Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

An Adaptive Sinusoidal Disturbance Rejection Controller for Single-Input-Single-Output Systems ⋆

W. C. Gan * L. Qiu ** J. Wang ***

* ASM Assembly Automation Ltd., Hong Kong. (e-mail: wcgan@asmpt.com)
** Department of Electrical and Electronic Engineering, The Hong Kong University of Science and Technology, Hong Kong. (e-mail: eeqiu@ece.ust.hk)
*** Department of Control Theory and Engineering, Harbin Institute of Technology, China.

Abstract: The design of an adaptive sinusoidal disturbance rejection controller for Single-Input-Single-Output (SISO) systems is presented in this paper. Sinusoidal disturbance rejection problems exist in industry applications such as velocity ripples in a CNC milling machine. The controller is first developed based on the Internal Model Principle (IMP) and the pole-zero placement technique, then a Gain Scheduled (GS) robust Two-Degree-of-Freedom (2DOF) regulator is constructed to eliminate the sinusoidal disturbances with known frequencies and to achieve a desirable tracking response simultaneously, but without estimating the amplitude and the phase values of the sinusoidal disturbances. Using the small gain theorem, the system stability radius is obtained for n slow time varying sinusoidal disturbances. Finally, the proposed controller is applied to eliminate the torque and velocity ripples in the Alternating Current (AC) Permanent Magnet (PM) motor control systems.

1. INTRODUCTION

Sinusoidal disturbance rejection problems always exist in industry applications such as velocity ripples in CNC milling machines (Liu and Chen [1994]) and precision mechanisms (Spindler [2004]). Therefore, sinusoidal disturbance rejection controller development is important in precision motion control industry. In this paper, an adaptive sinusoidal disturbance rejection controller is developed for Single-Input-Single-Output (SISO) systems and the disturbance frequencies are assumed to be known. The controller is first developed based on the Internal Model Principle (IMP) and the pole-zero placement technique, then a Gain Scheduled (GS) robust Two-Degree-of-Freedom (2DOF) regulator is constructed to eliminate the sinusoidal disturbances with known frequencies and to achieve a desirable tracking response simultaneously, but without estimating the amplitude and the phase values of the sinusoidal disturbances.

To validate the effectiveness of the controller, the proposed control algorithm will be applied to the torque and velocity ripple elimination of Alternating Current (AC) Permanent Magnet (PM) systems due to its importance in precision motion control industry (Chen and Paden [1993]; Liu and Chen [1994]; Petrovic et al. [2000]; Ferretti et al. [1998]).

In the past, there were many different techniques to eliminate the torque ripples of AC PM motor control systems. Broadly speaking, these techniques fall into two major categories. The first class consists of techniques that concentrate on the motor design so that it can eliminate the cogging and reluctance torque ripple generation of AC PM motors. Secondly, different adaptive control algorithms have been applied to eliminate the torque ripple of AC PM motor control systems (Ferretti et al. [1998]).

The common way to eliminate sinusoidal torque ripples in AC PM motor control systems is to purchase high-grade components and the total cost of an automation machine is thus much increased (Gan and Qiu [2004]). To replace the inefficient and high-cost solution, the development of a novel and cost-effective speed control algorithm to eliminate the torque ripples is discussed in this paper.

In the present literature (Gan and Qiu [2004]), the DC current offsets can be approximated by a sinusoidal function with a known frequency. The Internal Model Principle (IMP) and the pole-zero placement technique is then used to design a robust 2DOF speed regulator to eliminate the torque and velocity ripples with a single disturbance frequency. However, an AC PM motor control system usually has output sinusoidal ripples. For example, DC offsets are always present at the motor terminals, gain error exists between phase; in addition, gain nonlinearity is difficult to avoid in the current control loop. The above disturbances can be modeled as sinusoidal functions with one, two and three times of the rotor electrical frequency respectively (Gan et al. [2005]). In this paper, the extension work of (Gan and Qiu [2004]) is discussed and the proposed controller can be generalized to n sinusoidal disturbances. The stability radius of the system can also be derived with the small gain theorem. If the acceleration

⋆ This work is supported by Hong Kong Research Grants Council under Project HKUST6163/04E.

978-1-1234-7890-2/08/$20.00 © 2008 IFAC 15684 10.3182/20080706-5-KR-1001.0841

profile, is available to the controller, then the stability radius can be further enlarged.

