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A Model Reference Adaptive Variable Structure Controller for Reconfigurable Flight Control Systems

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Abstract: This paper presents a new design of reconfigurable flight controller, applying both variable structure control and model reference adaptive control. In case of control effectors failures, aircraft model will vary, and unknown disturbances will affect the characteristics of controller. To study this problem, a model reference adaptive variable structure control law is designed, which composes a model-following adaptive control part and a variable structure control part. According to the model matching condition, an update scheme is used in the adaptive control part to eliminate system errors caused by model mismatch. With an adaptive sliding-mode gain the variable structure control part can deal with uncertain bounded disturbances of flight system. An aircraft example with stabilsator failure is presented to demonstrate the feasibility of the proposed reconfigurable control law. Simulation results show the control law is effective for reconfigurable flight control systems.

1. INTRODUCTION

Model reference control (MRC) is one of the most popular adaptive methods. By adaptively tuning the control parameters, MRC methods can force a system to follow its reference model to achieve satisfactory performance, without an accurate system model and no need for parameter identification. Therefore, MRC is widely used in practice (Landau, 1985). One of the typical applications is reconfigurable flight control, which is the focus of this paper.

When aircraft failures occur, the uncertainty of aircraft model can significantly affect the quality of flight(MAYBECK, 1991 and BoSkovic, 2001). In this situation usually a reconfigurate controller is used to realize satisfactory flight control.

Reconfigurable flight control and system identification has been studied for years. In providing effective reconfiguration capabilities for hard and soft failures of sensors or actuators, a multiple model adaptive controller is shown to be effective (Maybeck, 1991). By introducing a forgetting factor to a recursive least-squares algorithm, Bodson and Groszkiewicz(1997) discusses a new algorithm for multivariable model reference control. Based on direct adaptive scheme with parameter estimation (Moon, 2005), a reconfigurable control method utilizing asymptotic model following conditions is also investigated in recent years. Another skill in model reference adaptive control design and control reconfiguration is neural networks (Calise, 1998 and Shin, 2004). In accommodation of control effector failures, a scheme based on the concept of multiple models, switching and tuning is built in BoSkovic and Mehra (2001), arriving at an adaptive reconfigurable formation control algorithm. Lastly, as documented in (KrishnaKumar, 2003), a Level 2 intelligent control architecture is beneficial to conquer damage situations of a C-17 aircraft, where the controller utilizes an adaptive critic to tune the parameters of a reference model.

This paper presents a new method for reconfigurable flight control design. Based on both variable structure control theory and adaptive control theory, the new controller is named model reference adaptive variable structure controller (MRAVSC). The controller is composed of two parts: the linear adaptive part and the variable structure part. Based on model matching conditions, an adaptive rule is developed for control parameters. Stability can be proved by Lyapunov function. The proposed MRAVSC is applicable for a variety of nonlinear systems with certain assumptions. The short-period of aircraft longitudinal model is chosen to be a test-bed.

In Section II, model reference reconfigurable control is described, and model matching condition is given. Section III carries the design of MRAVSC, whose reconfigurable property is demonstrated by a case study in Section IV. Concluding remarks come in Section V.

2. PROBLEM FORMATION AND MODEL REFRENCE CONTROL SCHEME

The discussion in this paper follows the typical literalized aircraft model used in (Hess, 2002):

$$\dot{x} = Ax + Bu + f(x,u,t) \tag{1}$$

where $A \in IR^{n\times n}, B \in IR^{n\times m}, m \geq n$. $x \in IR^n$, $u \in IR^m$ denote system state and input respectively, $f(x,u,t) \in IR^n$ represents parameter uncertainties and external disturbances. Such a class of plants is suitable to serve as aircraft models in a wide range of flight regimes. Model (1) is assumed to be a minimum phase system with

$$\left| det\left(BB^T\right)\right| > 0$$

Suppose the desired behaviours of a plant with no failures are specified by a reference model:

$$\dot{x}_m = A_m x_m + B_m u_m \tag{2}$$

where $x_m \in IR^n, u_m \in IR^m$, $A_m \in IR^{n\times n}, B_m \in IR^{n\times m}$, and the pair $\left(A_m, B_m\right)$ is controllable.

