Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Fault Detection and Accommodation Control for a Class of Nonlinear Systems

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Abstract: In this paper, a fault detection and accommodation scheme is designed, which is applicable to a large class of nonlinear systems. The nonlinear observer is designed to enable the detection of fault occurrences through comparison of observed states with their signatures. Under such circumstances, when a component fails, the fault accommodation control will be activated to compensate the effects of the fault function.

Keywords: Fault detection; robotic systems, nonlinear systems.

1. INTRODUCTION

Various approaches to fault detection have been reported over the last two decades. It has been shown that the use of adequate process models can allow early fault detection with normal measurable variables [1]. In [2], an expert system model is developed for fault detection. In [5], a linear observer is designed for detecting the cutting tool wear. In [3], an adaptive observer technique is proposed for actuator fault diagnosis. In [4], a dynamical model is presented to detect incipient fault. In [21], model following method is used for reconfigurable flight control problem against structural damage or system faults. However, the model-based fault detection schemes depend on the assumption that a mathematical characterization of the robotic system is available. In practice, this is not true since it is difficult to obtain an accurate model. Recently, online approximation approaches to nonlinear fault detection problem have been developed [6, 7]. Furthermore, in many applications, it is important not only to detect but also to accommodate any failures as quickly as possible. Visinsky *et al* [2] propose an expert system for fault tolerant control (FTC). In [9], adaptive methods for accommodating actuator failures are studied. In [14], a fault diagnosis and tolearant control approach is presented which is based on a simple first order system. In [10], stable adaptive controllers are applied to achieve fault-tolerant engine control. Most of these studies are focused on the single-input-single-output (SISO) systems [14, 9, 10]. The FTC problem that arises in multiple-input-multiple-output (MIMO) systems introduces additional complexities and is considered by several investigators [8, 22, 23, 15]. The work of [8, 22, 23, 12] is focused on control of MIMO linear systems with actuator or state failures. However, most physical systems are inherently nonlinear. For most practical applications, the linear control synthesis on FTC only guarantees stability in a region about operating point and possibly degradation in performance and instability over a large domain of operation. The literature on the FTC scheme of MIMO nonlinear systems can be found in [15, 16].

In this paper, we present a fault detection and accommodation control scheme based on a class of MIMO nonlinear systems. There are two main contributions in the paper. First, the matching condition and full states available in [15, 16] are removed completely. Second, the stability analysis of the fault accommodation control scheme is investigated for two different operating modes of the closed-loop system: 1)in the absence of faults, 2)after fault detection. A simulation example is given to illustrate the effectiveness of the proposed scheme.

2. PROBLEM STATEMENTS

Consider the following MIMO nonlinear system described by

$$\left.\begin{array}{rcl} x_i^{(n_i)} &=& f_i(\mathbf{x},t) + \sum_{j=1}^{m} g_{ij}(\mathbf{x})u_j + \eta_i(\mathbf{x},t) \\ && +\beta_i(t-T)\zeta_i(\mathbf{x}) \\ y_i &=& x_i \end{array}\right\},$$

where $x_i^{(n_i)} = d^{n_i}x_i/dt^{n_i}, \mathbf{x} = [x_1, ..., x_m^{(n_m-1)}]^T \in R^n$ with $n = n_1 + n_2 + ... + n_m$, is the overall state vector, $u_i \in R, i = 1, 2, ..., m$, are the inputs and $y_i \in R, i = 1, 2, ..., m$, are the outputs of the system. The nonlinear functions $f_i, g_{ij}, i, j = 1, 2, ..., m$, are assumed to be known and the functions $\eta_i, i = 1, 2, ..., m$, represent the system uncertainties. The terms $\zeta_i, i = 1, 2, ...m$, are unknown functions which represent the faults in the system respectively, $\beta_i(t-T), i = 1, 2, ..., m$, represent the time profiles of the faults, and T is the fault-occurrence time. The system (1) can also be written in the compact form

$$\left.\begin{array}{rcl} x^{(n)} &=& F(\mathbf{x},t) + G(\mathbf{x})u + \eta(\mathbf{x},t) + \mathcal{B}(t-T)\zeta(\mathbf{x}), \\ y &=& C\mathbf{x} \end{array}\right\} \quad (1)$$

where $x^{(n)} = [x_1^{(n_1)}, ..., x_m^{(n_m)}]^T \in R^m$, $u = [u_1, ..., u_m]^T \in R^m$, $y = [y_1, ...y_m]^T \in R^m$, and

$$F(\mathbf{x},t) = [f_1(\mathbf{x},t), f_2(\mathbf{x},t), ..., f_m(\mathbf{x},t)]^T, \quad (2)$$

$$G(\mathbf{x}) = \begin{bmatrix} g_{11}(\mathbf{x}) & ... & g_{1m}(\mathbf{x}) \\ \vdots & ... & \vdots \\ g_{m1}(\mathbf{x}) & ... & g_{mm}(\mathbf{x}) \end{bmatrix}, \quad (3)$$

$$\eta(\mathbf{x}, t) = [\eta_1(\mathbf{x}, t), \eta_2(\mathbf{x}, t), ..., \eta_m(\mathbf{x}, t)]^T, \quad (4)$$

$$\zeta(\mathbf{x}) = [\zeta_1(\mathbf{x}), \zeta_2(\mathbf{x}), ..., \zeta_m(\mathbf{x})]^T, \quad (5)$$

$$C = diag\{C_1, C_2, ..., C_m\},\ C_i = [1, 0, ..., 0]_{1\times n_i}. \quad (6)$$