This paper is organized as follows. In Section 2, the development of the adaptive sinusoidal disturbance rejection controller for SISO systems, is addressed in details. The IMP and the pole-zero placement technique is used to design a controller with the hypothesis that the disturbance frequencies are constants. Then the Gain Scheduled (GS) robust 2DOF speed regulator is developed for a time-varying disturbance frequencies in Section 3. The stability issue of the proposed GS robust 2DOF speed regulator is also addressed. In Section 4, the application of the proposed controller to AC PM motor systems is given. This includes a brief review on the vector control of AC PM motors and the modeling of the torque ripples are given. With the acceleration profile input, the stability radius of the system is enlarged. The simulation results are presented in Section 5 to support the proposed controller. Finally, some concluding remarks are given in Section 6.

2. ROBUST 2DOF REGULATOR DESIGN

Robust 2DOF regulators were discussed in (Wolovich [1994]). In the following we assume that the disturbance and reference may have different modes. In reference to Fig. 1, the plant $G(s)$ is assumed to be a general SISO system while the reference input $r(t)$, and the disturbance input $d(t)$, are assumed to have possibly different modes.

Let $G(s)$ be a SISO plant described by a strictly proper transfer function $G(s) = b(s)/a(s)$, where

$$a(s) = s^{n_a} + a_1 s^{n_a-1} + \cdots + a_{n_a}$$
$$b(s) = b_1 s^{n_a-1} + b_2 s^{n_a-2} + \cdots + b_{n_a}$$

and it is assumed that $a(s)$ and $b(s)$ are coprime. The general 2DOF controller shown in Fig. 1 can be written as

$$[\, K_1(s) \;\; -K_2(s)\,] = \frac{1}{k(s)} [\, q(s) \;\; -h(s)\,] \quad (1)$$

where

$$k(s) = s^{n_k} + k_1 s^{n_k-1} + \cdots + k_{n_k} \quad (2)$$
$$q(s) = q_0 s^{n_k} + q_1 s^{n_k-1} + \cdots + q_{n_k} \quad (3)$$
$$h(s) = h_0 s^{n_k} + h_1 s^{n_k-1} + \cdots + h_{n_k}. \quad (4)$$

Then the 2DOF control structure becomes the following one as shown in Fig. 2. The transfer function from the input reference to the output is given by

$$\frac{Y(s)}{R(s)} = \frac{b(s)q(s)}{a(s)k(s) + b(s)h(s)} = \frac{b(s)q(s)}{\delta(s)}$$

and the transfer function from the disturbance input to the output is given by

$$\frac{Y(s)}{D(s)} = \frac{b(s)k(s)}{\delta(s)}$$

where $\delta(s)$ is the closed loop characteristic polynomial of the system. Let the unstable modes of $r(t)$ be the roots of monic polynomial $m_r(s)$ and those of $d(t)$ be the roots of $m_d(s)$. Let the least common multiple of $m_r(s)$ and $m_d(s)$ be $m(s)$. It is well-known that the robust regulator

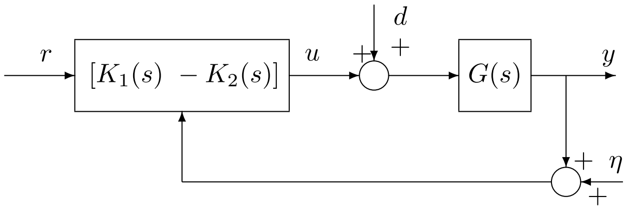

Fig. 1. A 2DOF controller structure.

problem is solvable, i.e., it is possible to design a controller so that the disturbance rejection and reference tracking are achieved, if and only if $m(s)$ and $b(s)$ are coprime. The detailed design of $q(s)$, $h(s)$ and $k(s)$ using IMP and pole placement technique can be found in (Gan and Qiu [2004]).

3. GAIN SCHEDULED CONTROLLER DESIGN

In this section, the design of a Linear Time Invariant (LTI) robust 2DOF controller for a constant speed reference is first discussed. Then a GS robust 2DOF controller for a slowly time-varying speed step reference is designed by modifying the LTI controller.

