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Robust Controller Design to Uncertain Nonlinear Tailless Aircraft

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Abstract: The vertical tail of an aircraft produces a large radar signature and the requirement for stealth in new military aircraft necessitates the design of tailless aircraft. Without the vertical tail results in lateral-directional response characteristics which are a great deal different from those of conventional aircraft. In this paper, a robust controller designed for the tailless aircraft is presented. The controller's basic structure consists of an inner-loop DI controller wrapped around an outer-loop robust μ-synthesis controller. The inner-loop controller equalizes the tailless aircraft dynamics across the flight envelope, and the outer-loop controller addresses the issues of stability, performance, and robustness to tailless aircraft uncertainties.

1. INTRODUCTION

Recently there has been considerable interest in the development of aircraft which have reduced vertical tail size or even no vertical tail compared with conventional aircraft. Eliminating the vertical tails can reduce airframe weight and radar cross section, improve aircraft lift to drag ratio, and improve aircraft agility. On the other hand, tailless configuration presents a challenge from a stability and control perspective. Absence of a vertical tail reduces directional stability and directional control power. Due to its configuration, the tailless aircraft dynamics are tightly coupled and highly nonlinear. Any yaw motion tends to be accompanied by a pitch motion. This coupling behavior requires the controller to consider the aircraft dynamics in all three axes at the same time.

Since tailless aircraft are inherently nonlinear system, applying linear design tools means that one must design several linear controllers, and then gain-schedule them over the operating regime of the aircraft. There are alternative techniques that can deal directly with the known nonlinearities of the tailless aircraft dynamics, using these nonlinearities in the controller to improve the system performance. These techniques are generally based on the feedback linearization approach, for example DI (Dynamic Inversion). The nonlinear nature of the tailless aircraft can be addressed with the DI approach. A fundamental assumption in DI is that plant dynamics are perfectly modeled and can be canceled exactly. In practice this assumption is not realistic, so it require some form of robust controller, for example μ-synthesis, to suppress undesired behavior due to plant uncertainties. This is the reason why the controller consisted of two parts, an inner loop designed using DI to linearize the nonlinearities of tailless aircraft dynamics, and an outer loop designed using μ-synthesis to meet performance requirements and to address parameter variations.

A manual flight control system based on linear ICE (Innovative Control Effectors) tailless model was described in Ngo (1996). The controller consisted of two parts, an inner loop controller was designed using DI to equalize the aircraft dynamics across the entire flight envelope, an outer loop was designed using μ-synthesis to meet performance requirements. Robustness analysis showed very good robustness to control derivative and stability derivative uncertainties. In Ito (2002), a controller which also consists two parts was designed to re-entry vehicle. Its outer loop was designed using LQG/LTR to meet performance, and the simulation showed good robustness to parametric uncertainties.

The paper is organized as follows. In section 2 we build a tailless aircraft model, and discuss the uncertainties which relate to the tailless aircraft. In section 3 we give the basic theory of the dynamic inversion and have a review about μ-synthesis. In section 4 we design the inner controller and outer controller for the tailless aircraft respectively. In section 5 we discuss the stability and performance of the tailless aircraft with the controller we have designed. Section 6 presents some brief conclusion.

2. THE TAILLESS AIRCRAFT MODEL

Security and proprietary consideration have resulted in a lack of mathematical models for tailless aircraft which are available to research. A tailless model mentioned in N.G.M (2004) is modified here for designing controller. For our purpose a brief description of the longitudinal dynamics are only given.

2.1 Assumptions

For the modeling of the aircraft dynamics, several assumptions are used:

- The aircraft is consider as a rigid body with six degrees of freedom; three translational and three rotational degrees.
- The mass of the aircraft is constant.
- The Earth is flat and therefore the global coordinate system is fixed to it. Consequently, the Earth is

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considered as an inertial system and Newton's second law can be applied.

- OX_b and OZ_b are planes of symmetry for the aircraft,therefore:$I_{xy} = \int yzdm$ and $I_{xy} = \int xydm$ are equal to zero and $I_{xz} = I_{zx}$ in the inertial tensor matrix.
- The acceleration due to gravity is considered as constant with a value equal to $9.81m/s^2$.

2.2 Axes System

For modeling the tailless aircraft, three primary axes systems are employed, which are described below:

(1) The first axes system is the Earth inertial axes system($\overrightarrow{e}^e$). Because $\overrightarrow{e}^e_1$ is pointed towards the North, $\overrightarrow{e}^e_2$ towards the East and $\overrightarrow{e}^e_3$ Downward, this reference frame is also known as the NED axes system. The inertial frame is required for the application of Newton's laws.
(2) The second axes system is the aircraft-carried inertial axes system ($\overrightarrow{e}^c$). This axis system is obtained if the Earth inertial axes frame is translated to the center of gravity of the aircraft with a vector.
(3) The body axes system ($\overrightarrow{e}^b$) is also an aircraft-carried axes system, with $\overrightarrow{e}^b_1$ pointed towards the nose of the aircraft, $\overrightarrow{e}^b_2$ towards the right wing and $\overrightarrow{e}^b_3$ to the bottom of the aircraft. The axis system is obtained through successive rotations of the aircraft-carried inertial frame with Tait-Bryant angles ψ, θ and ϕ. The velocity vectors along these axes are u, v and w and the angular velocity vectors are respectively: roll rate p, pitch rate q and yaw rate r.