We consider faults with time profiles modeled by

$$\mathcal{B}(t-T) = diag\{\beta_1(t-T), \beta_2(t-T), ..., \beta_m(t-T)\},$$
$$\beta_i(t-T) = \begin{cases} 0 & t < T \\ 1 - e^{-\theta_i(t-T)} & t \geq T \end{cases}, \quad (7)$$

where the fault-occurrence time T is unknown, and $\theta_i > 0$ is an unknown constant that represents the rate at which the fault in states and actuators evolves.

The paper has the following two objectives: 1) It can detect a fault when the monitored system fails to function normally; 2) After a fault is detected, it is required that the controller should be reconfigured to accommodate the fault. The basic assumptions for the problems stated are

A1) The fault function $\zeta(\mathbf{x})$ is uniformly continuous.

A2) The matrix $\frac{G(\mathbf{x})+G^T(\mathbf{x})}{2}$ is positive definite or negative definite, i.e., $\sigma\left(\frac{G(\mathbf{x})+G^T(\mathbf{x})}{2}\right) \geq b_\sigma > 0$ where $\sigma(.)$ represents the smallest singular value of the matrix inside the bracket and b_σ is its lower bound.

A3) The modeling uncertainty $\eta_i(\mathbf{x}, t)$ is bounded by a known function,i.e.,

$$|\eta_i(\mathbf{x}, t)| \leq \bar{\eta}_i(\mathbf{x}, t), \quad (8)$$

where the bounding function $\bar{\eta}(\mathbf{x}, t)$ is continuous and uniformly bounded.

A4) The desired trajectories $y_d = [y_{d1}, y_{d2}, ..., y_{dm}]^T$ are known bounded functions of time with bounded known derivatives.

Remark 2.1. The model (1) includes a large class of nonlinear mechanical systems. For example, consider the following mechanical systems described by

$$\ddot{q} = M^{-1}(q)[\tau - V_m(q, \dot{q}) - F(\dot{q}) - G(q) - \tau_d] + \mathcal{B}(t-T)\zeta(q, \dot{q}) \quad (9)$$

where $q = [q_1, q_2, ...q_n]^T \in R^n$ are the joint position of the subsystem $i \in [1, n]$; $M(q)$ are the symmetric positive definite inertia matrix; $V_m(q, \dot{q})$ represent Coriolis and centripetal forces; $F(\dot{q})$ are the dynamic frictional force matrix; τ_d are a load disturbance matrix; $G(q)$ are the potential energy terms; τ denote generalized input control of the system applied at the joints. The fault function is represented by the term $\mathcal{B}(t-T)\zeta(q, \dot{q}) \in R^n$, where $\zeta(q, \dot{q})$ is a vector which represents the fault in the system, $\mathcal{B}(t-T)$ represents the time profile of the fault, and T is the time of occurrence of the fault.

In order to be able to design the output feedback control, let us re-write system (1) as

$$\left.\begin{aligned} \dot{\mathbf{x}} &= A_0\mathbf{x} + b[F(\mathbf{x}, t) + G(\mathbf{x})u + \eta(\mathbf{x}, t) \\ &\qquad + \mathcal{B}(t-T)\zeta(\mathbf{x})] \\ y &= x = C\mathbf{x} \end{aligned}\right\}$$

where

$$A_0 = diag\{A_{01}, A_{02}, ..., A_{0m}\} \quad (10)$$

$$b = diag\{b_1, b_2, ..., b_m\} \quad (11)$$

$$A_{0i} = \begin{bmatrix} 0 & 1 & 0 & ... & 0 \\ 0 & 0 & 1 & ... & 0 \\ \vdots & \vdots & \vdots & ... & \vdots \\ 0 & 0 & 0 & ... & 0 \end{bmatrix}_{n_i\times n_i},\ b_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}_{n_i\times 1}, \quad (12)$$

$$x = [x_1, x_2, ..., x_m]^T. \quad (13)$$

The proposed fault control scheme makes use of the assumptions:

A5) (C, A_0) is observable.

