In this work we present a method to formulate the stable MPC that allows controlling unstable systems with a large domain of attraction. The proposed controller preserves, in general, the performance properties of the standard dual controller (i.e. the dual MPC that uses a LQR as terminal controller). These results are consequence of the inclusion of an appropriate set of slacked terminal constraints into the optimization problem of the dual MPC with a null local controller. Since the local controller is not optimal, the proposed controller does not achieve local optimality. However, simulations show that the input and output performances are quite similar to the one obtained with the standard dual MPC.

2. MODEL DESCRIPTION

Let us consider the (controllable and stabilizable) system

$$\begin{bmatrix} x^{nst}(k+1) \\ x^{st}(k+1) \end{bmatrix} = \begin{bmatrix} F^{nst} & 0 \\ 0 & F^{st} \end{bmatrix} \begin{bmatrix} x^{nst}(k) \\ x^{st}(k) \end{bmatrix} + \begin{bmatrix} B^{nst} \\ B^{st} \end{bmatrix} \Delta u(k), \qquad (1)$$

$$y(k) = \begin{bmatrix} I_{ny} & \Psi \end{bmatrix} \begin{bmatrix} x^{nst}(k) \\ x^{st}(k) \end{bmatrix},$$

where

$x^{nst} \in X^{nst} \subseteq R^{nns}$, $x^{st} \in X^{st} \subseteq R^{ns}$, $\Delta u(k) = u(k) - u(k-1) \in R^{nu}$.

Model (1) is an Output Prediction Oriented Model (OPOM) (see González et al. 2007, for the details of the model formulation). States x^{nst} are related to the non-stable modes of the system, containing both, the integrating poles induced by the incremental form of the model, and the original system unstable modes itself. In this way, it is possible to write $B^{nst} = \begin{bmatrix} B^i \\ B^{un} \end{bmatrix}$, $F^{nst} = \begin{bmatrix} I_{ny} & 0 \\ 0 & F^{un} \end{bmatrix}$, $x^{nst} = \begin{bmatrix} x^{i^T} & x^{un^T} \end{bmatrix}^T$, where I_{ny} is the identity matrix of dimension ny, $x^i \in X^i \subseteq R^{ny}$ are the integrating states, $x^{un} \in X^{un} \subseteq R^{nun}$ are the original unstable modes of the system, and $X^{nst} = X^i \times X^{un}$. On the other hand, x^{st} corresponds to the stable system modes. F^{nst} and F^{st} are diagonal matrices with components of the form $e^{r_i T}$ where r_i is a pole of the system, and T is the sampling period. The system has nns and ns non-stable and stables poles, respectively. In addition, $nns = nun + ny$, where nun is the number of unstable modes, and ny is the number of integrating modes, that is the number of system outputs. Matrix Ψ accommodates the influence of the non-stable and stable states into the output.

The input and input increment are constrained to be:

$u_{\min} \le u \le u_{\max}$,

$-\Delta u_{\max} \le \Delta u \le \Delta u_{\max}$.

To simplify the notation, we define a set U for the input increments Δu as follows:

$$U = \left\{ \Delta u : -\Delta u_{\max} \le \Delta u \le \Delta u_{\max} \text{ and } u_{\min} \le u(k-1) + \Delta u \le u_{\max} \right\}$$

where $u(k-1)$ is the past value of the input u. In addition, we assume that the states are constrained to belong to a set X, given by $X = X^i \times X^{un} \times X^{st}$. In the general case, we assume that X^{un} and X^{st} are unlimited (or large enough) and X^i is defined by the input constraints.

