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Robustness Analysis of Mobile Robot Velocity Estimation Using a Regular Polygonal Array of Optical Mice

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Abstract: This paper presents the robust localization of an omnidirectional mobile robot using a regular polygonal array of optical mice that are installed at the bottom of a mobile robot. First, the basic principle of the proposed localization scheme is explained. Second, the velocity kinematics from a mobile robot to an array of optical mice is derived as an overdetermined linear system. Third, for a given set of optical mouse readings, the least squares velocity estimation of a mobile robot is obtained as the simple average. Fourth, the robustness of the proposed least squares velocity estimation against measurement noise, partial malfunction, and imprecise installation is analyzed.

1. INTRODUCTION

In the near future, personal service robots are expected to come into human daily life as supporters in education, leisure, house care, health care, and so on. Most of them built on mobile platforms require the capability of autonomous navigation in unknown and/or dynamic environments. The key ingredients for autonomous navigation are viable techniques for map building, obstacle detection/avoidance, localization (Murphy, 2000; Thrun *et al*., 2005). The concern of this paper is a robust localization method for an omnidirectional mobile robot as a platform of personal service robots.

Typical sensors used for the localization of a commercial mobile robot include encoders, ultrasonic sensors, and cameras (Borenstein *et al*., 1996). However, encoders are vulnerable to wheel slip, ultrasonic sensors require the line of sight, and cameras usually mandate heavy computation. There have been several attempts to employ the optical mice for the localization of a mobile robot (O'Hara, 2001; Sorenson *et al*., 2003; Lee *et al*., 2004; Bonarini *et al*., 2004; Singh *et al*., 2004; Kim *et al*., 2006). In fact, the optical mouse is an inexpensive but high performance device with sophisticated image processing engine (Horn *et al*., 1981; Alden 2002). The velocity estimation of a mobile robot using a set of optical mice can overcome to some extent the aforementioned limitations of typical sensors.

For the localization of an omnidirectional mobile robot on the plane, three variables including two positional coordinates and the steering angle need to be determined. Since an optical mouse provides two positional information, the required number of optical mice should be more than or equal to two. Most of previous research (Sorenson *et al*., 2003; Lee *et al*., 2004; Bonarini *et al*., 2004; Singh *et al*., 2004) use two optical mice, while only a single optical mouse is used in (O'Hara, 2001). However, few attempt has been made to use more than two optical mice except (Kim *et al*., 2006).

In this paper, we present the robust localization of a mobile robot using the redundant number of optical mice arranged in a regular polygonal array. This paper is organized as follows. Section 2 explains the basic principle. Section 3 derives the velocity kinematics as an overdetermined system. Section 4 obtains the least squares velocity estimates as the simple average. Sections 5, 6, and 7 analyze the robustness of the proposed least squares velocity estimation against measurement noise, partial malfunction, and imprecise installation.

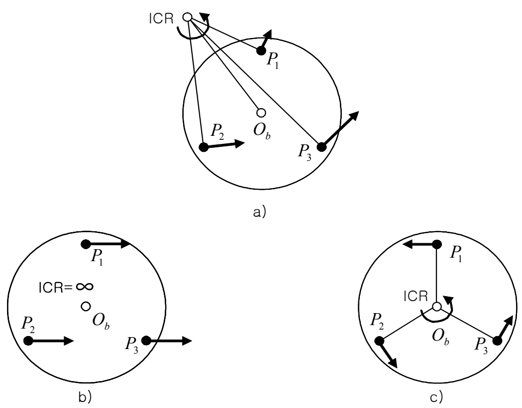

Fig. 1. The basic principle of the proposed localization system.

2. BASIC PRINCIPLE

To explain the basic principle of the proposed localization method, let us consider a regular triangular array of optical mice attached at the bottom of a mobile robot. Generally, the traveling pattern of a mobile robot can be specified in terms of the location of ICR (Instantaneous Center of Rotation) on the plane. For three different traveling patterns of a mobile robot, Fig. 1 shows the linear velocities observed by a regular triangular array of optical mice.