The objective of reconfigurable flight control is to design an input value $\Delta u(t)$ when failures occur, so as to eliminate the state error of light system. The tracking error vector is defined as

$$e = x - x_m \tag{3}$$

Differentiating (3) yields

$$\begin{aligned} \dot{e} &= \dot{x} - \dot{x}_m \\ &= Ax + Bu + f(x,u,t) - A_m x_m - B_m u_m \\ &= A_m e + (A - A_m)x + B\Delta u + (B - B_m)u_m + f(x,u,t) \end{aligned} \tag{4}$$

where $\Delta u = u - u_m$. To ensure the reference model being accurately followed, it suffices to demand

$$\begin{cases} e(t) = x - x_m = 0 \\ \dot{e}(t) = \dot{x} - \dot{x}_m = 0 \end{cases} \tag{5}$$

Any close-loop stable feedback design can be decomposed into two stages: a first stage involving plant stabilization (which can be omitted for stable plants); and a second stage involving a model reference scheme, which is automatically close-loop stable as only stable elements are used in this stage(Zames, 1981). According to Moon(2005) and Landau(1985), the model matching conditions are stated as

$$\begin{cases} A - A_m = BK_1 \\ B - B_m = BK_2 \\ f(x,u,t) = BK_3(x,u,t) \end{cases} \tag{6}$$

Substituting (6) into equation (4) gives

$$\begin{aligned} \dot{e} = A_m e + BK_1 x + B\Delta u + BK_2 u_m \\ + BK_3(x,u,t) \end{aligned} \tag{7}$$

To accomplish the objective of model reference, the following assumptions are needed to guarantee the equivalence in (6).

Assumption 1: In system (1) and (2), there exist vectors K_1, K_2 satisfying(Luo 1993 and Chou 2003)

$$\begin{aligned} rank(B) &= rank(B | f(x,u,t)) \\ &= rank(B | BK_1) \\ &= rank(B | BK_2) \end{aligned} \tag{8}$$

Assumption 2: The matrix $f(x,u,t)$ is composed of n vectors f_i , which are all bounded through some known functions of time and system state:

$$\left\| f_i(x,u,t) \right\| \le g_{0i}(t) + \sum_{j=1}^{n} g_{1j} \left\| x_j \right\| \tag{9}$$

3. ADAPTIVE VARIABLE STRUCTURE RECONFIGURABLE FLIGHT CONTROLLER DESIGN

To guarantee the reference model (2) being followed by the time-varying uncertain system (1), we refer to variable structure control (VSC), which is often used in deterministic control of uncertain systems (Zinober 1994).

Defining the error vector and its integral as the switching surface(or a sliding manifold), the sliding function is written as

$$s = e - \int_0^t (A_m + BK_4) e d\tau \tag{10}$$

where $K_4 \in R^{m\times n}$ is carefully selected to satisfy the inequality $Re\left[\lambda_{\max}(A_m + BK_4)\right] < 0$. By sliding mode theory, if K_4 and the controller are designed appropriately, the states will be driven to the switching surface $s = 0$ and stay there, then the controlled system will yield desired dynamic responses and eventually arrive at the state defined by (5).

Differentiating (10) yields

$$\begin{aligned} \dot{s} &= \dot{e} - (A_m + BK_4)e \\ &= B\left(K_1 x + \Delta u + K_2 u_m + K_3 - K_4 e\right) \end{aligned} \tag{11}$$

To guarantee that the switch surface $s_d = 0$ and $\dot{s}_d = 0$ can be reached in finite time, the equivalent controller is designed as

$$\Delta u_{equ} = -K_1 x - K_2 u_m - K_3 + K_4 e \tag{12}$$

The sliding mode is independent of uncertainties and model mismatch. The variable structure control is strongly robust and own good dynamic responses as long as the unknown term $f_i(x,u,t)$ is bounded.

