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Master-slave telecontrol of a class of underactuated mechanical systems with communication time-delay

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Abstract: This paper deals with the problem of time-delay telecontrol of a class of under-actuated mechanical systems (UMS) in a master-slave configuration. This problem has been solved by designing some discontinuous causal compensators which have proved by formal stability analysis to guarantee position coordination of the mechanisms and to allow good force reflection to the master side, satisfying in this way classical objectives in the telecontrol of systems. Moreover, the whole closed-loop system behaves stable and results robust to parametric uncertainties and external disturbances. The performance of the proposed control scheme is visualized through experiments developed in a local network.

1. INTRODUCTION

Distance control of mechanical systems is an active field of research due to important applications like teleoperation (Hokayem and Spong (2006)) and telerobotics (Oboe and Fiorni (1997)), where is desired that two or more mechanical systems work together to achieve a specific task. Commonly, it is required that the motion of a so-called slave mechanism tracks the motion of a master one, which in turn is governed by a human being or an autonomous controller (Xi and Tarn (2000)). This control problem has been considered to achieve different objectives like position coordination and force reflection. However, it turns complicated when the information between the mechanisms is delayed since its effect in the system has proved to cause several deterioration of performance and even instability. Under this situation, the whole system can be viewed as a time-delay system and the control design must be carried out carefully. In this sense, the approach for reducing the detrimental effect in the stability of the system due to delays, which consists in designing control schemes based on state prediction (Casavola *et al.* (2006)), could be non acceptable for some applications since there exists a strong tradeoff between stability and performance. An alternative approach, which does not comprise the system performance nor its stability, is to design controllers with causality properties (Estrada-García *et al.* (to appear)).

The available results reported for the telecontrol of mechanisms are far from being developed since they are for completely actuated systems. Then, one interesting and challenging issue is to extend these results to a general class of UMS. However, this is not an easy task since different UMS do not share the same difficulties from the control point of view.

The analysis and control of UMS -those with less control inputs than degrees of freedom (DOF), have proved to be a serious problem because of the reduction of the control space and the inherent non-holonomic (NH) constraints in their dynamics. In particular, the second-order NH UMS have proved to be the most challenging to control w.r.t. those having fisrt-order NH constraints. Some interesting results for the problems of stabilization and tracking have been developed for the so-called upper-actuated class (class-I) (see Spong (1998) and Grizzle *et al.* (2005)). Opposite to these contributions, the lower-actuated class (class-II) has not been completely developed since their (non-minimum) internal dynamics always remain Lagrangian and their dynamic equations do not satisfy the Brockett's necessary condition which translates into the incapability of designing continuous time-invariant control schemes (either static or dynamic) for stabilization or tracking (see Rayhanoglu *et al.* (1999)). Often, control schemes for the class-II include logarithmic, time-varying or discontinuous approaches.

Up to date, a lot of research in UMS has been developed (see Fantoni and Lozano (2002), Spong (1998) for some surveys) mainly motivated by applications where it is desired to stabilize ships, underwater vehicles, mobile platforms, aircrafts, mechanical manipulators, etc. Distance control of these classes of systems is very important in applications for handling remote objects with minimum control efforts, like exploration of remote environments not easily accesibles for human beings or the development of dangerous tasks. Although some results in the telecontrol of specific UMS (class-I) have been reported in Kwon *et al.* (2000) and Lee *et al.* (2006), the problem for a general class of UMS still remains open.

In this paper, the problem of master-slave bilateral telecontrol of UMS (class-I and class-II) with communication time-delays has been addressed and solved by designing some discontinuous causal compensators to guarantee position coordination of the mechanisms and to allow force reflection to the master side while keeping the whole closed-loop system stable with robustness features to parametric uncertainties and external disturbances. Experimental results of the proposed control scheme are given within a local network environment.

The general setup considered is as shown in Fig.1. Consider two identical UMS with n-DOF each. One of these, named the master, is driven by the external force u_r which is an autonomous controller and ensures convergence of the joint variables $(q_m, \dot{q}_m)$ to some desired trajectories $(q_r, \dot{q}_r)$. The other, named the slave, is driven by the control force u_{sc} in order to guarantee the tracking of the master motion in force and position. The environmental force u_e acts on the slave and is reflected to the master side through u_{mc}. This information allows a bilateral scheme and is considered important for decision making, which of course will depend on the application. The transported information from master to slave (and viceversa) is considered to be delayed a constant time Δ.