3. PROPOSED MPC

Based on González et al. (2007), we consider the following cost function:

$$V_k = \sum_{j=0}^{\infty} x(k+j/k)^T Q x(k+j/k) + \sum_{j=0}^{m-1} \Delta u(k+j/k)^T R \Delta u(k+j/k) \qquad (2)$$

where

$$x(k+j/k) = \begin{bmatrix} x^{nst}(k+j/k) - x^{sp} \\ x^{st}(k+j/k) \end{bmatrix}, \qquad x^{sp} = \begin{bmatrix} y^{sp} \\ 0 \end{bmatrix},$$

$x(k/k) = x(k)$, $\Delta u(k+j/k)$ is the control move computed at time k to be applied at time $k+j$, m is the control or input horizon, Q and R are positive weighting matrices of appropriate dimensions, and y^{sp} is the output reference. This cost represents the dual MPC cost when a null control law is implemented as the local controller. However, since an infinite horizon is used and the model defined in (1) has integrating and unstable modes, terminal constraints must be added to prevent the cost from becoming unbounded. These constraints can be written as:

$$F^{nst^m}\left(x^{nst}(k) - x^{sp}\right) + C^{nst}\Delta u_k = 0 \left(x^{nst}(k+m/k) - x^{sp} = 0\right) \qquad (3)$$

where

$$\Delta u_k = \begin{bmatrix} \Delta u(k/k)^T & \cdots & \Delta u(k+m-1/k)^T \end{bmatrix}^T,$$

$$C^{nst} = \begin{bmatrix} F^{nst^{m-1}} B^{nst} & F^{nst^{m-2}} B^{nst} & \cdots & B^{nst} \end{bmatrix},$$

$m \ge nns$. (we must have, at least, one degree of freedom per non-stable state)

In addition, we must add the input and input increments constraint

$$\Delta u(k+j/k) \in U, \quad j = 1, \cdots, m-1. \qquad (4)$$

At this point, it is possible to define the domain of attraction for the non-stable modes of the later formulation, using the m-steps stabilizable set to the origin:

$$St_m^{nst}\left(X^{nst}, \{0\}\right) = \left\{ x^{nst}(0) \in X^{nst} : \text{for all } k = 0, \cdots, m-1, \ \exists\, \Delta u(k) \in U \text{ such that } x^{nst}(k) \in X^{nst} \text{ and } x^{nst}(m) \in \{0\} \right\}. \qquad (5)$$

For a small control horizon m, the domain of attraction given by (5) is small. This implies that constraints (3) and (4) become infeasible for moderate disturbances in the non-stable states. One alternative to enlarge the small region where the controller is feasible is to include slack variables into the control problem. To do that, the cost defined in (2) can be rewritten as follows

$$V_k = \sum_{j=0}^{\infty} \left(x(k+j/k) + \overline{\delta}(k,j) \right)^T Q \left(x(k+j/k) + \overline{\delta}(k,j) \right) + \sum_{j=0}^{m-1} \Delta u(k+j/k)^T R \Delta u(k+j/k) + \delta_k^{nst^T} S^{nst} \delta_k^{nst} \qquad (6)$$

where

$\overline{\delta}(k,j) = \begin{bmatrix} F^{nst^j}\delta_k^{nst} \\ 0 \end{bmatrix}$, S^{nst} is a positive matrix of appropriate dimensions, and $\delta_k^{nst} \in \mathbb{R}^{nns}$ is the slack variable that is used to enlarge the terminal set. With the control cost defined in (6), the terminal constraints (3) become

$$F^{nst^m}\left(x^{nst}(k) - x^{sp}\right) + C^{nst}\Delta u_k + F^{nst^m}\delta_k^{nst} = 0 . \tag{7}$$

The slack variable and the corresponding penalization can be written as $\delta_k^{nst} = \begin{bmatrix} \delta_k^i \\ \delta_k^{un} \end{bmatrix}$ and $S^{nst} = \begin{bmatrix} S^i & 0 \\ 0 & S^{un} \end{bmatrix}$, in order to differentiate the integrating states from the unstable ones. Consider now the set Ω, given by

$$\Omega = \left\{ x^{nst} \in X^{nst} : x^i \in X^i \ and \ x^{un} \in \{0\} \right\}, \tag{8}$$

which can be shown to be a control invariant set for states x^{nst}. Then, consider that the initial non-stable state $x^{nst}(k)$ belongs to $St_j^{nst}\left(X^{nst},\Omega\right)$ but it does not belong to $St_{j-1}^{nst}\left(X^{nst},\Omega\right)$, with $m < j \le N$, for an (large) integer *N*. Then, we define the theoretical optimization problem that produces the stable MPC as:

Problem 1

$$\min_{\Delta u_k, \delta_k^{nst}} V_k$$

subject to: (4), (7) and

$$x^{nst}(k+1/k) \in St_{index}^{nst}\left(X^{nst},\Omega\right), \ index = \max(j-1, m) \tag{9}$$

where $St_i^{nst}\left(X^{nst},\Omega\right)$ is the *i*-step stabilizable set to Ω.