2.3 Uncertainty

Uncertainties of the tailless aircraft used in this paper are shown below:

(1) The unmodeled dynamic uncertainty. Due to deliberate neglect or lack of knowledge of the tailless aircraft, the model may have missed dynamics effects. It happens in high frequency range mostly. Here, we use input multiplicative uncertainty to represent this type uncertainty.

(2) Aerodynamic and control coefficients uncertainty. This type uncertainty is called parametric uncertainty in the robust control. More detail see Table 1.

(3) Actuator dynamic uncertainty and sensor noise. Actuators and sensors uncertainties are usually modeled as unstructured uncertainties with high pass filters whose magnitudes reflect the amount of uncertainties.

2.4 Longitudinal dynamics

The longitudinal dynamics are described using the states $x = [\theta \quad u \quad w \quad q]$, which are the pitch angle, velocity components in the body X and Z direction, pitch rate respectively.

The rigid body system dynamics are given as:

$$\dot{\theta} = q \tag{1}$$

$$\dot{u} = \frac{\bar{q}S}{m}(A_{uF} + \frac{2T_i \cos p_{ni}}{\bar{q}S}) - g\sin\theta - qw \tag{2}$$

$$\dot{w} = \frac{\bar{q}S}{m}(A_{wF} + \frac{2T_i \sin p_{ni}}{\bar{q}S}) + g\cos\theta + qu \tag{3}$$

$$\dot{q} = \frac{1}{I_{yy}}(\bar{q}S\bar{c}C_m + 2T_i \sin p_{ni} X_{cg}) \tag{4}$$

$$\alpha = \arctan(\frac{w}{u}) \tag{5}$$

$$A_{uF} = C_L \sin\alpha - C_D \cos\alpha \tag{6}$$

$$A_{wF} = -C_L \cos\alpha - C_D \sin\alpha \tag{7}$$

$\bar{q} = \frac{1}{2}\rho V^2$ is the dynamic pressure (ρ is the air density), S is reference area, m is the mass, T_i is the thrust force here which assumed is constant, this tailless aircraft has two engines, and p_{ni} is pitch angle of nozzle $i(i = 1, 2)$, C_L, C_D, C_m are dimensionless lift coefficient, drag coefficient, and pitch moment coefficient respectively, detail description is shown below, I_{yy} is the moment of inertia about the Y axis, $\bar{c}$ is mean aerodynamic chord, X_{cg} is the distance from the nozzle exit to the C.G(Center of Gravity). The control inputs are nozzle angle p_{ni}.

$$C_D = C_{D0} + C_{D1}\alpha + C_{D2}(\frac{u - u_{eq}}{u_n}) \tag{8}$$

$$C_L = C_{L0} + C_{L1}\alpha + C_{L2}q(\frac{b}{2V_t}) + C_{L3}(\frac{u - u_{eq}}{u_n}) \tag{9}$$

$$C_m = C_{m0} + C_{m1}(\frac{u - u_{eq}}{u_n}) + C_{m2}\alpha + C_{m3}q(\frac{\bar{c}}{2V_t}) \tag{10}$$

The values for the aerodynamic parameters C_i, u_n, u_eq can be found in ref N.G.M (2004)

3. DYNAMIC INVERSION AND μ-SYNTHESIS THEORY

In general, aircraft dynamics are expressed by

$$\dot{x} = F(x, u) \tag{11}$$

$$y = H(x) \tag{12}$$

where x is the state vector, u is the control vector, and y is the output vector. For conventional uses, the function F is affine in u. Above equation can be rewritten as

$$\dot{x} = f(x) + g(x)u \tag{13}$$

where f is a nonlinear state dynamic function and g is a nonlinear control distribution function. If we assume $g(x)$ is invertible for all values of x, the control law is obtained by subtracting $f(x)$ from both sides of equation(13) before multiplying both sides by $g^{-1}(x)$.

$$u = g^{-1}(x)[\dot{x} - f(x)] \tag{14}$$

The next step is to command the aircraft to specified states. Instead of specifying the desired states directly, we will specify the rate of the desired states, $\dot{x}$. By swapping $\dot{x}$ in the previous equation to $\dot{x_{des}}$, we get the final form of a DI control law.

$$u = g^{-1}(x)[\dot{x}_{des} - f(x)] \tag{15}$$

Although the basic DI process is simple, a few points need to be emphasized. First, although we assume $g(x)$ is invertible for all values of x, this assumption is not always true. For example, $g(x)$ is not generally invertible if there are more states than controls. Furthermore, even if $g(x)$ is invertible (i.e., $g(x)$ is small), the control input, u, become large; and this growth is a concern because of actuator saturation.