2. UNDERACTUATED DYNAMICS

The dynamics to be considered is composed of a pair of UMS with $n-$DOF each

$$H_i(q_i)\ddot{q}_i(t) + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = B_i u_i(t) \qquad (1)$$

where the vector $q_i(t) = [\, q_{i_a}(t) \;\; q_{i_u}(t)\,]^T$ and the matrix $B = [\, I_{l\times l} \;\; 0\,]^T$ have appropriate dimensions. The subscript $i = m, s$ denotes master and slave variables, while $q_{i_u} \in \mathbb{R}^p$ and $q_{i_a} \in \mathbb{R}^{l=n-p}$ denote the p-non-actuated and l-actuated DOF, respectively, with $n > l \geq p$. Besides, $u_i \in \mathbb{R}^l$ are the control vectors, $H_i(q_i) \in \mathbb{R}^{n\times n}$ are the inertia matrices, $C_i(q_i, \dot{q}_i) \in \mathbb{R}^{n\times n}$ are matrices of Coriolis and centrifugal forces and $G_i(q_i) \in \mathbb{R}^n$ are vectors of potential forces. As it is standard in robotics, (1) is assumed to have the following properties for all configurations of $q_i(t)$.

P1 $0 < \lambda_{min} \leq ||H_i(q_i)|| \leq \lambda_{max} < \infty$
P2 $||C_i(q_i, \dot{q}_i)\dot{q}_i|| \leq c_0||\dot{q}_i|| \leq c_1 < \infty$
P3 $||G(q_i)|| \leq g_0 < \infty$

where $\{c_0, c_1, g_0\}$ are constants; $\lambda_{min}, \lambda_{max}$ are minimum and maximal eigenvalues of $H_i(q_i)$ and $||\bullet||$ stands for the Euclidean norm.

The inputs of the mechanisms are defined as

$$u_m(t) \triangleq u_r(t) + u_{mc}(t) \qquad (2)$$

$$u_s(t) \triangleq u_e(t) + u_{sc}(t) \qquad (3)$$

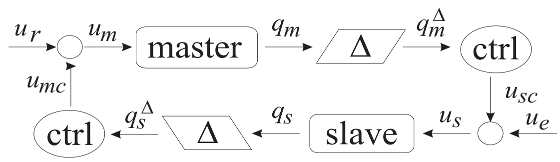

Fig. 1. The considered telecontrol setup.

where $u_r(t)$ and $u_e(t)$ denote external forces to the system, produced by a local reference and the environment respectively, and $u_{ic}(t)$ are the control inputs for each mechanism. The external force $u_e(t)$ can be viewed as a disturbance force acting on the slave(s), probably non-vanishing, with the following property: $0 \leq ||u_e(t)|| \leq \beta$, for some positive constant β.

Problem Statement: Given the system (1) design, if possible, causal compensators $u_{ic}(t)$ to achieve the following telecontrol objectives.

- In free motion (*i.e.* $u_e = 0$), there exist position coordination

$$\lim_{t\to\infty} ||q_s(t) - q_m(t-\Delta)|| = 0 \qquad (4)$$

  and force tracking

$$\lim_{t\to\infty} ||u_s(t) - u_m(t-\Delta)|| = 0 \qquad (5)$$

- In constrained motion (*i.e.* $u_e(t) \neq 0$), the external reference from the environment is reflected to the master side

$$\lim_{t\to\infty} ||u_{mc}(t) - u_e(t-\Delta)|| = 0 \qquad (6)$$

3. CONTROL SCHEME

The system (1) can be represented in the form of $2n$ equations of first order with respect to vectors q_i and $\dot{q}_i$. Let be the variables $x_i(t) := [\, q_i(t) \;\, \dot{q}_i(t)\,]^T$, then a state-space representation easily follows as

$$\dot{x}_i(t) = f\big(x_i(t)\big) + b\big(x_i(t)\big)u_i(t) \qquad (7)$$

where $f(x_i) \in \mathbb{R}^{2n}$ and $b(x_i) \in \mathbb{R}^{2n\times l}$. Notice from the properties P1-P3 that $f(x_i)$ and $b(x_i)$ remain bounded for all configurations of x_i. It is easy to show that

$$||b(x_i)|| \leq \lambda_{min}^{-1} \triangleq b_0 \qquad (8)$$

$$||f(x_i)|| \leq \left[\left(\frac{c_1}{c_0}\right)^2 + b_0^2(c_1 + g_0)^2\right]^{1/2} \triangleq f_0 \qquad (9)$$