When a mobile robot is rotating with ICR apart from the center of a mobile robot as shown in Fig. 1a), three velocity vectors are different in both direction and magnitude. When a mobile robot is moving straight as shown in Fig. 1b), corresponding to the case of ICR at infinity, three velocity vectors become the same in both direction and magnitude. When a mobile robot is rotating with ICR coincident with the center of a mobile robot as shown in Fig. 1c), three velocity vectors become different in direction but the same in magnitude. Theses observations tells that a different traveling pattern of a mobile robot results in a set of different velocity readings of an array of optical mice. Reversely, it is possible to estimate the linear and angular velocities of a traveling mobile robot from the velocity readings of optical mice.

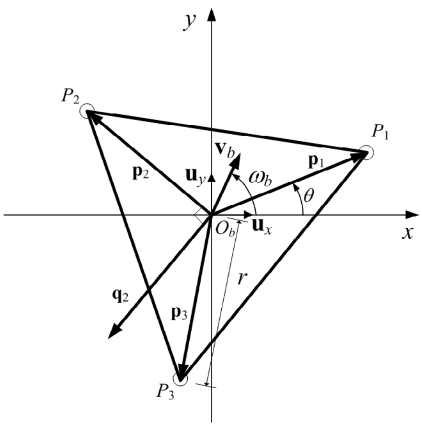

Fig. 2. A regular triangular array of optical mice with $N=3$.

3. VELOCITY KINEMATICS

Assume that N optical mice are installed at the vertices, P_i, $i=1,\cdots,N$, of a regular polygon that is centered at the center, O_b, of a mobile robot traveling on the xy plane. Fig. 2 shows an example of a regular triangular array of optical mice with $N=3$. Let $\mathbf{u}_x = [1\ \ 0]^t$ and $\mathbf{u}_y = [0\ \ 1]^t$ be the unit vectors along the x axis and the y axis, respectively. The position vector, $\mathbf{p}_i = [p_{ix}\ \ p_{iy}]^t$, $i=1,\cdots,N$, from O_b to P_i, can be expressed as

$$\mathbf{p}_i = \begin{bmatrix} p_{ix} \\ p_{iy} \end{bmatrix} = \begin{bmatrix} r\cos\{\theta+(i-1)\times\frac{2\pi}{N}\} \\ r\sin\{\theta+(i-1)\times\frac{2\pi}{N}\} \end{bmatrix} \tag{1}$$

where θ represents the heading angle of a mobile robot with the forwarding direction aligned with $\mathbf{p}_1$, and r represents the distal distance of each optical mouse. Due to the regular polygonal arrangement of optical mice, it holds that

$$\sum_{i=1}^{N} p_{ix} = \sum_{i=1}^{N} p_{iy} = 0 \tag{2}$$

regardless of the heading angle θ. For notational convenience, let $\mathbf{q}_i$, $i=1,2,3$, be the vector obtained by rotating $\mathbf{p}_i$ by 90^o counterclockwise.

In this paper, we assume that a mobile robot has the omnidirectional mobility on the xy plane. Let $\mathbf{v}_b = [v_{bx}\ \ v_{by}]^t$ and w_b be the linear velocity and the angular velocity at the center O_b of a mobile robot, respectively. And, let $\mathbf{v}_i = [v_{ix}\ \ v_{iy}]^t$, $i=1,\cdots,N$, be the linear velocity at the vertex P_i, which corresponds to the velocity readings of the i^{th} optical mouse. Then, there holds the following velocity relationship:

$$\mathbf{v}_b + \omega_b \mathbf{q}_i = \mathbf{v}_i \tag{3}$$

Premultiplied by $\mathbf{u}_x^t$ and $\mathbf{u}_y^t$, (3) gives

$$\mathbf{u}_x^t \mathbf{v}_b + \omega_b \mathbf{u}_x^t \mathbf{q}_i = \mathbf{u}_x^t \mathbf{v}_i \tag{4}$$
$$\mathbf{u}_y^t \mathbf{v}_b + \omega_b \mathbf{u}_y^t \mathbf{q}_i = \mathbf{u}_y^t \mathbf{v}_i \tag{5}$$

respectively. Referring to Fig. 2, (4) and (5) can be rewritten as

$$v_{bx} - w_b \times p_{iy} = v_{ix} \tag{6}$$
$$v_{by} + w_b \times p_{ix} = v_{iy} \tag{7}$$

From (6) and (7), the velocity mapping from a mobile robot to an array of optical mice can be represented as

$$\mathrm{A}\ \dot{\mathbf{x}} = \dot{\Theta} \tag{8}$$

where

$$\dot{\mathbf{x}} = \begin{bmatrix} v_{bx} \\ v_{by} \\ \omega_b \end{bmatrix} \tag{9}$$

$$\dot{\Theta} = \begin{bmatrix} \dot{\Theta}_1 \\ \dot{\Theta}_2 \\ \vdots \\ \dot{\Theta}_N \end{bmatrix} \in \mathrm{R}^{2N\times 1} \text{ with } \dot{\Theta}_i = \begin{bmatrix} v_{ix} \\ v_{iy} \end{bmatrix} \tag{10}$$

$$\mathrm{A} = \begin{bmatrix} \mathrm{A}_1 \\ \mathrm{A}_2 \\ \vdots \\ \mathrm{A}_N \end{bmatrix} \in \mathrm{R}^{2N\times 3} \tag{11}$$

with

$$\mathrm{A}_i = \begin{bmatrix} 1 & 0 & -p_{iy} \\ 0 & 1 & p_{ix} \end{bmatrix} \tag{12}$$

Note that the expression of A is quite simple as a function of the position vectors, $\mathbf{p}_i = [\,p_{ix}\ \ p_{iy}]^t$, $i = 1,\cdots,N$. It should be mentioned that such simplicity of A is effective for a general polygonal array of optical mice.

In the case of $N=1$, (8) represents two equations of three unknowns, including two linear velocity components, v_{bx} and v_{by}, and one angular velocity component, w_b. Thus, (8) becomes an underdetermined system, which implies that the mobile robot velocity cannot be uniquely determined from the optical mouse readings. However, for $N \geq 2$, (8) becomes an overdetermined system consisting of $2N$ equations, for which the least squares solution can be sought. From now on, it is assumed that the number of optical mice in use is greater than or equal to two, that is, $N \geq 2$.

4. VELOCITY ESTIMATION

Typically, the velocity readings of an optical mouse suffer from two kinds of the inaccuracy caused by both systematic and nonsystematic errors (Borenstein *et al.*, 1996). Here, we assume that the calibration has been performed to compensate all systematic errors involved. Under this assumption, the measurement model of a array of optical mice can be expressed as

$$\dot{\Theta} = \mathrm{A}\ \dot{\mathrm{x}} + \mathrm{n} \tag{13}$$

where

$$\mathrm{n} = [\,n_1\ n_2\ \cdots\ n_{2N}]^t \in \mathrm{R}^{2N\times 1} \tag{14}$$

with n_i, $i = 1,\cdots,2N$, being independent zero mean measurement noise with constant covariance.