A6) $F(\mathbf{x}, t)$ is Lipschitz in $\mathbf{x}$ i.e., $||F(\mathbf{x}, t) - F(\hat{\mathbf{x}}, t)|| \leq L_F||\mathbf{x} - \hat{\mathbf{x}}||$, and $G(\mathbf{x})$ is Lipschitz in x i.e., $||G(\mathbf{x}) - G(\hat{\mathbf{x}})|| \leq L_G||x - \hat{x}||$.

A7) The modeling uncertainty is bounded by a known constant, i.e., $||\eta(\mathbf{x}, t)|| \leq \bar{\eta}$.

3. FAULT DETECTION SCHEME

In this section, the proposed fault detection system and its constituent components will be elaborated in detail. The construction of a nonlinear estimation model is first designed. Utilizing this estimation model, a time-varying threshold bound is developed so that it can serve to give a warning signal when a fault occurs.

We consider the following nonlinear model as an observer.

$$\hat{x}^{(n)} = \Lambda\tilde{x}^{(n-1)} + F(\mathbf{x}) + G(\mathbf{x})u, \quad (14)$$

where $\hat{x}^{(n)}$ denotes the estimated state vector, $\tilde{x}^{(n-1)} = x^{(n-1)} - \hat{x}^{(n-1)}$, and $\Lambda = diag\{\lambda_1, \lambda_2, ..., \lambda_m\}(\lambda_i > 0)$ is a constant matrix. The next step in the construction of the fault detection scheme is the design of the algorithm for monitoring a fault occurrence. Based on the estimation model (14), a fault estimation algorithm is presented. Since $\mathcal{B}(t-T)\zeta(\mathbf{x})$ is zero when $t < T$, each component $\bar{x}_i(t), i = 1, 2, ..., m$, of the state estimation error is given by

$$\bar{x}_i(t) = e^{-\lambda_i t}\bar{x}_i(0) + \int_0^t e^{-\lambda_i(t-\tau)}\eta_i(\mathbf{x}, t)d\tau = \varpi_i. \quad (15)$$

The threshold bound is defined as ϖ_i. The decision that a fault has occurred is made when at least one component of the estimation error $|\bar{x}_i(t)|$ exceeds its corresponding threshold bound ϖ_i. The fault detection time is defined as T_d.

4. ROBUST CONTROL FOR THE HEALTHY SYSTEMS

Before the fault occurrence, we first consider a robust controller of the system (1) when there is no fault issue.

For given a desired trajectory $y_{di}(t) \in R$, we define the errors $e_i(t) = y_i - y_{di}$. Then, the filtered tracking error is given by

$$\dot{s}_1 = (\frac{d}{dt} + k_1)^{n_1-1}e_1, ..., \dot{s}_m = (\frac{d}{dt} + k_m)^{n_m-1}e_m, \quad (16)$$

where $k_1, ..., k_m$ are positive constants to be selected. From the result of [20], it is well-known that for $s_i(t) = 0$, we have a set of linear differential equations whose solutions $e_i, i = 1, 2, ..., m$, converge to zero with constants $(n_i - 1)/k_i, i = 1, 2, ..., m$. Thus, the system equation can be written as

$$\dot{S}(t) = F(\mathbf{x}) + G(\mathbf{x})u + v + \eta(\mathbf{x}, t) + \mathcal{B}(t-T)\zeta(\mathbf{x}), \quad (17)$$

where $S = [s_1, s_2, ..., s_m]^T$ and $v = [v_1, v_2, ..., v_m]^T$ with $v_i = -y_{di}^{(n_i)} + k_i^{n_i-1}\dot{e}_i + (n_i-1)k_i^{n_i-2}\ddot{e}_i + ... + (n_i-1)k_i e_i^{(n_i-1)}$.

In the absence of any faults, the original system (17) becomes as

$$\dot{S}(t) = F(\mathbf{x}) + G(\mathbf{x})u + v + \eta(\mathbf{x}, t), \quad (18)$$

The following original control law is assumed

$$u = G^{-1}(\mathbf{x})[-F(\mathbf{x}) - v - \Lambda S - \frac{1}{2}\delta||\bar{\eta}(\mathbf{x}, t)||^2 S], \quad (19)$$

where Λ is the same as in (14) and the parameter $\delta > 0$ is a design constant. It is easy to prove that the proposed controller (19) can achieve the following theorem.

Theorem 4.1. Consider the system (1) without the presence of faults $(0 \le t \le T)$ and the controller described by (19). Suppose Assumptions **A1-A3** are satisfied. Then, the tracking errors S are UUB.

5. ACCOMMODATION CONTROL AFTER FAULT DETECTION

Fault accommodation scheme is typically achieved through reconfiguration of the feedback control system. In this subsection, we develop a nonlinear fault accommodation controller and analyze the properties of the proposed scheme.