Also, notice that $\text{rank}\big(b_i\big) = l$ for all x_i; then there exists the pseudoinverse of b_i denoted as $\bar{b}_i$. Since $l < n$, the product $b_i^T b_i$ is nonsingular and the pseudoinverse is given by $\bar{b}_i = (b_i^T b_i)^{-1} b_i^T$, which is unique and satisfies the so-called Moore-Penrose conditions (see Nakamura (1991)).

Any trajectory which could be tracked by the mechanisms must be produced by a reference model because of the dynamical constraints in the system. At first, assume there is available a set of factible trajectories x_r with time derivatives $\dot{x}_r$, which are both bounded for all t such that $||x_r||_\infty \leq r_0$ and $||\dot{x}_r||_\infty \leq r_1$, for positive constants r_0 and r_1. Now define the tracking errors

$$\tilde{x}_m(t) \triangleq x_m(t) - x_r(t) \quad (10)$$

$$\tilde{x}_s(t) \triangleq x_s(t) - x_m(t-\Delta) \quad (11)$$

and the hyperplanes

$$\sigma_i(t) \triangleq \tilde{x}_i(t) + \gamma_i \int_{t_0}^{t} \tilde{x}_i(\tau)d\tau = 0 \quad (12)$$

with scalars γ_i. Let $x(t)$ be some (vector) variable defined at time t, k be a constant such that $k \in \mathbb{N}$ and Δ be a (constant) time-delay. Then any $x(t)$ delayed k-times will be denoted from now on as $x^{k\Delta} := x(t - k\Delta)$. For notational simplicity, the subscript $j = r, m$ and the superscript k are used throughout the paper and behave as follows. If $i = m$, then $j = r$ and $k = 0$. If $i = s$, then $j = m$ and $k = 1$.

Proposition 3.1. (Peñaloza-Mejía et al. (2007)): Given a set of (bounded) factible trajectories x_r and their time derivaties $\dot{x}_r$, the master tracks them by using

$$u_r = \bar{b}_m(\dot{x}_r - f_m + v_m) \quad (13)$$

$$v_m = -\gamma_m\tilde{x}_m - \eta_m sgn(\sigma_m) - \alpha_m\sigma_m \quad (14)$$

while the slave tracks the master motion with

$$u_{sc} = \bar{b}_s(\dot{x}_m^{\Delta} - f_s + v_s) \quad (15)$$

$$v_s = -\gamma_s\tilde{x}_s - \eta_s sgn(\sigma_s) - \alpha_s\sigma_s \quad (16)$$

where $sgn(\sigma_i) := [\, sgn(\sigma_{i_1}) \ \dots \ sgn(\sigma_{i_{2n}})\,]^T$ and $sgn(\bullet)$ stands for the signum function. Then the force reflection to the master side is always achieved with

$$u_{mc} = \eta_s \bar{b}_m^{2\Delta} sgn^{\Delta}(\sigma_s) \quad (17)$$

The objectives (4)-(6) are satisfied with (13)-(17) if the control parameters are chosen as

$$\left.\begin{array}{l} \alpha_i > 0, \quad \gamma_i > 0, \quad \Lambda_i \geq 0 \\ \eta_m = \Lambda_m + \eta_m^0 + \eta_s \\ \eta_s = \Lambda_s + \eta_s^0 + \beta b_0 \end{array}\right\} \quad (18)$$

where η_i^0 is given in the proof. Moreover, the resulting closed-loop system remains stable and results robust to parametric uncertainties and external disturbances.