From (13), the least squares estimation can be obtained by (Bar-shalom *et al.*, 2001)

$$\hat{\dot{\mathrm{x}}} = \mathrm{B}\ \dot{\Theta} \tag{15}$$

where

$$\hat{\dot{\mathrm{x}}} = \begin{bmatrix} \hat{v}_{bx} \\ \hat{v}_{by} \\ \hat{\omega}_b \end{bmatrix} \tag{16}$$

$$\mathrm{B} = (\mathrm{A}^t \mathrm{A})^{-1} \mathrm{A}^t \in \mathrm{R}^{3\times 2N} \tag{17}$$

which is the generalized inverse of A. (15) represents the estimated velocity of a mobile robot from the noisy optical mouse readings which minimizes the quadratic error, given by $\|\mathrm{A}\ \dot{\mathrm{x}} - \dot{\Theta}\|^2$. Note that the lease squares estimation obtained above becomes equivalent to the maximum likelihood estimation under the additional assumption that the measurement noise n_i, $i = 1,\cdots,2N$, are Gaussian (Bar-shalom *et al.*, 2001).

For a general polygonal array of optical mice, it can be shown that

$$\mathrm{A}^t\mathrm{A} = \begin{bmatrix} N & 0 & -\sum_{i=1}^{N} p_{iy} \\ 0 & N & \sum_{i=1}^{N} p_{ix} \\ -\sum_{i=1}^{N} p_{iy} & \sum_{i=1}^{N} p_{ix} & \sum_{i=1}^{N} \|\mathbf{p}_i\|^2 \end{bmatrix} \tag{18}$$

Since

$$\frac{1}{N} \times \|\sum_{1}^{N} \mathbf{p}_i\|^2 < \sum_{1}^{N} \|\mathbf{p}_i\|^2 \tag{19}$$

the inverse of $\mathrm{A}^t\mathrm{A}$ always exists independent of the heading angle θ of a mobile robot, which guarantees the observability of the measurement model, given by (13).

For a regular polygonal array of optical mice, for which (2) holds, we have

$$\mathrm{A}^t\mathrm{A} = \begin{bmatrix} N & 0 & 0 \\ 0 & N & 0 \\ 0 & 0 & Nr^2 \end{bmatrix} \tag{20}$$

Using (1), (11), (12), and (20), (17) can be obtained by

$$\mathrm{B} = [\,\mathrm{B}_1\ \mathrm{B}_2\ \cdots\ \mathrm{B}_N\,] \in \mathrm{R}^{3\times 2N} \tag{21}$$

where

$$\mathrm{B}_i = \frac{1}{N} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -\frac{1}{r}\sin\{\theta + \frac{(i-1)2\pi}{N}\} & \frac{1}{r}\cos\{\theta + \frac{(i-1)2\pi}{N}\} \end{bmatrix} \tag{22}$$

Finally, for given velocity readings of N optical mice, $\dot{\Theta} = [\,v_{1x}\ v_{1y}\ v_{2x}\ v_{2y}\ \cdots\ v_{Nx}\ v_{Ny}]^t$, the linear and angular velocity components of a mobile robot, $\hat{\dot{\mathrm{x}}} = [\,\hat{v}_{bx}\ \hat{v}_{by}\ \hat{\omega}_b]^t$, can be obtained, from (15), (21), and (22), as follows:

$$\begin{aligned} \hat{v}_{bx} &= \frac{1}{N}\sum_{i=1}^{N} v_{ix} \\ \hat{v}_{by} &= \frac{1}{N}\sum_{i=1}^{N} v_{iy} \\ \hat{w}_b &= \frac{1}{N}\sum_{i=1}^{N} w_i \end{aligned} \tag{23}$$

where

$$w_i = \frac{1}{r}\left[-\sin\left\{\theta + \frac{(i-1)2\pi}{N}\right\}\times v_{ix} + \cos\left\{\theta + \frac{(i-1)2\pi}{N}\right\}\times v_{iy}\right] \tag{24}$$

which represents the angular velocity experienced by the i^{th} optical mouse. Regarding The velocity estimation based on (23), the following remarks need to be made. First, the angular velocity estimate, $\hat{w}_b$, is dependent on the heading angle θ, while the linear velocity estimates, $\hat{v}_{bx}$ and $\hat{v}_{by}$, are not. Second, each of three velocity estimates is determined as the simple average of the corresponding velocity components read from all the optical mice. Such computational simplicity is attributed to the arrangement of optical mice in a regular polygonal array centered at the center of a mobile robot.