We assume that the fault function $\zeta(\mathbf{x})$ can be approximated by a general one layer neural network (NN) [18] as

$$\zeta(\mathbf{x}) = W^{*T}\Phi(\mathbf{x}) + \xi, \quad (20)$$

where the bounded function approximation error ξ satisfies $||\xi|| \le \xi_M$ with constant ξ_M. Therefore, the fault accommodation control law is reconfigured by

$$u = G^{-1}(\mathbf{x})[-F(X) - v - \Lambda S - \frac{1}{2}\delta||\bar{\eta}(X, t)||^2 S - \hat{W}^T\Phi(\mathbf{x})] \quad (21)$$

with the learning rule

$$\dot{\hat{W}} = \Upsilon\Phi(\mathbf{x})S^T - \rho\Upsilon(\hat{W} - W_a), \quad (22)$$

where $\Upsilon = \Upsilon^T > 0, \rho > 0$, and W_a is a design constant vector. Define the Lyapunov function $V = S^2 + \mathrm{tr}(\tilde{W}^T\Upsilon^{-1}\tilde{W})$. The time derivative of V is given by

$$\dot{V} \le -2\lambda_{min}(\Lambda)||S||^2 + \delta^{-1} + 2\rho\mathrm{tr}[\tilde{W}^T(\hat{W} - W_a)] - 2S^T\mathcal{B}(t-T)\xi + 2S^T\Theta(t-T)W^{*T}\Phi(\mathbf{x}), \quad (23)$$

where we have used that

$$\mathcal{B}(t-T)\zeta(X) - \hat{W}^T\Phi(\mathbf{x}) = \tilde{W}^T\Phi(\mathbf{x}) - \Theta(t-T)W^{*T}\Phi(\mathbf{x}) + \mathcal{B}(t-T)\xi. \quad (24)$$

By completion of squares, it follows that

$$2\mathrm{tr}[\tilde{W}^T(\hat{W} - W_a)] \le -||\tilde{W}||_F^2 + ||W^* - W_a||_F^2. \quad (25)$$

where $||\cdot||_F$ is the Frobenius norm. Using the inequality $2\alpha^T\beta \le \frac{1}{2}\alpha^T\alpha + 2\beta^T\beta$, we have

$$-2S^T\mathcal{B}(t-T)\xi \le \frac{1}{2}\lambda_{min}||S||^2 + 2\lambda_{min}^{-1}\xi_M^2$$

$$2S^T\Theta W^{*T}\Phi \le \frac{1}{2}\lambda_{min}(\Lambda)||S||^2 + 2\lambda_{min}^{-1}max_{1\le i\le n}[e^{-2\theta_i(t-T)}]||W^{*T}\Phi||^2.$$

where $\lambda_{min} = \lambda_{min}(\Lambda)$. Substituting the above inequalities into (23) yields

$$\dot{V} \le -\lambda_{min}||S||^2 - \rho||\tilde{W}||_F^2 + \rho||W^* - W_a||_F^2 + 2\lambda_{min}^{-1}\xi_M^2 + 2\lambda_{min}^{-1}max_{1\le i\le n}[e^{-2\theta_i(t-T)}]||W^{*T}\Phi(q,\dot{q})||^2 + \delta^{-1}.$$

Let

$$\mu = 2\lambda_{min}^{-1}(\Lambda)\xi_M^2 + 2\lambda_{min}^{-1}(\Lambda)max_{1\le i\le n}[e^{-2\theta_i(t-T)}]||W^{*T}\Phi||^2 + \delta^{-1}.$$

Hence, we obtain the following conditions for $\dot{V} \le 0$

$$||S|| > \sqrt{\frac{\rho||W^* - W_a||_F^2 + \mu}{\lambda_{min}(\Lambda)}},$$

$$\text{or, } ||\tilde{W}||_F > \sqrt{\frac{\rho||W^* - W_a||_F^2 + \mu}{\rho}}.$$

This demonstrates that $S, \tilde{W}$ are uniformly bounded. Thus, we have the following theorem.

Theorem 5.1 Consider the system (1) in the presence of faults and the nonlinear fault accommodation scheme described by (21) and (22). Suppose Assumptions **A1-A3** are satisfied. Then, the tracking error S and NN weights $\tilde{W}$ are UUB.

6. EXTENSION TO OUTPUT FEEDBACK CONTROL DESIGN

In the preceding section, all the results have been obtained under the assumption that the full state of the system is measured. We now remove this assumption and consider more realist problems where only a part of the state is available for measurement.