Proof: The closed-loop system is obtained by putting (13)-(16) in (7) with the use of (2) and (3). After some simple algebraic manipulations this yields

$$\dot{\sigma}_i = \dot{\tilde{x}}_i + \gamma_i\tilde{x}_i = -\alpha_i\sigma_i - \eta_i sgn(\sigma_i) + T_i + \tilde{v}_i \quad (19)$$

$$\tilde{v}_i = [b_i\bar{b}_i - I][\dot{x}_j^{k\Delta} - f_i + v_i] \quad (20)$$

$$T_m = \eta_s b_m \bar{b}_m^{2\Delta} sgn^{\Delta}(\sigma_s) \quad (21)$$

$$T_s = b_s u_e \quad (22)$$

which remain bounded by

$$||T_m|| \leq \eta_s||b_m|| \cdot ||\bar{b}_m^{2\Delta}|| \cdot ||sgn^{\Delta}(\sigma_s)||_{\infty} = \eta_s \quad (23)$$

$$||T_s|| \leq ||u_e|| \cdot ||b_s|| = \beta b_0 \quad (24)$$

$$||\tilde{v}_i|| \leq ||\dot{x}_j^{k\Delta}|| + ||v_i|| + ||f_i|| := \eta_i^0 \quad (25)$$

Taking the temporal derivative of the Lyapunov function $V = \frac{1}{2}\sum_i \sigma_i^T\sigma_i$ along the trajectories of (19), yields

$$\dot{V} = \sum_i \sigma_i^T(-\alpha_i\sigma_i - \eta_i sgn(\sigma_i) + T_i + \tilde{v}_i) \quad (26)$$

Simplifying and taking upper bounds

$$\begin{aligned}\dot{V} \leq& -\alpha_s||\sigma_s||^2 + (-\eta_s + \beta b_0 + \eta_s^0)||\sigma_s|| \\ &-\alpha_m||\sigma_m||^2 + (-\eta_m + \eta_m^0 + \eta_s)||\sigma_m|| \end{aligned} \quad (27)$$

At this point, notice that as long as the tracking errors $\tilde{x}_i$ are bounded in $t = t_0$, it is always possible to choose

$$\eta_i^0 = r_1 + \gamma_i||\tilde{x}_i(t_0)|| + f_0 \quad (28)$$

Since the control parameters are chosen in accordance with (18), the time derivative (27) reduces to

$$\dot{V} \leq -\sum_i \alpha_i||\sigma_i||^2 - \sum_i \Lambda_i||\sigma_i|| \quad (29)$$

which is negative definite and guarantees the existence of a sliding mode. Then the system motion is driven to the surfaces $\sigma_i = 0$ in finite time. Once on the surfaces, the closed-loop dynamics is given by

$$\dot{\sigma}_i = \dot{\tilde{x}}_i + \gamma_i\tilde{x}_i = 0 \quad (30)$$

and states that

$$\tilde{x}_i \to 0 \text{ as } t \to \infty \quad (31)$$

since $\gamma_i > 0$. This proves (4).

From (11) and (7) one has

$$\dot{\tilde{x}}_s = f(\tilde{x}_s + x_m^{\Delta}) + b(\tilde{x}_s + x_m^{\Delta})u_s - f(x_m^{\Delta}) - b(x_m^{\Delta})u_m^{\Delta} \quad (32)$$

which is reduced by (31) to $b(x_m^{\Delta})u_s \to b(x_m^{\Delta})u_m^{\Delta}$, or equivalently to $\bar{b}(x_m^{\Delta})b(x_m^{\Delta})u_s \to \bar{b}(x_m^{\Delta})b(x_m^{\Delta})u_m^{\Delta}$. Then it is concluded that

$$u_s \to u_m^{\Delta} \text{ as } t \to \infty \quad (33)$$

since the product $\bar{b}(\bullet)b(\bullet)$ is always equal to $I_{l\times l}$. Therefore, (5) is clearly achieved.