Remarks:

* It can be shown that for systems with stable and non-stable modes that remain controllable at the steady state corresponding to the desired output reference, Problem 1 is always feasible with a domain of attraction given by $X^{at} = \left\{ x \in X : x^{nst} \in St_N^{nst}\left(X^{nst},\Omega\right) \right\}$, where N is a finite number, large enough, such that $St_{N+1}^{nst}\left(X^{nst},\Omega\right) \approx St_N^{nst}\left(X^{nst},\Omega\right)$. Also, it can be shown that if weight S^i is sufficiently large, then the control sequence obtained from the solution to Problem 1 at successive time steps drives the output of the closed loop system asymptotically to the reference value.

* Constraint (9) forces the non-stable state to go from one stabilizable set to the next one. Once the non-stable state reaches the *m*-step stabilizable set, the slack of the unstable states, δ_k^{un}, *can be* zeroed.

* Given that the effect of the (slacked) non-stable modes on the cost was zeroed at the end of the control horizon, the cost (6) can be reduced to a two finite terms, using the Lyapunov equation.

* The proposed controller is offset free, since the velocity form of the model adds an integrating state to the observer. In addition, it can steer the system to an operating point where an input saturates, if at the resulting steady state the unstable states remain controllable. Furthermore, if the steady states input is not feasible, the controller will do the best possible (that is, steer the inputs to its bounds) preserving always the feasibility. This property, which is derived from the use of the slack variables, allows a safety operation in real systems.

3.1 Implementation of the MPC algorithm

To implement the proposed algorithm, two initial problems must be solved: it is necessary to compute the stabilizable sets off-line, and it is necessary to ask, at every time step, where the current non-stable state is (i.e. which stabilizable set it belongs to). In the case of having boxes constraints for the inputs, input increments and states (that is, upper and lower bounds), these sets can be computed in a rather simple way, using existing algorithm. Given that all these sets are polyhedron, one can ask if one state belongs or not to the set using a simple and quickly routine. Given a large enough integer *N*,

$$\begin{array}{l}
if \ \ \text{isinset}\left(x^{nst}(0), St_{N-1}^{nst}\left(X^{nst},\Omega\right)\right) = 1 \\
\quad if \ \ \text{isinset}\left(x^{nst}(0), St_{N-2}^{nst}\left(X^{nst},\Omega\right)\right) = 1 \\
\qquad \vdots \\
\qquad\quad if \ \ \text{isinset}\left(x^{nst}(0), St_{m}^{nst}\left(X^{nst},\Omega\right)\right) = 1 \\
\qquad\qquad j = m \\
\qquad\quad else \ \ j = m+1 \\
\qquad \vdots \\
\quad else \ \ j = N-1 \\
else \ \ j = N
\end{array} \tag{10}$$

where "$\text{isinset}(x, X)$" is true if the vector *x* is an element of the polyhedron *X*.

3.2 Case of having one unstable state per input (nu=nun)

In this Sub-section we consider the case (that does occur in most of the real systems) in which each input affects only one unstable state x^{un}. In this case, the behavior of each unstable state can be analyzed independently. It can be shown that the slack reduction performed by constraint (9) can be obtained by means of a norm optimization, which simplifies the whole formulation. The idea is that for the one-dimensional case the successive non-stable stabilizable sets can be associated with the norm of the slack variables of the unstable states. Then, to minimize the norm is equivalent to steer the non-stable states from one stabilizable set to the next one. To properly implement the MPC algorithm, let us consider the following two-stage optimization problem:

Problem 2 a)

$$\min_{\Delta u_{a,k}, \delta_k^{un}} V_{a,k} = \delta_k^{un^T} S^{un} \delta_k^{un} \tag{11}$$

subject to

$$\Delta u_a(k+j/k) \in U,\ j = 0, 1, \cdots, m-1$$

$$F^{un^m} x^{un}(k) + C^{un} \Delta u_{a,k} + F^{un^m} \delta_k^{un} = 0 \quad (12)$$