6.1 Fault Detection Scheme

To detect the fault, the following observer is constructed:

$$\dot{\hat{\mathbf{x}}} = A_0\hat{\mathbf{x}} + L_0(y - \hat{y}) + b[F(\hat{\mathbf{x}}, t) + G(\hat{\mathbf{x}})u] \quad (26)$$

$$\hat{y} = \hat{x} = C\hat{\mathbf{x}} \quad (27)$$

Define the state and output estimation errors by $\tilde{\mathbf{x}} = \mathbf{x} - \hat{\mathbf{x}}$ and $\tilde{y} = y - \hat{y}$ respectively. It can be easily derived that the dynamics of residual generator is governed by

$$\dot{\tilde{\mathbf{x}}} = \bar{A}\tilde{\mathbf{x}} + b[G(\mathbf{x})u - G(\hat{\mathbf{x}})u] + b[F(\mathbf{x},t) - F(\hat{\mathbf{x}},t)] + b\eta(\mathbf{x},t) \quad (28)$$

$$\tilde{y} = C\tilde{\mathbf{x}} \quad (29)$$

where $\bar{A} = A_0 - L_0 C$. The gain matrix L_0 is chosen so that $\bar{A}$ is stable. We consider the Lyapunov function $\mathcal{V}_0 = \tilde{\mathbf{x}}^T P \tilde{\mathbf{x}}$ and its derivative is given by

$$\dot{\mathcal{V}}_0 = \tilde{\mathbf{x}}^T(\bar{A}^T P + P\bar{A})\tilde{\mathbf{x}} + 2\tilde{\mathbf{x}}^T Pb[G(\mathbf{x})u - G(\hat{\mathbf{x}})u] + 2\tilde{\mathbf{x}}^T Pb[F(\mathbf{x},t) - F(\hat{\mathbf{x}},t)] + 2\tilde{\mathbf{x}}^T Pb\eta(\mathbf{x},t) \quad (30)$$

Note that the last three terms satisfy the inequalities $2\tilde{\mathbf{x}}^T Pb[G(\mathbf{x})u - G(\hat{\mathbf{x}})u] \le \tilde{\mathbf{x}}^T P^2\tilde{\mathbf{x}} + L_G^2||b||^2||\tilde{y}||^2||u||^2$, $2\tilde{\mathbf{x}}^T Pb[F(\mathbf{x},t) - F(\hat{\mathbf{x}},t)] \le \gamma\tilde{\mathbf{x}}^T P^2\tilde{\mathbf{x}} + \gamma^{-1}||b||^2 L_F^2||\tilde{\mathbf{x}}||^2$, $2\tilde{\mathbf{x}}^T Pb\eta(\mathbf{x},t) \le ||\tilde{\mathbf{x}}||^2 + ||Pb||^2\bar{\eta}^2$ where we have used Assumption **A 6**, $x - \hat{x} = y - \hat{y}$ and $\gamma = ||b||^2 L_F^2$. This implies that

$$\dot{\mathcal{V}}_0 \le -\frac{\lambda_{min}(Q)}{\lambda_{max}(P)}\mathcal{V}_0 + L_G^2||b||^2 \parallel \tilde{y} \parallel^2 \parallel u \parallel^2 + ||Pb||^2\bar{\eta}^2 \quad (31)$$

In the above analysis, it is assumed that

$$\bar{A}^T P + P\bar{A} + (||b||^2 L_F^2 + 1)P^2 + I + Q \le 0 \quad (32)$$

By using *Lemma 3.2.4* of [25], we have

$$\mathcal{V}_0 \le e^{-\frac{\lambda_{min}(Q)}{\lambda_{max}(P)}t}\mathcal{V}_0(0) + \int_0^t e^{-\frac{\lambda_{min}(Q)}{\lambda_{max}(P)}(t-\tau)}[L_G^2||b||^2 \parallel \tilde{y} \parallel^2 \parallel u \parallel^2 + ||Pb||^2\bar{\eta}^2]d\tau = \varpi_{out} \quad (33)$$

Using $||\tilde{\mathbf{x}}||^2 \le \frac{1}{\lambda_{min}(P)}\mathcal{V}_0$, one obtains

$$||\tilde{y}|| \le ||C||\sqrt{\varpi_{out}} = \varpi \quad (34)$$

In the above result, $\mathcal{V}_0(0)$ can be replaced by a conservative estimate δ_0, where $|\mathcal{V}_0(0)| \le \delta_0$. Since the first term contains an exponential function $e^{-\frac{\lambda_{min}(Q)}{\lambda_{max}(P)}t}$, the replacement will not affect the threshold seriously. The fault detection can be carried out as

$$\begin{cases} ||\tilde{y}|| \le \varpi, \text{ no fault occurs} \\ ||\tilde{y}|| > \varpi, \text{ fault has occurred} \end{cases}. \quad (35)$$

6.2 Robust Control for the Healthy System

For the design of fault accommodation control, we need to introduce a state error system. Define the state error $E = \mathbf{x} - \mathbf{x}_d$ where $\mathbf{x}_d = [y_{d1}, ..., y_{dm}^{(n_m-1)}]^T \in R^n$. System (10) may be expressed as

$$\dot{E} = AE + b[F(\mathbf{x},t) + G(\mathbf{x})u - y_d^{(n)} + KE + \eta(\mathbf{x},t) + \mathcal{B}(t-T)\zeta(\mathbf{x})] \quad (36)$$

$$e = CE \quad (37)$$

where $A = A_0 - bK$, $y_d^{(n)} = [y_{d1}^{(n_1)}, ..., y_{dm}^{(n_m)}]^T$ and $e = y - y_d$. The constant matrix K in (36) is chosen so that $C[sI - A]^{-1}b$ is strictly positive real (SPR).