In constrained motion, from the system behavior on the surfaces $\sigma_i = 0$ in (19), one has

$$\begin{aligned}\eta_s sgn(\sigma_s) &= T_s + \tilde{v}_s \\ &= b(x_s)u_e + [b(x_s)\bar{b}(x_s) - I][\dot{x}_m^{\Delta} - f_s + v_s]\end{aligned} \quad (34)$$

Multiplying by $\bar{b}(x_s)$ the above equation and simplifying terms, one has $\eta_s\bar{b}(x_s)sgn(\sigma_s) = u_e$, which after shifting Δ times and using (11), turns into

$$\eta_s\bar{b}(\tilde{x}_s^{\Delta} + x_m^{2\Delta})sgn^{\Delta}(\sigma_s) = u_e^{\Delta} \quad (35)$$

Because $\tilde{x}_s \to 0$ from (31), on (35) is implied that

$$\eta_s\bar{b}(x_m^{2\Delta})sgn^{\Delta}(\sigma_s) \to u_e^{\Delta} \quad (36)$$

Then it follows from (17) and (36) that

$$u_{mc} \to u_e^{\Delta} \text{ as } t \to \infty \quad (37)$$

and (6) has been proved.

Eq. (31) states that $x_i \to x_j^{k\Delta}$ since $\tilde{x}_i \to 0$; therefore $x_s \to x_m^{\Delta} \to x_r^{\Delta}$. Due to the fact that x_r is bounded, the aforementioned chain of implications guarantees that the whole closed-loop system remains stable. ■

Remark 3.2. Chattering avoidance. Chattering and the need for discontinuous control, which are inherent in classical sliding modes, constitute the main criticisms in using this approach in mechanical systems since the rapidly changing control actions induce fatigue in mechanical parts and the system could be damaged in a short time. One way to avoid this situation is to use, for instance, the continuous approximation $tanh(\sigma_i/\epsilon_i) \approx sgn(\sigma_i)$ where the parameter ϵ_i gives the width of the switching region

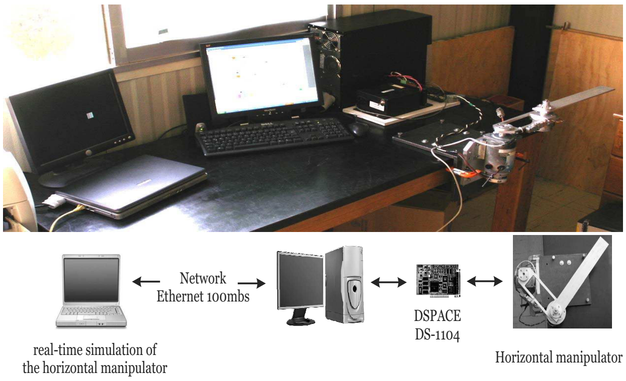

Fig. 2. Picture and diagram of the experimental setup.

near to the switching surface σ_i. By using this approximation, a sliding mode does not exists anymore but the system will remain moving around the switching surfaces within a bounded region $\delta(\epsilon)$, which could be as small as desired provided that $\epsilon_i << 1$. This will induce a small drift in the tracking errors (which can be proved to be ultimately uniformly bounded) and correspond in real life implementations to the practical zero or the resolution from sensors. With the use of the continuous approximation, the closed-loop system (19) can be written as

$$\dot{\sigma}_i = -\alpha_i \sigma_i + d(t, \sigma_i) \tag{38}$$

where $d(t, \sigma_i) := -\eta_i tanh(\sigma_i/\epsilon_i) + T_i + \tilde{v}_i$. Then the nominal system $\dot{\sigma}_i = -\alpha_i \sigma_i$ has a globally exponentially stable equilibrium point at $\sigma_i = 0$ since $\alpha_i > 0$. Notice that the term $d(t, \sigma_i)$ is bounded by (23), (24) and (25).

The boundedness of the tracking errors comes naturally from proving that σ_i is ultimately uniformly bounded, which can be easily done by using Lemma 9.2 in Khalil (2002) for the stability analysis of the perturbed system (38).

Remark 3.3. Robustness. Sliding modes have been considered often in the technical literature for the robust control of mechanical systems since the sliding behavior is insensitive to model uncertainties and disturbances (Utkin (1999)). In the proposed scheme, it is important to notice that the vector $\tilde{v}_i$ is bounded. It can be claimed that measurement errors and model uncertainties (unknown but bounded) can be put into this vector. Then the closed-loop system will result robust to these disturbances for a proper choice of η_i, since this parameter depends directly on the upper bound of $\tilde{v}_i$. Under this situation, an additional term in $d(t, \sigma_i)$ is added. However, stability of the closed loop system and bounded errors are concluded by studying the stability of the perturbed system (38).