5. MEASUREMENT NOISE

The redundant number of optical mice helps to reduce the effect of the measurement noise accompanying the velocity readings from optical mice. Suppose that a mobile robot stands still without moving, that is, $v_{bx} = v_{by} = 0.0$ [$m/\sec$] and $w_b = 0.0$ [rad/sec]. Now, the optical mouse readings come solely from unbiased random noise which are assumed to be independent and identical. For instance, the mean and the standard deviation of random noise are given respectively by

$$\begin{aligned} \mathrm{E}[v_{1x}] &= \mathrm{E}[v_{2x}] = \cdots = \mathrm{E}[v_{Nx}] = 0 \\ \mathrm{std}[v_{1x}] &= \mathrm{std}[v_{2x}] = \cdots = \mathrm{std}[v_{Nx}] = \sigma \end{aligned} \tag{25}$$

For the mobile robot velocity estimation based on (23), it can be shown that for instance,

$$\begin{aligned} \mathrm{E}[v_{bx}] &= 0 \\ \mathrm{std}[v_{bx}] &= \frac{\sigma}{\sqrt{N}} \end{aligned} \tag{26}$$

(26) tells that the greater the number of optical mice, the smaller the velocity estimation error, under the same level of measurement noise. As the number of optical mice is increased from $N(\geq 2)$ to $(N+1)$, the percent improvement in accurate velocity estimation is given by

$$\mathrm{PI} = \left(1 - \sqrt{\frac{N}{N+1}}\right) \times 100 \tag{27}$$

Note that the percent improvement is more significant for the smaller number of optical mice.

6. PARTIAL MALFUNCTION

The sucess of the velocity estimation based on (23) is heavily dependent on whether all the optical mice function properly. It is very important to detect and isolate malfunctioning optical mice, if any, from the velocity estimation. Here, we propose a simple but effective means to cope with the partial malfunction by using the redundant number of optical mice. The basic rationale is that the velocity estimation based on (23) can be considered as a process of building consensus among all the optical mice of equal privilege.

For a given mobile robot velocity estimate, $\hat{\mathrm{x}}$, the residual of the i th optical mouse is given by (Golub, 1996)

$$\Delta \dot{\Theta}_i = \hat{\dot{\Theta}}_i - \dot{\Theta}_i \tag{28}$$

where

$$\hat{\dot{\Theta}}_i = \mathrm{A}_i \, \hat{\mathrm{x}} \tag{29}$$

which represents the expected velocity readings of the i th optical mouse. For the i th optical mouse, we consider the magnitude of the residual $\Delta \dot{\Theta}_i$, given by

$$\rho_i = \| \hat{\dot{\Theta}}_i - \dot{\Theta}_i \| \tag{30}$$

which represents the discrepancy between the expected and the actual velocity readings. Using (10), (30) can be expressed as

$$\rho_i^2 = (\hat{v}_{ix} - v_{ix})^2 + (\hat{v}_{iy} - v_{iy})^2 \tag{31}$$

which can be easily computed.

The residual magnitude ρ_i can be interpreted as a measure of inconformity of the i th optical mouse to the consensus reached by all the optical mice. If the degree of inconformity exceeds a prespecified threshold, then it would be reasonable to isolate the problematic optical mice in order to search for a new consensus among the remaining ones. Supposing that the k th optical mouse is malfunctioning, the new velocity estimation can be made under

$$\mathrm{A} = \begin{bmatrix} \mathrm{A}_1 \\ \vdots \\ \mathrm{A}_{k-1} \\ \mathrm{A}_{k+1} \\ \vdots \\ \mathrm{A}_N \end{bmatrix} \in \mathrm{R}^{2(N-1)\times 3} \tag{32}$$