For the healthy system, i.e., $\mathcal{B}(t-T)\zeta(\mathbf{x}) = 0$, the feedback control is required to cause output vector y to track reference y_d. Since the state of system (10) is not available, the following observer is proposed

$$\dot{\hat{\mathbf{x}}} = A_0\hat{\mathbf{x}} + L(y - \hat{y}) + b[F(\hat{\mathbf{x}},t) + G(\hat{\mathbf{x}})u + \Xi_1] \quad (38)$$

where $\Xi_1 = \frac{L_G}{b_\sigma}\tilde{y}[k_e \parallel e \parallel + ||F(\hat{\mathbf{x}},t)|| + \parallel y_d^{(n)} \parallel] + (\frac{2}{\lambda_{min}(Q_1)}L_F^2 + \frac{1}{2})\tilde{y} + \frac{3L_G^2}{2b_\sigma^2}\tilde{y}[k_e^2||e||^2 + ||F(\hat{\mathbf{x}},t)||^2 + ||y_d^{(n)}||^2]$. The gain matrix L is chosen so that $C[sI - (A_0 - LC)]^{-1}b$ is SPR. When equation (38) is subtracted from (10) it results in the following observation error system

$$\dot{\tilde{\mathbf{x}}} = \bar{A}\tilde{\mathbf{x}} + b[G(\mathbf{x})u - G(\hat{\mathbf{x}})u + F(\mathbf{x},t) - F(\hat{\mathbf{x}},t) + \eta(\mathbf{x},t) - \Xi_1] \quad (39)$$

where $\bar{A} = A_0 - LC$. Using (38), we now propose the following controller for the healthy system

$$u = \frac{e}{e^T G(\hat{\mathbf{x}})e}[-k_e||e||^2 - e^T F(\hat{\mathbf{x}},t) + e^T y_d^{(n)}] \quad (40)$$

where $k_e > 0$ with $k_e > \frac{2}{\lambda_{min}(Q_1)}L_F^2 + \frac{1}{\lambda_{min}(Q_2)}||K||^2 + 1$ (Q_1, Q_2 will be given below). Our task is to stabilize (10),(38) with respect to the Lyapunov function $\mathcal{V}_3 = \mathcal{V}_1 + \mathcal{V}_2$ where $\mathcal{V}_1 = \tilde{\mathbf{x}}^T P_1\tilde{\mathbf{x}}$ and $\mathcal{V}_2 = E^T P_2 E$.

Since $C[sI - \bar{A}]^{-1}b$ is SPR, from *Lemma 3.5.4* of [25], there exists $P_1 > 0$ such that $\bar{A}^T P_1 + P_1\bar{A} = -Q_1, b^T P_1 = C$. Similarly, since $C(sI - A)^{-1}b$ is also SPR, we have $A^T P_2 + P_2 A = -Q_2, b^T P_2 = C$. The derivative of $\mathcal{V}_1$ is

$$\begin{aligned}\dot{\mathcal{V}}_1 &= \tilde{\mathbf{x}}^T(\bar{A}^T P_1 + P_1\bar{A})\tilde{\mathbf{x}} + 2\tilde{y}^T[G(\mathbf{x})u - G(\hat{\mathbf{x}})u + F(\mathbf{x},t) - F(\hat{\mathbf{x}},t) + \eta(\mathbf{x},t)] - 2\tilde{y}^T\Xi_1 \\ &\le -\frac{3}{4}\lambda_{min}(Q_1)||\tilde{\mathbf{x}}||^2 + \bar{\eta}^2 - 3\frac{L_G^2}{b_\sigma^2}||\tilde{y}||^2[k_e^2||e||^2 + ||F(\hat{\mathbf{x}},t)||^2 + ||y_d^{(n)}||^2]\end{aligned} \quad (41)$$

where we have used the facts that

$$e^T G(\hat{\mathbf{x}})e = e^T(\frac{G(\hat{\mathbf{x}}) + G^T(\hat{\mathbf{x}})}{2})e \ge b_\sigma||e||^2$$

and Assumptions A6-A7.