Problem 2 b)

$$\min_{\Delta u_{b,k}, \delta_k^{un}, \delta_k^i} V_{b,k} = \sum_{j=0}^{\infty} \left(x(k+j/k) + \overline{\delta}(k,j) \right)^T Q \left(x(k+j/k) + \overline{\delta}(k,j) \right) + \sum_{j=0}^{m-1} \Delta u_b(k+j/k)^T R \Delta u_b(k+j/k) + \delta_k^{i\,T} S^i \delta_k^i \quad (13)$$

subject to:

$$\Delta u_b(k+j/k) \in U,\ j = 0, 1, \cdots, m-1$$

$$F^{i^m} \left(x^i(k) - y \right)^{sp} + C^i \Delta u_{b,k} + F^{i^m} \delta_k^i = 0 \quad (14)$$

$$F^{un^m} x^{un}(k) + C^{un} \Delta u_{b,k} + F^{un^m} \delta_k^{un} = 0, \qquad \delta_k^{un} = \delta_k^{un^*}.$$

Where $\delta_k^{un^*}$ is the optimal slack obtained in Problem 2 a),

$C^{un} = \left[F^{un^{m-1}} B^{un} \quad F^{un^{m-2}} B^{un} \quad \cdots \quad B^{un} \right]$, and

$C^i = \left[F^{i^{m-1}} B^i \quad F^{i^{m-2}} B^i \quad \cdots \quad B^i \right]$.

Remark:

* It can be shown that for systems with stable and non-stable modes in which each input affects only one unstable state x^{un}, the sequential solution of problems 2 a) and 2 b) is always feasible with a domain of attraction given by $X^{at} = \left\{ x \in X : x^{nst} \in St_N^{nst}\left(X^{nst}, \Omega \right) \right\}$. Also, if the weight S^i is sufficiently large, then the control sequence obtained from the solution to Problems 2a and 2 b) at successive time steps drives the output of the closed loop system asymptotically to the reference value.

4. DOMAIN OF ATTRACTION

Let us assume that a large number N exists, such that $St_N^{nst}\left(X^{nst}, \Omega \right) \approx St_{N+1}^{nst}\left(X^{nst}, \Omega \right) \triangleq St_\infty^{nst}\left(X^{nst}, \Omega \right)$. Then, the domain of attraction of the proposed controller is given by $St_\infty^{nst}\left(X^{nst}, \Omega \right)$ for the non-stable states x^{nst} - which is the largest possible set, and by X^{st} for the stable states x^{st} (unlimited in the usual case of unconstrained states).

On the other hand, the domain of attraction of the standard dual MPC is given by the m-stablizable set to the terminal set, $St_m\left(X, O_\infty^K \right)$, where O_∞^K is the terminal set. This terminal set, in turn, is given by the maximal positive invariant set under the unconstrained control law $\Delta u = -Kx$, where K is obtained form the algebraic Riccati equation:

$$O_\infty^K = \left\{ x(0) \in X : (A - BK)^j x(0) \in X, \; K(A - BK)^j x(0) \in U \;\; j = 0,1,\cdots \right\}. \quad (15)$$

Note that this set is contained into the input admissible subset of X:

$$O_\infty^K = \left\{ x(0) \in X : Kx(0) \in U \right\}. \quad (16)$$

Then, it is clear that the terminal set O_∞^K produces a limited domain of attraction for all the states x^{nst} and x^{st} (because the terminal controller is unconstrained, and then it must be assured that $\Delta u = -Kx \in U$ for all predictions beyond k+m).