Remark 3.4. Delay estimation. Notice that it is not necessary to estimate or to measure the delay Δ in order to compute the causal control laws. By just sending the signals to the slave and receiving it back from it, the control scheme results independent of Δ.

4. EXPERIMENTS

Some experiments are now presented: they consist in the telecontrol of a pair of 2-DOF master-slave horizontal manipulators with communication time-delay. The slave mechanism is a physical prototype while the master is real-time simulated in a laptop. Both are interconnected and exchange real-time data within a local network (switched ethernet at 100mbs). Time-delay is considered in the (bidirectional) information transport and was implemented by using buffers.

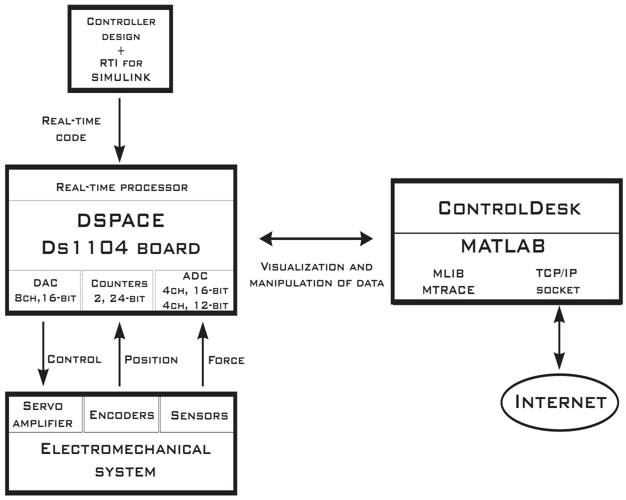

Fig. 3. The real-time platform developed with the dSPACE DS1104 board.

4.1 Platform

The complete picture and diagram of the experimental setup is shown in Fig. 2, which has easy-to-use rapid control prototyping features and Internet capabilities. It was first developed in Peñaloza-Mejía *et al.* (2005) and the main component in this proposal was a digital signal processor (DSP) controller board from dSPACE (DS1104) (some technical specifications are overviewed in Fig 3). The operation of the setup with its Internet capabilities is as follows.

The controller for the mechanism is designed from SIMULINK considering the information to be received and transmitted as data variables. Then the RTI generates the real-time code which is uploaded to the DSP. The MTRACE library allows these variables to be shared by the MATLAB workspace and the real-time processor; then the complete integration is achieved within the MATLAB workspace by installing a TCP/IP socket and using the MLIB/MTRACE libraries to identify and modify such shared variables online.

Exchange of data between both workstations in Fig. 2 is done through the network connection by installing a TCP/IP socket within the MATLAB environment. The key is to deploy the TCP/IP communication and timer tools from set of functions of the so-called IOLIB library (*tcp.m* and *itimer.m*) together with the MLIB/MTRACE functions (*mlib.m* and *gettracevar.m*) in a pair of M-files. By doing this, communication and control for both stations in a soft real-time fashion are achieved. The architecture for each workstation is shown in Fig. 3. Please refer to Peñaloza-Mejía *et al.* (2005) for more details.

4.2 Underactuated Manipulators

As mentioned before, the master mechanism is real-time simulated while the slave mechanism is a physical proto-

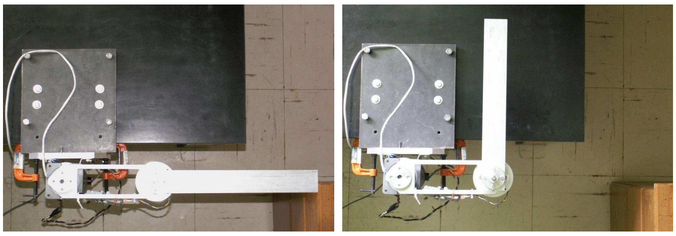

Fig. 4. (a) initial and (b) final position of the slave manipulator in the desired task.

type, as shown in Fig. 2. The manipulators are underactuated systems of class-II and are constrained to a horizontal plane such that there is no potencial field acting on their dynamics.