Using (36) and SPR condition, the derivative of the Lyapunov function $\mathcal{V}_2$ is computed as

$$\dot{\mathcal{V}}_2 = E^T(A^T P_2 + P_2 A)E + 2e^T[G(\mathbf{x}) - G(\hat{\mathbf{x}})]u + 2e^T[G(\hat{\mathbf{x}})u + F(\mathbf{x},t) - y_d^{(n)} + KE + \eta(\mathbf{x},t)] \quad (42)$$

Taking a similar proof procedure as in the derivative of $\mathcal{V}_1$, we have

$$\begin{aligned}\dot{\mathcal{V}}_2 &\le -\lambda_{min}(Q_2)||E||^2 + 2\frac{L_G||\tilde{y}||}{b_\sigma}||[-k_e||e||^2 - e^T F(\hat{\mathbf{x}},t) + e^T y_d^{(n)}]|| - 2k_e||e||^2 \\ &\quad + 2e^T[F(\mathbf{x},t) - F(\hat{\mathbf{x}},t)] + 2e^T KE + 2e^T\eta(\mathbf{x},t) \\ &\le -\frac{\lambda_{min}(Q_2)}{2}||E||^2 + \frac{3L_G^2}{b_\sigma^2}||\tilde{y}||^2[k_e^2||e||^2 + ||F(\hat{\mathbf{x}},t)||^2 \\ &\quad + ||y_d^{(n)}||^2] - \lambda_e||e||^2 + \frac{\lambda_{min}(Q_1)}{4}||\tilde{\mathbf{x}}||^2 + \bar{\eta}^2\end{aligned}$$

where $\lambda_e = 2k_e - \frac{4}{\lambda_{min}(Q_1)}L_F^2 - \frac{2}{\lambda_{min}(Q_2)}||K||^2 - 2$. Combining $\dot{\mathcal{V}}_1$ with $\dot{\mathcal{V}}_2$, the derivative of the Lyapunov function $\mathcal{V}_3$ is

$$\dot{\mathcal{V}}_3 \leq -\frac{\lambda_{min}(Q_1)}{2}||\tilde{\mathbf{x}}||^2 - \frac{\lambda_{min}(Q_2)}{2}||E||^2 - \lambda_e||e||^2 + 2\bar{\eta}^2$$

which can be used to show that $\tilde{\mathbf{x}}, E$ are UUB.

6.3 Accommodation Control After Fault Detection

When a fault is detected, the accommodation in the control system can be achieved through adding a NN approximator into the normal controller. In this section, we use the same control policy to compensate for the faults. The proposed control law is given as follows:

$$u = \frac{e}{e^T G(\hat{\mathbf{x}})e}[-k_e||e||^2 - e^T F(\hat{\mathbf{x}},t) + e^T y_d^{(n)} - e^T \hat{W}^T\Phi(\hat{\mathbf{x}})] \quad (43)$$

with adaptive law $\dot{\hat{W}} = \Upsilon\Phi(\hat{\mathbf{x}})\tilde{y}^T - \rho\Upsilon(\hat{W} - W_a)$. The reconfigured observer is given by

$$\dot{\hat{\mathbf{x}}} = A_0\hat{\mathbf{x}} + L(y - \hat{y}) + b\{F(\hat{\mathbf{x}},t) + G(\hat{\mathbf{x}})u + \Xi_1 + \hat{W}^T\Phi(\hat{\mathbf{x}}) + \frac{1}{2}\tilde{y} + [1 + \frac{2L_G}{\lambda_e b_\sigma}||\hat{W}^T\Phi(\hat{\mathbf{x}})||]\frac{L_G}{b_\sigma}\tilde{y}||\hat{W}^T\Phi(\hat{\mathbf{x}})||\}$$

The resulting observation error equation is

$$\dot{\tilde{\mathbf{x}}} = \bar{A}\tilde{\mathbf{x}} + b\{G(\mathbf{x})u - G(\hat{\mathbf{x}})u + F(\mathbf{x},t) - F(\hat{\mathbf{x}},t) + \eta(\mathbf{x},t) - \Xi_1 + \mathcal{B}(t-T)\zeta(\mathbf{x}) - \hat{W}^T\Phi(\hat{\mathbf{x}}) - \frac{1}{2}\tilde{y} - [1 + \frac{2L_G}{\lambda_e b_\sigma}||\hat{W}^T\Phi(\hat{\mathbf{x}})||]\frac{L_G}{b_\sigma}\tilde{y}||\hat{W}^T\Phi(\hat{\mathbf{x}})||\Big\}$$

Taking a similar proof procedure as in $\dot{\mathcal{V}}_3$, the derivative of $\mathcal{V} = \mathcal{V}_3 + tr(\tilde{W}^T\Upsilon^{-1}\tilde{W})$ is

$$\dot{\mathcal{V}} \leq -\frac{\lambda_{min}(Q_1)||\tilde{\mathbf{x}}||^2}{2} - \frac{\lambda_{min}(Q_2)||E||^2}{2} - (\rho - \frac{2\bar{\Phi}^2}{\lambda_e})||\tilde{W}||_F^2 + \rho||W^* - W_a||_F^2 + (1 + \frac{4}{\lambda_e})||\bar{g}||^2 + 2\bar{\eta}^2$$

where we have used the fact that $||\Phi(\hat{\mathbf{x}})|| \leq \bar{\Phi}$ and the relationship $\mathcal{B}(t-T)\zeta(\mathbf{x}) - \hat{W}^T\Phi(\hat{\mathbf{x}}) = \tilde{W}^T\Phi(\hat{\mathbf{x}}) + \bar{g}$ with $\bar{g} = W^{*T}[\Phi(\mathbf{x}) - \Phi(\hat{\mathbf{x}})] - \Theta(t-T)W^{*T}\Phi(\mathbf{x}) + \mathcal{B}(t-T)\xi$. We can show that $\tilde{\mathbf{x}}$ and E are UUB under the accommodation control (43).