Since $St_\infty^{nst}\left(X^{nst}, \Omega \right)$ is by definition the maximal set of non-stable states that can be steered to the origin, then $St_\infty^{nst}\left(X^{nst}, \Omega \right) \supseteq St_m^{nst}\left(X^{nst}, O_\infty^K \right)$, where $St_m^{nst}\left(X^{nst}, O_\infty^K \right)$ is the orthogonal projection of $St_m\left(X, O_\infty^K \right)$ into the non-stable states space. In other words, the m-stabilizable set to the terminal set O_∞^K, projected onto the non-stable subspace is included into the maximal stabilizable set for the non-stable states to the origin (generally, the later set is quite larger than the first). This means that the proposed controller has a larger domain of attraction for the non-stable modes of the system (In fact, it has the largest domain of attraction for the non-stable states that the system allows). In addition, the standard dual MPC presents a limited domain of attraction for the stable modes x^{st}, even if X^{st} is unlimited.

Recall that the proposed controller is not optimal, in the sense that it does not use an optimal control as terminal controller. Then, even for the unconstrained case, the predictions are not coincident with the actual behave, and it does not behave as LQR. However, as it will be shown in the next section, its performance is similar to the standard dual MPC case.

5. ILUSTRATIVE EXAMPLE

In order to evaluate domain of attraction and the performance of the proposed controller, we consider in this section a jacketed continuous stirred tank reactor (CSTR), studied in Henson and Seborg, 1997. A discrete state space model of the CSTR was presented in Pannochia and Kerrigan, 2005, using a sampling time of T=0.1 min and considering deviation variables. The model is as follows:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.7776 & -0.0045 \\ 26.6185 & 1.8555 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} -0.0004 \\ 0.2907 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}, \quad (17)$$

where the states x_1 and x_2 represent the reactant concentration and the reactor temperature, respectively; and the input u represents the coolant liquid temperature. The output controlled variable y corresponds to the reactor temperature. The input constraints imposed to the system are as follows (in this example no state constraints are considered):

$$u_{\min} = -15, \; u_{\max} = 15 \quad (18)$$

Using model (17) and trough an adequate transformation, the following velocity model is obtained:

$$x(k+1)=\begin{bmatrix}1 & 0 & 0\\ 0 & 1.7297 & 0\\ 0 & 0 & 0.9034\end{bmatrix}x(k)+\begin{bmatrix}1.2599\\ 0.7635\\ -0.2937\end{bmatrix}\Delta u(k)$$

$$y(k)=\begin{bmatrix}-0.6082 & 1 & -0.9994\end{bmatrix}x(k)$$

where can be seen an original unstable state, an integrating state induced by the velocity form of the model (these states are the non-stable state x^{nst}), and a stable state (this state is x^{st}). Notice that the input constraints of the original state space model (Eq. (18)) was adapted to the velocity model by means of an appropriate transformation. In addition, we consider an input increment constraint given by $\Delta u_{\max}=2.5$.

For the proposed MPC controller (that is derived from Problem 2a and 2b) the tuning nominal parameters are: Q=0.5, R=1, S^{i}=50, S^{un}=5000, m=2. As it has been said, the domain of attraction of the standard dual MPC controller depends on the aggressiveness of the terminal LQR controller, and on the control horizon m. A set of parameters that represent a reasonable trade-off between the dimension of the domain of attraction and the performance of the controller is: Q=0.5, R=1, m=2. The LQR controller used as terminal controller, which was obtained from the Riccati equation, is: $K=\begin{bmatrix}-0.2566 & 2.7127 & 0.0177\end{bmatrix}$. Fig. 1 shows the resulting domain of attraction for the non-stable states x^{nst} of the two MPC controllers (in order to make a comparison, it is only shown here the non-stable state domain of attraction. However, it should be noted that the main advantage of the proposed method could be seen in the stable state domain of attraction, which is unlimited for the simulated case). For the proposed MPC controller, the domain of attraction is given by the maximal stabilizable set to the set Ω, which in this case is given by $St_{N_{\max}}^{nst}\left(X^{nst},\Omega\right)\approx St_{N_{\max}+1}^{nst}\left(X^{nst},\Omega\right)$, with N_{max}=8. To test the performance of both controllers (adopting the nominal parameters), we simulate the closed loop system for different kinds of unmeasured disturbances. First, we simulate a state disturbance that corresponds to $x(0)=\begin{bmatrix}-15.7 & 1.08 & 0\end{bmatrix}^T$. In the same Fig. 1 is shown the non-stable state evolution for both controllers. In the left hand side of Fig. 1 can be seen that for the proposed starting state the standard dual MPC cannot steer the non-stable state to the origin without violating some of the constraints. In this case, the input constraint (the input is associated with the first state) should not be respected in the first two time steps.