$$\mathrm{B} = [\, \mathrm{B}_1 \cdots \mathrm{B}_{k-1} \; \mathrm{B}_{k+1} \cdots \mathrm{B}_N \,] \in \mathrm{R}^{3\times 2(N-1)} \tag{33}$$

which results in

$$\begin{aligned} \hat{v}_{bx} &= \frac{1}{N-1} \sum_{i=1, i\neq k}^{N} v_{ix} \\ \hat{v}_{by} &= \frac{1}{N-1} \sum_{i=1, i\neq k}^{N} v_{iy} \\ \hat{w}_b &= \frac{1}{N-1} \sum_{i=1, i\neq k}^{N} w_i \end{aligned} \tag{34}$$

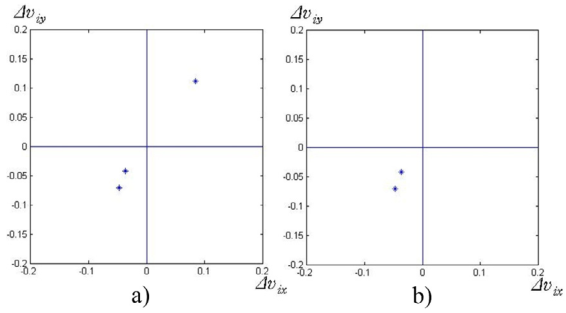

Fig. 3. Partial malfunction: a) before remedy and b) after remedy.

Notice that two velocity readings from the malfunctioning optical mouse are ignored simultaneously.

With $\Delta\dot{\Theta}_i = [\Delta v_{ix}\ \Delta v_{iy}]^t$, Fig. 3a) illustrates a situation in which one optical mouse is suspected of malfunction out of a total of three optical mice. Fig. 3b) illustrates a situation in which the malfunctioning optical mouse is excluded from the new velocity estimation. It should be mentioned that the redundant number of optical mice contributes to the robustness of the velocity estimation against partial malfunction as well as measurement noise.

7. IMPRECISE INSTALLATION

In practice, it may be rather difficult to install optical flow sensors in a regular polygonal array symmetric with respect to the center of a mobile robot without installation error. Let us examine the sensitivity of the velocity estimation based on (23) to imprecise installation of optical flow sensors. Suppose that the position error vector, $\delta\mathbf{p}_i$, of the i th optical flow sensor is described by

$$\delta\mathbf{p}_i = \begin{bmatrix} \delta p_{ix} \\ \delta p_{iy} \end{bmatrix} \tag{35}$$

where δp_{ix} and δp_{iy}, $i = 1, \cdots, n$, represent the deviation from the vertices of a regular polygon due to imprecise installation.

In the presence of installation error, the velocity kinematics, given by (8), can be expressed

$$(\mathrm{A} + \delta\mathrm{A})(\dot{\mathrm{x}} + \delta\dot{\mathrm{x}}) = \dot{\Theta} + \delta\dot{\Theta} \tag{36}$$

where

$$\delta\dot{\mathrm{x}} = \begin{bmatrix} \delta v_{bx} \\ \delta v_{by} \\ \delta\omega_b \end{bmatrix} \in \mathrm{R}^{3\times1} \tag{37}$$

$$\delta\mathrm{A} = \begin{bmatrix} 0 & 0 & -\delta p_{1y} \\ 0 & 0 & \delta p_{1x} \\ 0 & 0 & -\delta p_{2y} \\ 0 & 0 & \delta p_{2x} \\ \vdots & \vdots & \vdots \\ 0 & 0 & -\delta p_{Ny} \\ 0 & 0 & \delta p_{Nx} \end{bmatrix} \in \mathrm{R}^{2N\times3} \tag{38}$$

(38) represents the perturbation on A owing to imprecise installation, and (37) represents the resulting error in velocity estimation of a mobile robot.