7. SIMULATION EXAMPLE

To illustrate the performance of the proposed control scheme, we consider the following linear motor

$$\ddot{q} = M^{-1}[\tau - V(q,\dot{q}) - F(\dot{q}) - G(q) - \tau_d] + \mathcal{D}(t-T)\zeta(q,\dot{q}) \quad (44)$$

where q is the position, M is the mass, τ is the torque, $V(q,\dot{q})$ denotes the Coriolis and centripetal force, $F(\dot{q})$ and $G(q)$ denote the frictional and ripple forces respectively, and τ_d includes other residual uncertainties and disturbances in the system.

The function $V(q,\dot{q})$ is assumed to be $D\dot{q}$ with $D = 0.05$, while the friction $F(\dot{q})$ and G are assumed to be

$$F(\dot{q}) = f_1 sgn(\dot{q}) + f_2\dot{q} + f_3 exp(-(\dot{q}/\dot{q}_s)^2)sgn(\dot{q}) \quad (45)$$

and $G(q) = g_1 sin(w_1 q + \phi_1)$, respectively.

If a fault occurs due to a tangle of complex factors, the tracking performance will be degraded significantly. In the simulation, the fault is assumed to be a nonlinear variation described by

$$\zeta(q,\dot{q}) = [1 - e^{-10(t-T)}] \times (q^2\dot{q} + 10). \quad (46)$$

The fault depends on the current position and velocity of the mechanical system. The failure is assumed to be triggered at $t = 5s$. Figure 1 shows the position control when a fault modeled by (46) occurs without fault accommodation scheme. It is observed that the tracking error is increased significantly after the occurrence of the fault. Note that from the figure 1 the error between the state and the estimation has exceeded the threshold bound. Now we consider a reconfigured controller after the fault is detected. In this case, the fault is detected at $T = 5.0121$ when the residual error is above the threshold bound and the fault accommodation control is activated. The NN learning model used in this simulation has its parameters first fixed at $\Gamma = I, \eta = 0.1$. Figure 2 shows the plot of the fault accommodation control with the developed NN learning. It can be observed that the NN learning can compensate provides the fault function after detecting the system failure.

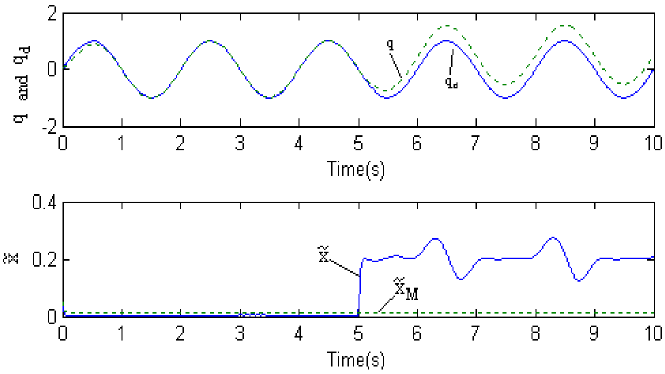

Fig. 1. Position control with fault occurrence

8. CONCLUSIONS

In this paper, the fault detection and accommodation schemes have been proposed for a class of MIMO nonlinear systems. Using an online approximation approach, we have been able to relax the parametric fault requirements of traditional adaptive control without considering the dynamic uncertainty as part of the fault. Stability results were obtained by considering two situations.

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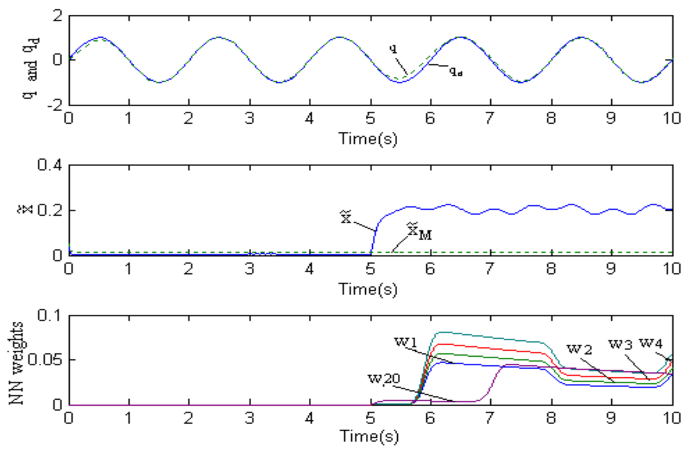

Fig. 2. Position fault accommodation control with fault occurrence (N=120)

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