The time response of the input u, the input increment Δu and the output y are shown in Fig. 2, for the proposed controller and for the dual MPC. In the same figure, it can be seen that for the dual MPC the input u surpasses the max bound at the beginning of the simulation.

Next we simulate a non-stable state disturbance given by $x(0)=\begin{bmatrix}15 & 2.38 & 0\end{bmatrix}^T$, which corresponds to a feasible state for the two controllers. The non-stable state evolution can be seen in Fig. 1, and the input u, the input increment Δu and the output y time response can be seen in Fig. 3. Notice that the proposed MPC and the standard dual MPC have a similar performance for a disturbances that correspond to feasible states. This fact represents a relevant point; since the main disadvantage of the proposed MPC controller is that its implicit control law is not optimal in the feasible region.

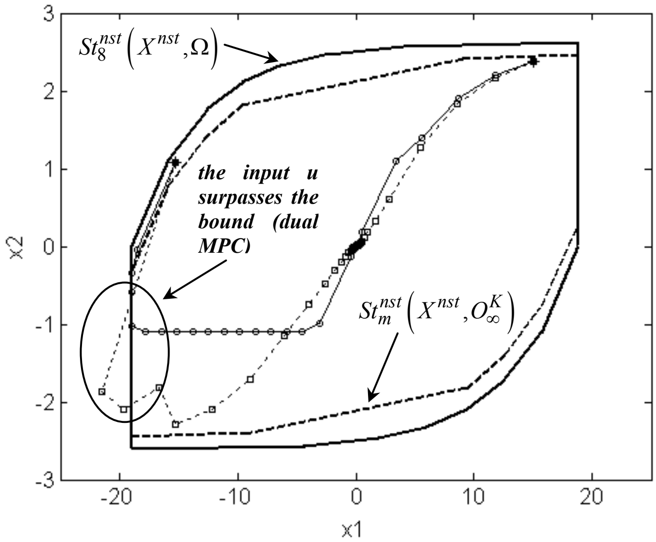

Fig. 1. Domain of attraction of the proposed MPC (solid line), and domain of attraction for the standard dual MPC (dashed line), considering only the non-stable state space. Non-stable states (x^{nst}) evolution: proposed MPC (solid line and circles), standard dual MPC with nominal parameters (dashed line and squares).

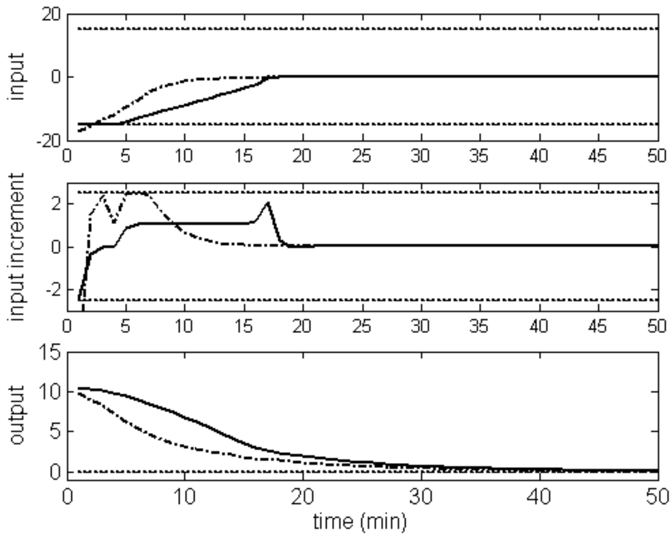

Fig. 2. Input and output evolutions for the initial state $x(0)=\begin{bmatrix}-15.7 & 1.08 & 0\end{bmatrix}^T$. Proposed MPC (solid line), standard dual MPC (dashed-dotted line).