The problem of accomplishing robust tracking and disturbance rejection is called the robust regulator problem. The key idea is that the controller should include the unstable modes of the reference and disturbance according to the IMP. We also propose to use a 2DOF controller structure to achieve better transient responses and simpler designs. A 2DOF controller has a structure as shown in Fig. 1 with η denotes the sensor noise. One of its advantages, in comparison with the usual one degree of freedom or unity feedback structure, which amounts to setting $K_1(s) = K_2(s)$, is that the tracking performance depends mainly on $K_1(s)$, the robustness and the disturbance rejection performance depends only on $K_2(s)$. Hence $K_1(s)$ and $K_2(s)$ can be designed with different considerations (Zhou and Zhang [2000]).

3.1 Design of LTI 2DOF Controller with Constant Frequencies

The 2DOF regulator structure employed in our analysis is shown in Fig. 2. Here we follow the design procedure for the robust 2DOF regulator using pole-zero placement technique as in (Gan and Qiu [2004]). Since the reference $r(t)$ is a step reference, it follows that $m(s) = s$. The disturbance $d(t)$ contains n sinusoidal functions, it follows that $m_d(s) = s(s^2+\omega_1^2)(s^2+\omega_2^2)\cdots(s^2+\omega_n^2)$. Therefore $m(s)=s(s^2+\omega_1^2)(s^2+\omega_2^2)\cdots(s^2+\omega_n^2)$. It follows that $m(s)a(s)$ and $b(s)$ are coprime and a solution to the robust

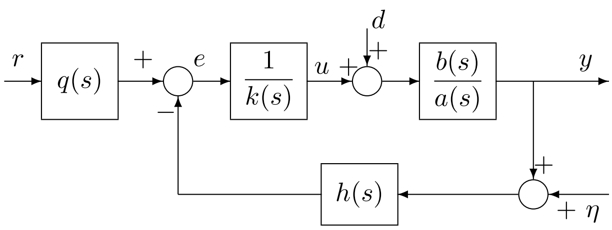

Fig. 2. A 2DOF regulator structure.

regulator problem based on the IMP exists. Since $n_a = 1$, we can choose $n_g = 0$. This leads to a controller of order equal to n_m, which is the lowest possible to achieve robust regulator. Hence

$$\begin{aligned} k(s) &= s(s^2+\omega_1^2)(s^2+\omega_2^2)\cdots(s^2+\omega_n^2) \\ &= s^{2n+1} + k_1 s^{2n-1} + \cdots + k_n s, \end{aligned} \tag{5}$$

and $h(s)$, $q(s)$ have the following forms

$$\begin{aligned} h(s) &= h_0 s^{2n+1} + h_1 s^{2n} + \cdots + h_{2n+1} \\ q(s) &= q_0 s^{2n+1} + q_1 s^{2n} + \cdots + q_{2n+1}. \end{aligned}$$

Choose the closed loop poles $\alpha_1,\ \alpha_2,\ \cdots,\ \alpha_i,\ \cdots,\ \alpha_{2n+1}$ according to the disturbance rejection specification and the remaining closed loop poles α_{2n+2} according to the transient tracking response specification so that the closed loop characteristic polynomial is

$$\begin{aligned} \delta(s) &= (s+\alpha_1)(s+\alpha_2)\cdots\ (s+\alpha_{2n+2}) \\ &= s^{2n+2} + \delta_1 s^{2n+1} + \delta_2 s^{2n} + \cdots + \delta_{2n+2}. \end{aligned}$$

Then by equating the coefficients of both sides of

$$\begin{aligned} \delta(s) &= k(s)a(s) + b(s)h(s) \\ &= s^{2n+2} + (a+bh_0)s^{2n+1} + (k_1+bh_1)s^{2n} \\ &\quad + \cdots + (k_i + bh_{2i-1})s^{2n+2-2i} \\ &\quad + (ak_i + bh_{2i})s^{2n+1-2i} + \cdots + bh_{2n+1}, \end{aligned} \tag{6}$$

we can get

$$\begin{aligned} h_0 &= \frac{1}{b}(\delta_1 - a),\ \ h_1 = \frac{1}{b}(\delta_2 - k_1) \\ &\vdots \\ h_{2i} &= \frac{1}{b}(\delta_{2i+1} - ak_i),\ \ h_{2i+1} = \frac{1}{b}(\delta_{2i+2} - ak_{i+1}) \\ &\vdots \\ h_{2i} &= \frac{1}{b}(\delta_{2i+1} - ak_i),\ \ h_{2n+1} = \frac{1}{b}\delta_{2n+2}. \end{aligned} \tag{7}$$