Their first joints q_{i_1} attached to inertial frames are considered actuated while the second joints q_{i_2} are not-actuated. The active joint of the slave mechanism is driven with a 90V dc servomotor from Minessota Electric Technology and the 25A8 PWM servoamplifier from Advanced Motion Control, configured in torque mode. Position data of the two joints are obtained with 1024ppr optical encoders from Bell Motion Systems.

Their dynamic equations are

$$\begin{aligned}&(a_1 + 2a_2 \cos q_{i_2})\ddot{q}_{i_1} + (a_3 + a_2 \cos q_{i_2})\ddot{q}_{i_2} - a_2\dot{q}_{i_2}^2 \sin q_{i_2}\\ &\quad -2a_2 \sin q_{i_2}\dot{q}_{i_1}\dot{q}_{i_2} = u_i\\ &(a_3 + a_2 \cos q_{i_2})\ddot{q}_{i_1} + a_3\ddot{q}_{i_2} + a_2\dot{q}_{i_1}^2 \sin q_{i_2} = 0\end{aligned} \tag{39}$$

The physical parameters for both manipulators have been taken from an experimental prototype and have the following values: $a_1 = 0.05653\ kg \cdot m^2$, $a_2 = 0.01081\ kg \cdot m^2$ and $a_3 = 0.01341\ Kg \cdot m^2$.

4.3 Trajectory planning

Regarding that the mechanisms cannot arbitrarily move in space due to their non-holonomic constrains, the external reference $u_r(t)$ must drive the motion of the master to desired factible trajectories which can track the slave. In Rosas-Flores *et al.* (2001), a class of parametric trajectories are proposed for positioning a class of UMS to a desired configuration with zero final velocity. For the considered mechanisms, these are given by

$$\begin{aligned}q_{r_1} &= q_1(t_0) + \frac{1}{2}(q_1(t_f) - q_1(t_0))(1 + tanh(a_4 t - a_5))\\ &\quad +\omega_1 sech(a_6 t - a_7) - \omega_2 sech(a_8 t - a_9)\\ 0 &= (a_3 + a_2 \cos q_{r_2})\ddot{q}_{r_1} + a_3\ddot{q}_{r_2} + a_2\dot{q}_{r_1}^2 \sin q_{r_2}\end{aligned} \tag{40}$$

where $a_4, ...a_9$, ω_1, ω_2 are constants that must be chosen in accordance to the desired configuration.

4.4 Implementation

By defining the state variables $x_r = [q_{r_1}, q_{r_2}, \dot{q}_{r_1}, \dot{q}_{r_2}]^T$ and $x_i = [q_{i_1}, q_{i_2}, \dot{q}_{i_1}, \dot{q}_{i_2}]^T$, a state-space representation is obtained and the control laws for (39) are determined, which result well-defined for all configurations of x_i. By taking advantage of the result in Rosas-Flores *et al.* (2001), it is desired to perform a simple task to stabilize the mechanisms to a given (desired) configuration with zero final velocity. The initial and final (desired) configuration for the slave mechanism is shown in Fig. 4. Then the trajectories (40) have been used for driving the manipulators with a maximal joint velocity of $\dot{q}_i(t) \in [-5, 5]\ rad/s$.

Parameter	Value
$q_r(t_0)$	$(0, 0)$ rad
$q_r(t_f)$	$(0, \pi/2)$ rad
$q_m(t_0)$	$(0, 0)$
$q_s(t_0)$	$(-0.0715, -0.0223)$
(ω_1,ω_2)	$(-0.62048, 0.95523)$
(α_m,α_s)	$(2, 15)$
(γ_m,γ_s)	$(5, 5)$
(η_m,η_s)	$(95, 30)$
(a_4, a_5, a_6)	$(1.6, 2.5, 7.2)$
(a_7, a_8, a_9)	$(9.0, 7.2, 27.0)$
(t_0, t_f, Δ)	$(3s, 8s, 0.350s)$