Premultiplied by $(\mathrm{A} + \delta\mathrm{A})^t$, (36) becomes

$$(\mathrm{A}^t + \delta\mathrm{A}^t)(\mathrm{A} + \delta\mathrm{A})(\dot{\mathrm{x}} + \delta\dot{\mathrm{x}}) = (\mathrm{A}^t + \delta\mathrm{A}^t)(\dot{\Theta} + \delta\dot{\Theta}) \tag{39}$$

Assuming that the installation error and the resulting estimation error are small enough, we have

$$\delta\mathrm{A}^t\,\delta\mathrm{A} \simeq 0_{2\times2} \tag{40}$$

$$(\delta\mathrm{A}^t\mathrm{A} + \mathrm{A}^t\delta\mathrm{A})\,\delta\dot{\mathrm{x}} \simeq 0_2 \tag{41}$$

$$\delta\mathrm{A}^t\,\delta\dot{\Theta} \simeq 0_2 \tag{42}$$

Under the assumption of (40), (41), and (42), (39) can be approximated as

$$\mathrm{P}\,\dot{\mathrm{x}} + \mathrm{P}\,\delta\dot{\mathrm{x}} + \delta\mathrm{P}\,\dot{\mathrm{x}} = \mathrm{A}^t\,\dot{\Theta} + \delta\mathrm{A}^t\,\dot{\Theta} + \mathrm{A}^t\,\delta\dot{\Theta} \tag{43}$$

or

$$\mathrm{P}\,\delta\dot{\mathrm{x}} + \delta\mathrm{P}\,\dot{\mathrm{x}} = \delta\mathrm{A}^t\,\dot{\Theta} + \mathrm{A}^t\,\delta\dot{\Theta} \tag{44}$$

where

$$\mathrm{P} = \mathrm{A}^t\mathrm{A} = \begin{bmatrix} N & 0 & 0 \\ 0 & N & 0 \\ 0 & 0 & Nr^2 \end{bmatrix} \tag{45}$$

$$\delta\mathrm{P} = \delta\mathrm{A}^t\mathrm{A} + \mathrm{A}^t\delta\mathrm{A} = \begin{bmatrix} 0 & 0 & -\sum_{i=1}^{N}\delta p_{iy} \\ 0 & 0 & \sum_{i=1}^{N}\delta p_{ix} \\ -\sum_{i=1}^{N}\delta p_{iy} & \sum_{i=1}^{N}\delta p_{ix} & 2\times\sum_{i=1}^{N}(p_{ix}\delta p_{ix} + p_{iy}\delta p_{iy}) \end{bmatrix} \tag{46}$$

Finally, from (44), the effect on the least squares velocity estimate owing to imprecise installation of optical flow sensors can be approximated by

$$\delta\dot{\mathrm{x}} \simeq \delta\dot{\mathrm{x}}_1 + \delta\dot{\mathrm{x}}_2 + \delta\dot{\mathrm{x}}_3 \tag{47}$$

where

$$\delta\dot{\mathrm{x}}_1 = -\mathrm{P}^{-1}\,\delta\mathrm{P}\,\dot{\mathrm{x}} \tag{48}$$

$$\delta\dot{\mathrm{x}}_2 = \mathrm{P}^{-1}\,\delta\mathrm{A}^t\,\dot{\Theta} \tag{49}$$

$$\delta\dot{\mathrm{x}}_3 = \mathrm{P}^{-1}\,\mathrm{A}^t\,\delta\dot{\Theta} \tag{50}$$

8. CONCLUSION

In this paper, we proposed the robust localization of an omnidirectional mobile robot using a regular polygonal array of optical mice that are installed at the bottom of a mobile robot. First, the basic principle of the proposed method was explained. Second, the velocity kinematics from a mobile robot to an array of optical mice was derived as an overdetermined system. Third, for a given set of optical mouse readings, the least squares velocity estimate of a mobile robot was obtained as the simple average. Fourth, the robustness analysis of the proposed least squares velocity estimation against measurement noise, partial malfunction, and imprecise installation was made.

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