Finally, as $m_r(s) = s$, we can arbitrarily assign the roots of $q(s)$. Here we choose the $2n+1$ roots of $q(s)$ to be exactly the same as the roots of $\delta(s)$ subject to the constraint

$$\begin{aligned} q_{2n+1} &= h_{2n+1} \\ q(s) &= \frac{h_{2n+1}}{\prod_{i=1}^{2n+1}\alpha_i}(s+\alpha_1)\cdots(s+\alpha_{2n+1}). \end{aligned} \tag{8}$$

The coefficients of $q(s)$, q_i $(i = 1, 2\cdots n)$ are obtained. The transfer function is finally given by $\dfrac{Y(s)}{R(s)} = \dfrac{\alpha_{2n+2}}{s+\alpha_{2n+2}}$.

3.2 Slowly Time-Varying Frequencies

When the sinusoidal disturbances have varying frequencies, we need to include internal modes which vary with the disturbance frequencies; other parameters of the controller in general also need to be changed with time to ensure that the closed loop system is stable. The disturbance frequencies ω_i and k_i in (5), (6) and (7) are replaced by $\omega_i(t)$ and $k_i(t)$ $(i = 1, 2\cdots n)$ respectively. Using the following observer canonical realization to implement the regulator with $\hat{x}(t) = [\, x_1(t)\ x_2(t)\ \cdots\ x_{2n+1}(t)\,]'$ and v(t)=$[\, r(t)\ \ y(t)+\eta(t)\,]$', we get

$$\begin{aligned} \dot{\hat{x}}(t) &= A_K(t)\hat{x}(t) + [\, B_{K1}\ \ B_{K2}\,]v(t) \\ u(t) &= C_K(t)\hat{x}(t) + [\, D_{K1}\ \ D_{K2}\,]v(t) \end{aligned}$$

where

$$A_K(t) = \begin{pmatrix} N_{2n\times1}(t) & I_{2n\times2n} \\ 0 & 0_{1\times2n} \end{pmatrix}$$

$$B_K(t) = \begin{pmatrix} q_1 & -h_1(t) \\ q_2 - q_0 n_1(t) & -h_2(t) + h_0(t)k_1(t) \\ q_3 & -h_3(t) \\ \vdots & \vdots \\ q_{2i} - q_0 n_i(t) & -h_{2i}(t) + h_0(t)k_i(t) \\ q_{2i+1} & -h_{2i+1}(t) \\ \vdots & \vdots \\ q_{2n+1} & -h_{2n+1}(t) \end{pmatrix}$$

$$\begin{aligned} C_K(t) &= [\, 1\ 0\ \cdots\ 0\ 0\,]_{1\times(2n+1)} \\ D_K(t) &= [\, q_0\ \ -h_0(t)\,] \end{aligned}$$

and $N_{2n\times1}(t) = [\, 0\ k_1(t)\ 0\ k_2(t)\ \cdots\ 0\ k_n(t)\,]'$. Fig. 3 shows the block diagram of the GS robust 2DOF speed regulator and the linearized AC PM motor system plant can be represented by the following state space equations (Vidyasagar [1985])

$$\begin{aligned} \dot{x}(t) &= -ax_{2n+2}(t) + b[u(t) + d(t)] \\ y(t) &= x_{2n+2}(t). \end{aligned} \tag{10}$$

Let

$$\begin{aligned} \tilde{x}(t) &= [\, x_1(t)\ x_2(t)\ \cdots\ x_{2n+2}(t)\,]' \\ w(t) &= [\, r(t)\ d(t)\ \eta(t)\,]' \\ \tilde{y}(t) &= [\, u(t)\ y(t)\,]', \end{aligned}$$

the closed loop state space equations can be written as follows

$$\begin{aligned} \dot{\tilde{x}}(t) &= A(t)\tilde{x}(t) + B(t)w(t) \\ \tilde{y}(t) &= C(t)\tilde{x}(t) + D(t)w(t). \end{aligned}$$

The problem now boils down to choose the transformation matrix $P(t)$ to transform $A(t)$ into an observer canonical form so as to facilitate the stability analysis by the small gain theorem. Let z(t)=$[\, z_1(t)\ z_2(t)\ \cdots\ z_{2n+2}(t)\,]$' be the new state variables. We have $\tilde{x}(t) = P(t)z(t)$ and