Based on the equivalent controller, the MRAVSC for the reconfigurable flight control system (C.H. Chou, 2003) is formulated as

$$\Delta u = \Delta u_{adp} + \Delta u_{vsc} \tag{13}$$

$$\Delta u_{adp} = -K_1^* x - K_2^* u_m - K_3^* + K_4 e \tag{14}$$

$$\Delta u_{vsc} = -B^T s \left\| s^T B \right\|^{-1} \left[g_0 + g_1 \left\| x \right\| \right] \tag{15}$$

where Δu_{adp} is the nonlinear adaptive control part, by which the reference model is followed given proper adaptive control parameters K_1^* , K_2^* , K_3^* to be designed later; Δu_{vsc} denotes the VSC part, which compensates the bounded system disturbance derived from the parameter uncertainties and external disturbances. The architecture of the MRAVSC (13) is depicted in Fig. 1.

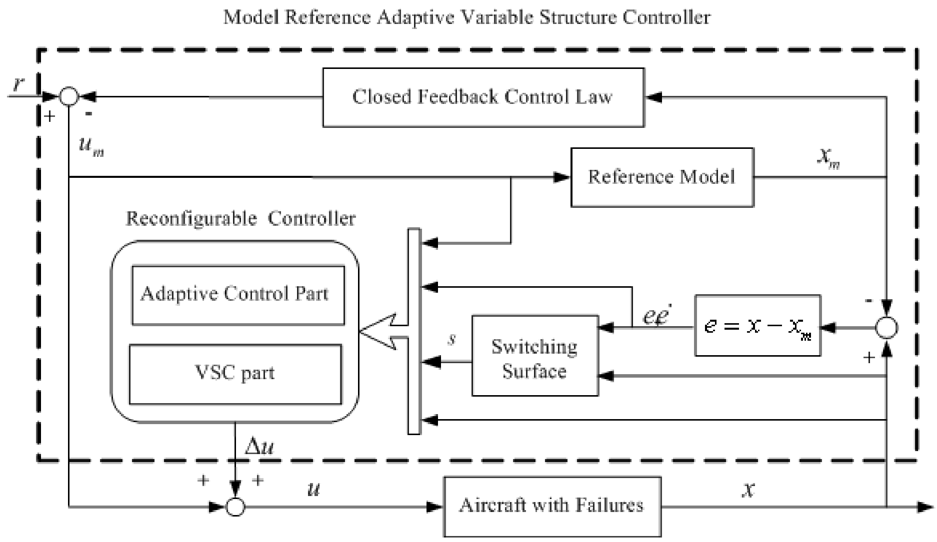

Fig. 1. The architecture of model reference adaptive variable structure controllers

To realize the model reference adaptive control, we define three generalized errors as below:

$$\begin{cases} \Delta K_1 = K_1 - K_1^* \\ \Delta K_2 = K_2 - K_2^* \\ \Delta K_3 = K_3 - K_3^* \end{cases} \tag{16}$$

Equation (11) can now be rewritten as

$$\dot{s} = B\Delta K_1 x + B\Delta K_2 u_m + B\Delta K_3 - B^T s \left\| s^T B \right\|^{-1} \left[g_0 + g_1 \|x\| \right] \tag{17}$$

Theorem: Consider the reconfigurable control law (13) with the matching conditions (6). If the proposed control parameter matrices satisfy the adaptive rules

$$\begin{cases} \dot{K}_1^* = -\gamma_1 B^T s x^T \\ \dot{K}_2^* = -\gamma_2 B^T s u_m^T \\ \dot{K}_3^* = -\gamma_3 B^T s \end{cases} \tag{18}$$

then the dynamic sliding mode (10) implies the stable equilibrium, and tracking errors of the reconfigurable flight system (1) will be eliminated.