In order to compute the velocity of the joint manipulators, the exact differentiator introduced in Levant (2003) was used. The main reasons in the use of this method to compute velocity, compared with traditional techniques like observers or computational approximations, are that this differentiator has finite time convergence, does not depend on the system model, satisfies trivially the separation principle, has an easy implementation and results suitable for experimentation purposes since incorporates an algorithm for sampled signals. The slave control scheme has been programmed in SIMULINK and then downloaded to the DSP by using the RTI interface. The master control has been programmed with the virtual manipulator by using a $4^{th}/5^{th}$ order Runge-Kutta-Fehlberg method at fixed-step of $1ms$. Besides, with the purpose of having softer control signals in both stations, the continuous function $tanh(\sigma_i/\epsilon_i)$ was used instead of $sgn(\sigma_i)$ and it was set $\epsilon_i = 0.1$. It was also considered a perturbation at the slave side which satisfies $||u_e||_\infty \leq 1.5$.

4.5 Results

Several experiments were carried out and some results are shown in Fig. 5, where a parametric uncertainty of 15% in the slave side is considered with a non-vanishing u_e, forcing the system to show its closed-loop behavior when going abruptly from free motion to constrained motion. In the graphics, it can be seen that the slave tracks accurately the master motion in force and position, while the reflection force to the master is successfully accomplished. It is also shown small variations in the control signals at the end of experiments, which are related to the chattering phenomena inherent to sliding mode. It can be claimed that this variations and the small drift error shown at the end of the experiment in the second joints are produced by the use of the continuous approximation instead of the discontinuous function. As mentioned, the trajectories remain near to the switching surfaces around the region imposed by $\delta(\epsilon)$. Then the proposed control schemes allow to achieve objectives (4)-(6) and to perform the given task, in free movement and in constrained motion.

5. CONCLUSION

A causal control scheme for the telecontrol of a class of UMS under constant time-delay communication has been proposed, which has proved to ensure stability of the whole system as well as good position coordination and force

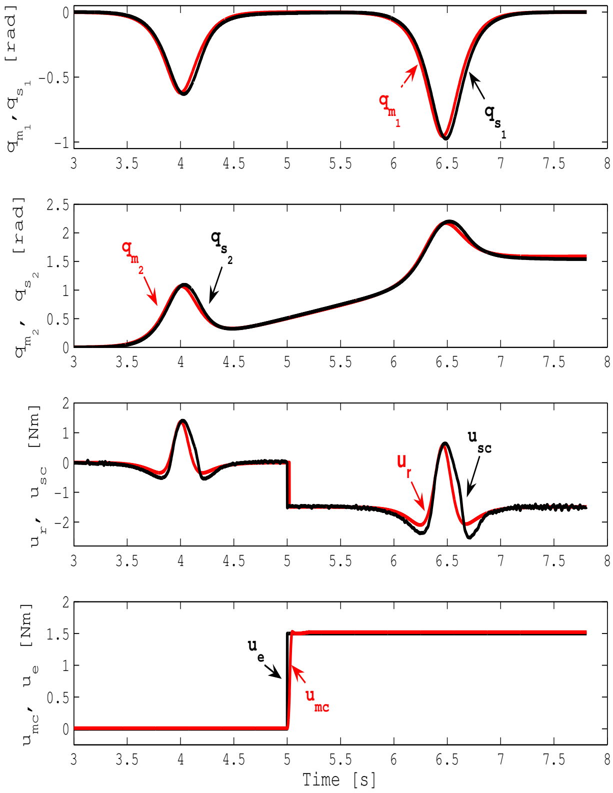

Fig. 5. Experimental results with 15% of parametric uncertainty at the slave side.

reflection, satisfying in this way traditional objectives in the distance control of systems.

The proposed scheme has been evaluated through experiments carried out in a local network and the corresponding results match with the expectations in the developed theoretical framework.

It can be said that time-delays do not always affect the system performance if the control schemes take them into account properly. In the proposed scheme, time-delays in the discontinuous feedback produce a non-causal solution which ensures some robustness features to parametric uncertainties and external disturbances.

There is a lot of interest to implement the proposed control scheme in a real-life environment. In this sense, some experiments are currently carried out on the public Internet network.

It is also considered an extension to the teleoperation case by taking into account some dynamic and structural discrepancy between a completely actuated master mechanism and an underactuated slave one such that a human being could be able to generate directly the factible trajectories and to feel force reflection in order to improve the development of complex tasks.

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