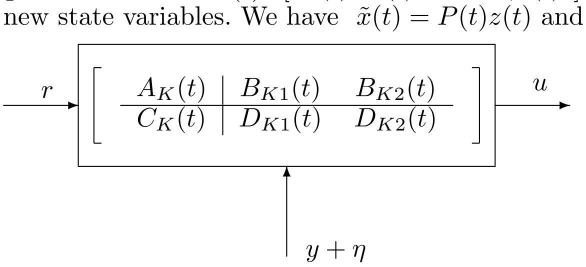

Fig. 3. The GS robust 2DOF speed regulator.

$$P(t)=\begin{pmatrix} 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & -k_n(t) \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & -k_i(t) \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & -k_1(t) \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & b \end{pmatrix}.$$

The closed loop system state space equations can be transformed into the following

$$\begin{aligned}\dot{z}(t) &= A_z(t)z(t)+B_z(t)\tilde{u}(t)\\ \tilde{y}(t) &= C_z(t)z(t)+D_z(t)\tilde{u}(t)\end{aligned}$$

where $$\mathrm{A}_z(t)=\begin{pmatrix} 0 & 0 & 1 & \cdots & -\delta_{2n+2} \\ 1 & 0 & 0 & \cdots & -\delta_{2n+1}-\dot{k}_1(t) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & -\delta_{2i} \\ \vdots & \vdots & \vdots & \cdots & -\delta_{2n+1-2i}-\dot{k}_i(t) \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & -\delta_1 \end{pmatrix}$$

$$B_z(t)=\begin{pmatrix} q_{2n+1} & 0 & -h_{2n+1}(t) \\ q_{2n} & n_n(t) & -h_{2n}(t) \\ q_{2n-1} & 0 & -h_{2n-1}(t) \\ q_{2n-2} & n_{n-1}(t) & -h_{2n-2}(t) \\ \vdots & \vdots & \vdots \\ q_{2i} & n_{n-i+1}(t) & -h_{2i}(t) \\ q_{2i-1} & 0 & -h_{2i-1}(t) \\ \vdots & \vdots & \vdots \\ bq_0 & b & -bh_0(t) \end{pmatrix}$$

$$C_z(t)=\begin{pmatrix} 0 & 0 & 1 & 0 & \cdots & 0 & -bh_0(t) \\ 0 & 0 & 0 & 0 & \cdots & 0 & b \end{pmatrix}$$

$$D_z(t)=\begin{pmatrix} q_0 & 0 & -h_0(t) \\ 0 & 0 & 0 \end{pmatrix}.$$

Here $A_z(t)$ is a $(2n+2)\times(2n+2)$ matrix. Suppose I_{kj} is the entry of the the first $2n+1$ columns in A_z, where

$$I_{kj}=\begin{cases} 1 & k=2i, j=2n-2i+2 \\ & k=2i-1, j=2i+1 \quad (i=1,2\cdots n) \\ 0 & \text{otherwise} \end{cases}$$

and $B_z(t)\in R^{(2n+2)\times 3}$ and $C_z(t)\in R^{2\times(2n+2)}$. Next let

$$\begin{aligned}L(t) &= P^{-1}(t)\dot{P}(t)\\ &= \left[\,0\ -\dot{k}_1(t)\ 0\ \cdots\ -\dot{k}_n(t)\ 0\ 0\,\right]'\end{aligned}$$

and $\mathrm{M}=\mathrm{P}^{-1}(t)A_z(t)P(t)$. Therefore,

$$M=\begin{pmatrix} 0 & 0 & 1 & \cdots & -\delta_{2n+2} \\ 1 & 0 & 0 & \cdots & -\delta_{2n+1} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \cdots & -\delta_{2i} \\ \vdots & \vdots & \vdots & \cdots & -\delta \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & -\delta_1 \end{pmatrix}.$$

Here M is a $(2n+2)\times(2n+2)$ matrix.

According to the small gain theorem (Vidyasagar [1985]), the associated autonomous system

$$\begin{aligned}\dot{z}(t) &= A_z(t)z(t)\\ &= \left(M+RL(t)\left[\,0\ \cdots\ 0\ 1\,\right]_{1\times(2n+2)}\right)z(t)\end{aligned}$$

is internally uniformly exponentially stable if the following condition is satisfied:

$$\begin{aligned}\|L(t)\|_\infty &< \left\|\left[\,0\ \cdots\ 0\ 1\,\right]_{1\times(2n+2)}(sI-M)^{-1}R\right\|_\infty^{-1}\\ &= \left\|\frac{\left[\,0\ s\ \cdots\ 0\ s^{2i-1}\ \cdots\ s^{2n-1}\ 0\ 0\,\right]}{\delta(s)}\right\|_\infty^{-1}\\ &= r_s \end{aligned}\qquad(11)$$