Proof: Consider the continuously differentiable Lyapunov function candidate

$$V = \frac{1}{2}\left[s^T s + tr\left(\frac{\Delta k_1^T \Delta k_1}{\gamma_1} \right) + tr\left(\frac{\Delta k_2^T \Delta k_2}{\gamma_2} \right) + tr\left(\frac{\Delta k_3^T \Delta k_3}{\gamma_3} \right) \right] \tag{19}$$

Evidently, this function is positive definite. Differentiating V yields

$$\dot{V} = s^T \dot{s} + tr\left(\frac{\Delta k_1^T \Delta \dot{k}_1}{\gamma_1} \right) + tr\left(\frac{\Delta k_2^T \Delta \dot{k}_2}{\gamma_2} \right) + tr\left(\frac{\Delta k_3^T \Delta \dot{k}_3}{\gamma_3} \right) \tag{20}$$

Substituting (17) (18) into (20) gives

$$\begin{aligned} \dot{V} &= tr\left[\frac{\Delta k_1^T}{\gamma_1} \left(\Delta \dot{k}_1 + \gamma_1 B^T s x^T \right) \right] + tr\left[\frac{\Delta k_2^T}{\gamma_2} \left(\Delta \dot{k}_2 + \gamma_2 B^T s u_m^T \right) \right] \\ &\quad + tr\left[\frac{\Delta k_3^T}{\gamma_3} \left(\Delta \dot{k}_3 + \gamma_3 B^T s \right) \right] - B^T s \left\| s^T B \right\| \left[g_0 + g_1 \|x\| \right] \\ &= -B^T s \left\| s^T B \right\|^{-1} \left[g_0 + g_1 \|x\| \right] \end{aligned} \tag{21}$$

As long as $s \neq 0$, $\dot{V}$ is negatively definite. By (19),

$$\lim_{t \to \infty} s \to 0$$

is guaranteed. It indicates that the system's tracking errors will be driven to zero. ■

When K_1, K_2, K_3 in the matching conditions are observable and obtainable,(14) can be replaced by

$$\Delta u_{adp} = -K_1 x - K_2 u_m - K_3 + K_4 e \tag{22}$$

4. EXAMPLE

An open-loop aircraft flight with trimmed condition exhibit two longitudinal modes of motion in steady-state, namely the short-period and the phugoid. The short period mode is normally fast and oscillatory, occurring at nearly constant speed with slight change in flight-path angle. As the short period dominates the aircraft's response to the pilot inputs, it is the primary mode of interest when performing manual flight control. Only angle of attack, pith and pitch rate vary in short period. Let

$$x(t) = \left[\alpha(t) \quad q(t) \right]^T$$
$$u(t) = \left[\delta_s \quad \delta_t \right]^T$$

where $\alpha(t)$ is angle of attack (deg), $q(t)$ is pitch rate (deg/sec), δ_s is the stabilator angle (deg) and δ_t is the thrust vector angle (deg). For the short-period the longitudinal motion is approximated by (Stevens, 2003 and Adarras, 1992)

$$\dot{x} = Ax + Bu + f(x,u,t) \tag{23}$$

We adopt the flight parameter of Ref. (R.A. Hess, 2002). Suppose the nominal flight condition is 0.6 Mach number at an altitude of 30,000 ft, and the parameters are

$$A_m = \begin{bmatrix} -0.5088 & 0.994 \\ -1.131 & -0.2804 \end{bmatrix}$$

$$B_m = \begin{bmatrix} -0.9277 & -0.01787 \\ -6.575 & -1.525 \end{bmatrix}$$

$$f_m = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

For the damaged vehicle (50% reduction in stabilator area),

$$A = \begin{bmatrix} -0.48 & 0.997 \\ -0.4639 & -0.01787 \end{bmatrix}$$

$$B = \begin{bmatrix} -0.4639 & -0.01787 \\ -3.2875 & -1.525 \end{bmatrix}$$

$$f = \begin{bmatrix} \sigma x_1 \\ 0.05 x_1 x_2 \sin(t) \end{bmatrix}$$

where $\sigma \in [-0.1 \quad 0.1]$ is a random number. The reconfigurable control input Δu is designed according to (13)(14)(15)(18) after we set

$$g_0 = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}, \; g_1 = \begin{bmatrix} 0.15 & 0.0 \\ 0.0 & 0.1 \end{bmatrix}$$

$$\gamma_1 = \gamma_2 = \gamma_3 = 1, K_4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