Finally, Fig. 4 shows the non-stable states evolution (proposed MPC controller) for the starting points $x(0)=\begin{bmatrix}15 & 2.38 & 0\end{bmatrix}^T$, and the stabilisable sets

$St_j^{nst}\left(X^{nst},\Omega\right)$ for j=2,3,4. Notice that the control actions obtained from Problem 2a) and 2 b) steer the non-stable state from one stabilizable set to the next, until it reaches $St_m^{nst}\left(X^{nst},\Omega\right)$, as was established in the Sub-section 3.2.

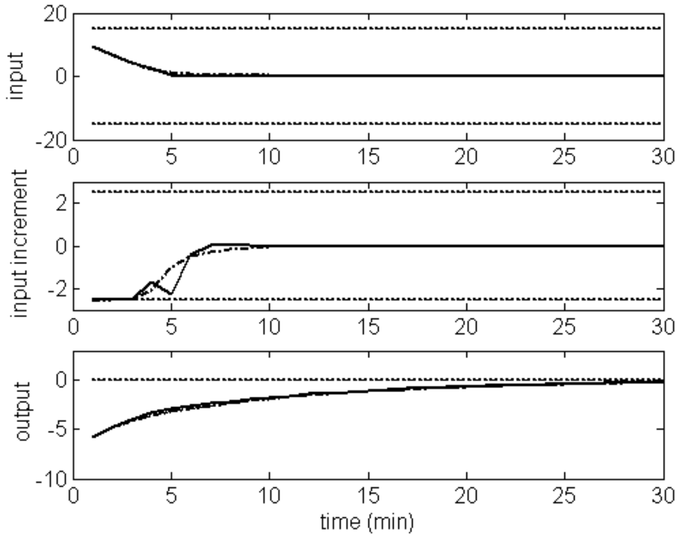

Fig. 3. Input and output evolutions for the initial state $x(0)=\begin{bmatrix}15 & 2.38 & 0\end{bmatrix}^T$. Proposed MPC (solid line), standard dual MPC (dashed-dotted line).

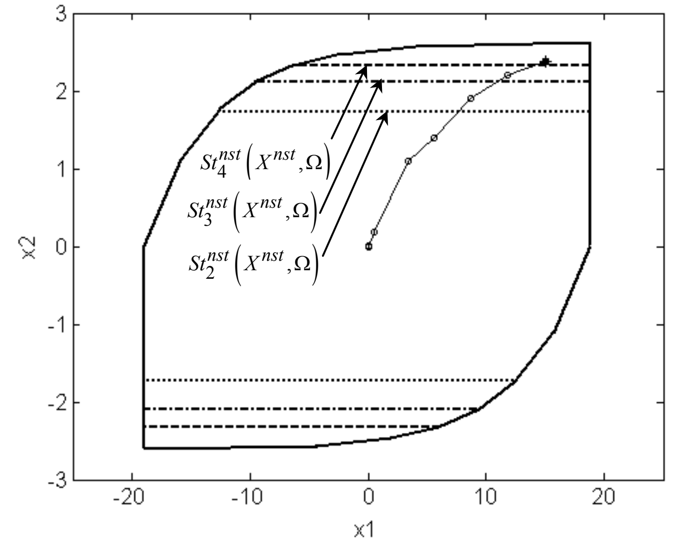

Fig. 4. Non-stable states (x^{nst}) evolution for the proposed MPC (solid line and circles) and Stabilisable sets: $St_4^{nst}\left(X^{nst},\Omega\right)$ (dashed line), $St_3^{nst}\left(X^{nst},\Omega\right)$ (dash-dotted line), $St_2^{nst}\left(X^{nst},\Omega\right)$ (dotted line).

6. CONCLUSIONS

In this paper a different formulation of the stable MPC is presented, which includes an appropriate set of slack variables into the optimization problem. The formulation allows a larger domain of attraction in comparison with a standard dual MPC. In addition, it guarantees recursive feasibility when the system is guided to a point in which the input saturates, or even, surpasses the bounds (this is not showed in the simulations), and it guarantees an offset-free operation without the necessity of a target calculation stage. Despite the proposed controller has not a local optimality, it shows a relatively good performance, and similar in many cases to the standard dual MPC that uses a LQR as a local controller.

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