where R is a $(2n+2)\times(2n+2)$ matrix. In the $2i$th row, the $2i$th column is 1, the other entries are zeros and r_s is defined as the stability radius. The internal stability of the closed loop system is preserved as long as $\|L(t)\|_\infty$ is less than r_s in (11). Since the matrices $B_z(t)$, $C_z(t)$ and $D_z(t)$ associated with the closed loop system are bounded, it follows from, the closed loop system is also bounded-input bounded-output stable if (11) is satisfied. If the acceleration profile, is available to the controller, then the stability radius can be further enlarged by modifying the feedback gains $h_{2i}(t)$ as $\mathrm{h}_{2i}(t)=\frac{1}{b}(\delta_{2i+1}-ak_i(t)-\dot{k}_i(t))$.

4. MODELING OF AC PM MOTORS AND DISTURBANCES

A three phase AC PM motor current input model in the $d-q$ frame is given by the following equations (Novotny and Lipo [1998]),

$$\tau_e(t)=\frac{3}{2}\frac{P}{2}\left[\lambda_m i_q(t)-(L_q-L_d i_d(t)i_q(t))\right]\qquad(12)$$

and

$$\tau_e(t)-\tau_l(t)=J_m\frac{d\omega(t)}{dt}+B_m\omega(t)\qquad(13)$$

where the parameters and variables have the following meanings

P	number of poles (even number);
L_d, L_q	$d-q$ frame stator inductances;
J_m	moment of inertia;
B_m	friction constant;
λ_m	constant magnetic flux;
$i_d(t), i_q(t)$	$d-q$ frame stator currents;
$\tau_e(t)$	electro-mechanical torque;
$\tau_l(t)$	load torque;
$\omega(t)$	rotor mechanical speed;
$\omega_e(t) = \frac{P}{2}\omega(t)$	rotor electrical speed.

The vector control technique suggests to set $i_d(t) = 0$. This converts the nonlinear AC PM motor system into a linear system and the torque is linearly proportional to the input $i_q(t)$,

$$\tau_e(t) = \frac{3}{2}\frac{P}{2}\lambda_m i_q(t). \tag{14}$$

4.1 Modeling of the Sinusoidal Disturbances

The sinusoidal ripples are always present in the output of the AC PM motor (Gan and Qiu [2004]). For example, periodic disturbances may be shown as several sinusoidal functions with known frequencies Therefore, we can assume that there are n sinusoidal disturbances in SISO systems and the frequencies of these n sinusoidal disturbances are already known. In this case, the disturbance input, $d(t)$, is given by

$$\begin{aligned} d(t) = &C + A_1 \sin(\omega_1(t)t + \phi_1) + A_2 \sin(\omega_2(t)t + \phi_2) \\ &+ \cdots + A_n \sin(\omega_n(t)t + \phi_n) \end{aligned} \tag{15}$$

where C is an unknown constant, the disturbance frequency $\omega_i(t)$ $(i = 1, 2 \cdots n)$ is a known time-varying function.

Our goal is to design a speed controller so that the output speed tracks a constant reference or a time-varying step reference and rejects the disturbances. The design objectives of the proposed controller are to have an order as low as possible, a good transient response, and a fast disturbance rejection response even when the system parameters vary slightly.

5. SIMULATION RESULTS

Table 1. motor parameters

J_m	$0.144 \times 10^{-4} kg \cdot m^2$
B_m	$5.416 \times 10^{-4} Nm/rad \cdot s^{-1}$
λ_m	$0.0283Wb$
P	8
$K_t = \frac{3}{2}\frac{P}{2}\lambda_m$	$0.1698Nm/A$

A 50 W AC PM motor is used in our simulations tests and the motor parameters are listed in TABLE 1. The gain difference between motor driver phases and the offset currents are always present in the AC PM motor system, so the torque ripples can be modeled as two sinusoidal functions whose frequencies depend on the motor speed,$\tau_{\text{off}}(t) = A_{d_1} \sin(\omega_{d_1}(t)t - \phi_{d_1}) + A_{d_2} \sin(\omega_{d_2}(t)t - \phi_{d_2})$ where $\omega_{d_1}(t) = \omega_e(t) \approx \frac{P}{2}\omega_r(t)$ and $\omega_{d_2}(t) = \frac{1}{2}\omega_e(t) \approx \frac{P}{4}\omega_r(t)$ are defined as the disturbance frequencies respectively.