The reference model input u_m and the reference input r are chosen as

$$u_m = \begin{bmatrix} -0.1788 & -0.4193 \\ -0.0114 & -0.0961 \end{bmatrix} x_m + r \; , \; r = \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}$$

The computer simulation results are shown in Fig.2-Fig.7.

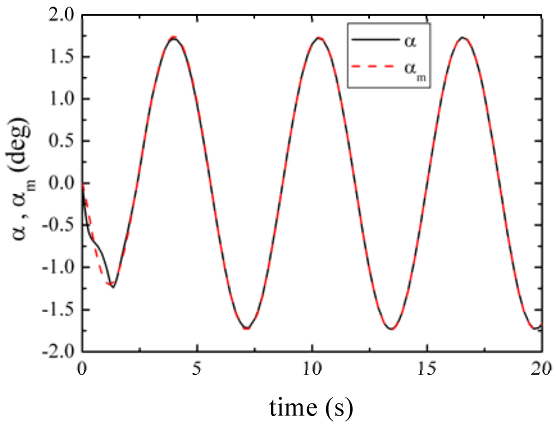

Fig. 2. Comparison between angles of attack

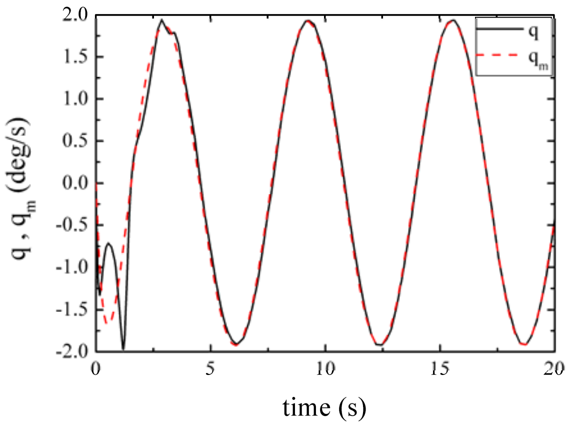

Fig. 3. Comparison between rates of pitch

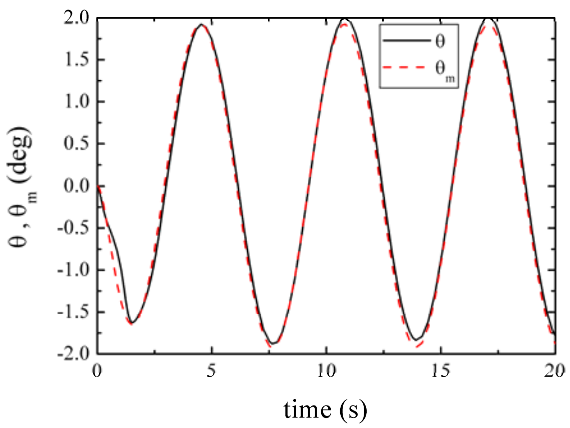

Fig. 4. Comparison between angles of pitch

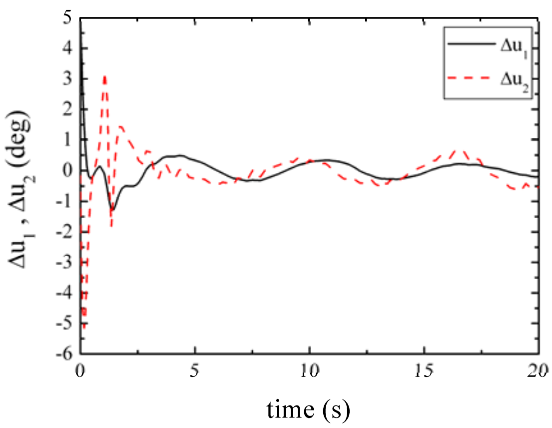

Fig. 5. Time-history of reconfigurable control input

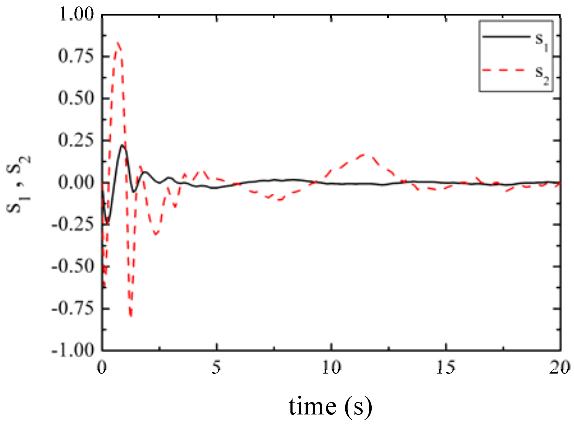

Fig. 6. Time-history of sliding mode

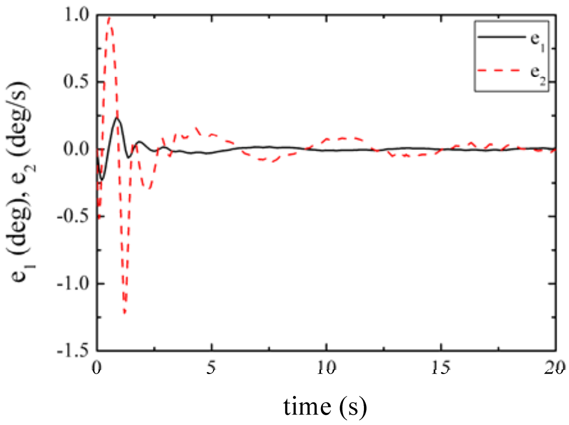

Fig. 7. Time-history of tracking errors

Fig.2-Fig4 show comparison between the dynamic response of attack angle, pitch rate and pitch angle and their reference signals respectively. As can be seen, the controller achieves excellent performance as adaptive tracking is realized. Reconfigurable control is realized when stabilator damage happens. Though reduction in stabilator area is serious, precise tracking is still achieved, and flight performance is rarely affected. Fig. 5 gives the reconfigurable control input. The reconfigurable control input adaptively provides compensate for system error due to stabilator damage. Fig.6 presents the trajectory of sliding mode. The sliding mode reaches the switching surface quickly. The dynamic performance of tracking errors is shown in Fig 7. Starting from large initial values, system error becomes bounded quickly before they are driven to zero asymptotically. Variation of the error reveals that the reconfigurable control law is adaptive and strongly robust.

Results in Fig.2-Fig.7 show the effectiveness of our design. Thanks to the sliding function (10), VSC controller plays a key role in the MRAVSC, contributing to the reduction of errors and the improvement of flight performance.

5. CONCLUDING REMARKS

Combining the model reference adaptive control formulation and adaptive variable structure control method, a new method for reconfigurable flight control is proposed in this paper. A Lyapunov function is constructed for the stability analysis of the controller.

When aircraft failures occur the aircraft model is affected by parameter uncertainties and external disturbance. In our design, direct adaptive control is developed to make the system achieve stable tracking of the reference model; adaptive variable structure control is adopted to restrain the system disturbances.

To verify the controller, the short-period of longitudinal modes in an open loop aircraft flight is discussed as an example. The demand of reconfigurable flight control can be satisfied. This makes MRAVSC a novel and effective technique for reconfigurable control.

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