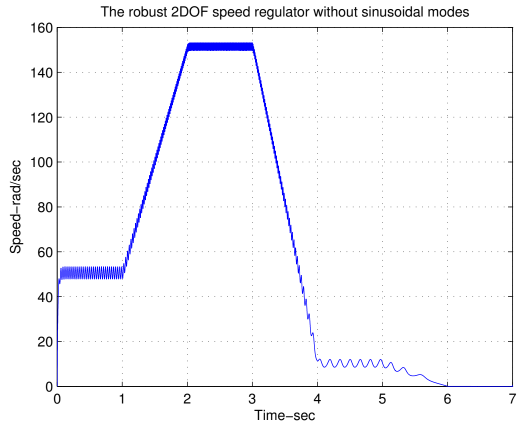

Fig. 4. Speed output with a general 2DOF regulator.

According to the design procedure listed in Section 3, one of the possible solutions is to choose the six closed loop poles at 140, 150, 160, 170, 180 and 190, and the five closed loop zeros at 150, 160, 170, 180, and 190 respectively. Other choices of the pole and zero locations are possible but too fast closed loop poles may lead to the signal saturation problem and the current loop dynamics may not be neglected if the bandwidth of the speed loop is close to that of the current loops. The GS robust 2DOF speed regulator is designed to satisfy the motion specifications with zero overshoot and rise time< 60ms.

The stability of this GS robust 2DOF speed regulator depends on the stability radius r_s. Let $\psi = [\ 0\ -(\omega_{d_1}^2(t) + \omega_{d_2}^2(t))'\ 0\ -(\omega_{d_1}^2(t)\omega_{d_2}^2(t))'\ 0\ 0\ ]$, according to the small gain theorem

$$\begin{aligned} \|\psi\|_\infty &= \sup_{t>0} \max_i |\psi_i(t)| \\ &< \left\| \frac{[0\ s\ 0\ s^3\ 0\ 0]}{s^6 + \delta_1 s^5 + \delta_2 s^4 + \delta_3 s^3 + \delta_4 s^2 + \delta_5 s + \delta_6} \right\|_\infty^{-1} \\ &= r_s. \end{aligned}$$

We can then evaluate the stability radius using the following formula, $\|\psi\|_\infty = \sup_{t>0} | -(\omega_{d_1}^2(t)\omega_{d_2}^2(t))'|$.

For the simulations, two profiles with the maximum values of $\sup_{t>0} | -(\omega_{d_1}^2(t)\omega_{d_2}^2(t))'| < r_s$ and $\sup_{t>0} | -(\omega_{d_1}^2(t)\omega_{d_2}^2(t))'| > r_s$ are employed to test the proposed GS robust 2DOF speed regulator.

In order to demonstrate the effectiveness of the use of the IMP, another robust 2DOF regulator is designed without including the sinusoidal disturbance modes to track the first input profile. This can be done by simply assigning $k(s) = s$ and the dominant closed loop pole of this robust 2DOF regulator is placed at -140, the same one of the GS robust 2DOF speed regulator. Fig. 4 shows the speed output is contaminated with ripples by the two sinusoidal disturbances. On the other hand, with the help of the sinusoidal modes inside the speed regulator, the output speed response of the GS robust 2DOF speed regulator achieves a desirable tracking response without any velocity ripple contamination as shown in the Fig. 5.

For the second input profile, we first test it without the acceleration profile input. The speed response is shown in Fig. 6 and the output speed response is not good in general and at $t = 3s$ as the value of $\sup_{t>0} |-(\omega^2_{d_1}(t)\omega^2_{d_2}(t))'| > r_s$, the output speed becomes unstable and oscillatory. However, if the improved GS robust 2DOF speed regulator is implemented with the acceleration profile input, a comparatively good speed response is obtained and shown in Fig. 7, and the stability of the whole system remains. Smooth transitions can still be maintained during the high speed reference change at $t = 2s$ and $t = 3s$. This simulation result shows the stability radius of the overall system can be enlarged with the availability of the acceleration input profile.

Fig. 6. Speed output without the acceleration profile input.

6. CONCLUSIONS

In summary, an adaptive sinusoidal disturbance rejection controller for SISO systems is developed and can be used to eliminate the ripples due to sinusoidal disturbances, as long as the disturbance frequencies do not change too fast. When the condition in (11) is satisfied, implementing with $\dot{k}_i(t)$ in $h_{2i}(t)$, an infinity stability radius can be achieved. The proposed controller is not limited to the application of AC PM motor systems, but can be applied to any servo problems with n sinusoidal disturbances with known frequencies.

REFERENCES

Spindler K. J., R. J. Carlin and S. A. Cinnamon (2004), Closed-loop phase compensation controller, *US Patent No: US 6,690,989*.

Chen D. and B. Paden (1993), An adaptive linearization of hybrid step motors: Stability analysis, *IEEE Trans. on Automatic Control*, vol. 38, no. 6, pp. 874–887.

Liu Y. and D. Chen (1994), Adaptive rejection of velocity-ripple from position transducer in a motion control system, *Proc. 33rd IEEE Conf. Decision and Control*, vol. 1, pp. 690–695.

Petrovic V., R. Ortega, A. M. Stankovis, and G. Tadmor (2000), Design and implementation of an adaptive controller for torque ripple minimization in PM synchronous motors, *IEEE Trans. on Power Electronics*, vol. 15, pp. 871–880.

Ferretti G., G. Magnani and P. Rocco (1998), Modeling, identification, and compensation of pulsating torque in permanent magnet AC motors, *IEEE Trans. on industrial Electronics*, vol. 45, pp. 912–920.

Gan W. C. and L. Qiu (2004), Torque velocity ripple elimination of AC permanent magnet motor control systems using the internal model principle, *IEEE/ASME Trans. on Mechatronics*, vol. 9, pp. 436–447.

Gan W. C., S. W. Tam, K. K. C. Chen and G. P. Widdowson (2005), Analysis of motor driver uncertainties in linear permanent magnet motor control systems, *Proc. of The Fifth International Symposium on Linear Drives for Industry Applications, LDIA'05*.

Pillay P. and R. Krishnan (1998), Modeling of permanent magnet motor drives, *IEEE Trans. on Industry Electronics*, vol. 13, pp. 388–400.

Novotny D. W. and T. A. Lipo (1998), *Vector Control and Dynamics of AC Drives*, New York: Oxford.

Wolovich W. A. (1994), *Automatic Control Systems*, Fort-Worth, TX: Sauders College Publishing.

Zhou K. M. and R. Zhang (2000), A new controller architecture for high performance, robust, and fault tolerant control, *IEEE Trans. on Automatic Control*, vol. 40, no. 10, pp. 4120–4125.

Vidyasagar M. (1985), *Control System Synthesis*, Cambridge, MA: MIT Press.

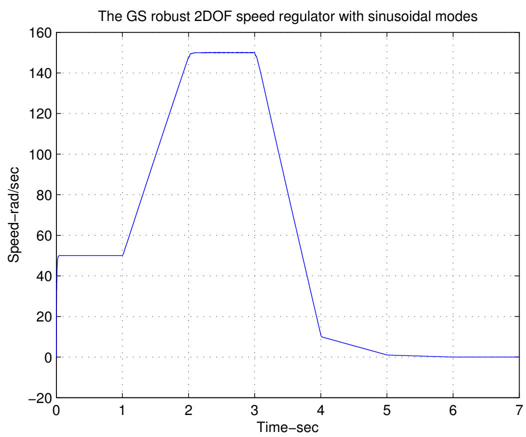

Fig. 5. Speed output with the GS robust 2DOF regulator.

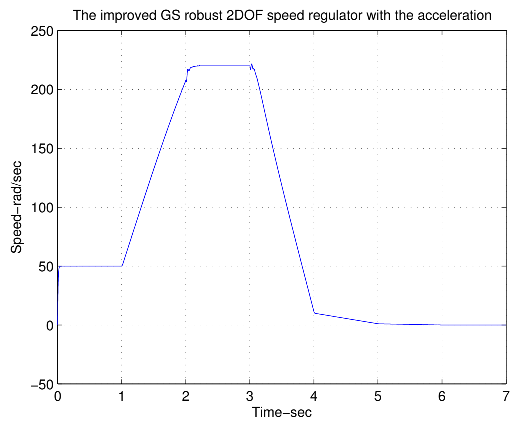

Fig. 7. Speed output with the acceleration profile input.