DI is also essentially a special case of model-following. While it is similar to other model-following controllers, a DI controller requires exact knowledge of model dynamics to achieve good performance. Robustness issues therefore play a significant role during the design process. To overcome these difficulties, a DI controller is normally used as an inner-loop controller in combination with an out-loop controller designed using μ-synthesis.

The general controller synthesis and analysis problem description as proposed by Stein (1991) is shown in figure 1.

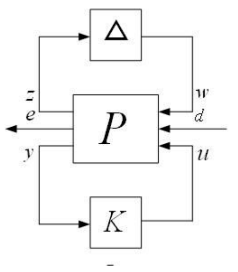

Fig. 1. General Analysis and Synthesis framework

The generalized system P has three input/output pairs: $y(t)$ and $u(t)$ (measurements and control inputs from the controller K), $e(t)$ and $d(t)$ (performance signals and external inputs respectively), $z(t)$ and $w(t)$ through which unit-norm perturbations in Δ are fed back into system.

Given the controller K, which might be obtained from any synthesis method, the generalized closed loop system for analysis as depicted in fig. 2 is given by $M(s) = F_l(P(s), K(s))$.

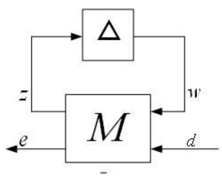

Fig. 2 Analysis part General interconnection structure

Robust performance of the system $M(s)$ is characterized by the transfer function from d to e with the perturbation Δ acting as a Linear Fractional Transformation(LFT):

$$\frac{e}{d} = F_u(M, \Delta) = [M_{22} + M_{21}\Delta[I - M_{11}\Delta]^{-1}M_{12}] \quad (16)$$

From eq. (16) we immediately see that robust stability is imposed by M_{11} in the fractional part. It may become singular for some Δ.

The function μ is used to assess well-definedness of (16) along the frequency axis:

$$\mu_{\triangle}(M) := \frac{1}{min\{\bar{\sigma}(\Delta) : \triangle \in \Delta, det(I - M\Delta) = 0\}} \quad (17)$$

In words $\mu_{\triangle}$ is the reciprocal of the smallest $\triangle$ (where we use $\bar{\sigma}$ as the norm) we can find in the set Δ that makes the matrix $I - M\triangle$ singular. If no such $\triangle$ exists, $\mu_{\triangle}$ is taken to be zero.

If M is stable, the following theorem apply Stein (1991):

(1) **Nominal performance is satisfied if and only if**
$$\|M_{22}(j\omega)\|_{\infty} < 1 \quad (18)$$
(2) **Robust stability is satisfied if and only is**
$$\mu_{\triangle}(M_{11}(j\omega)) < 1 \quad \forall\omega \quad (19)$$
(3) **Robust performance is satisfied if and only if**
$$\mu_{\triangle}[M(j\omega)] < 1 \quad \forall\omega \quad (20)$$

A tutorial on μ-synthesis and more thorough treatment of the theory can be found in Jean (1997).

4. CONTROLLER DESIGN

Designing controller consists of two tasks. The first task is to equalize the tailless aircraft dynamics throughout the flight envelope via the dynamic inversion. The second task involves finding a single controller using μ-synthesis that will satisfied the required performance and robustness.

4.1 Inner Loop Design

In the inner loop, we attempt to modify the tailless dynamics at different condition to the desired dynamics using the dynamic inversion method. The output variable we are interested in is the pitch angle θ, for which holds: $\dot{\theta} = q$. For the pitch rate q, the relationship with the nozzle pitch angle input p_{ni} is the moment equation (4). Inverting this equation, and introducing the new control variable $\dot{q}_c = \ddot{\theta}_c$ and $Sp_{ni} = \sin p_{ni}$ gives:

$$Sp_{ni} = \frac{I_{yy}\ddot{\theta}_c - \bar{q}S\bar{c}C_m}{2T_iX_{cg}} \quad (21)$$

Note that C_m and T_i are nonlinear terms. In the ideal case we exactly know the model components, so that substitution of (4) into (21) gives:

$$\dot{q} = \ddot{\theta}_c, \quad or : \theta = \frac{1}{s^2}\ddot{\theta}_c \quad (22)$$

The relation between our new input, and the pitch angle has become a double integrator.

We are able to impose desired command response of θ by placing the two integrator poles at desired locations in the left half plane. In a classical sense, we could do that with the simple control law:

$$\ddot{\theta}_c = K_p(\theta_c - \theta) - K_d q \quad (23)$$

We choose $K_p = \omega^2$, and $K_d = 2\zeta\omega$, so that with (22):

$$\theta = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2}\theta_c \quad (24)$$

Setting $\omega = 1rad/s$ and $\zeta = 1$ ensures the rise time $t_r < 5s$ and settling time $t_s < 20s$. The closed loop system is depicted in fig.3.

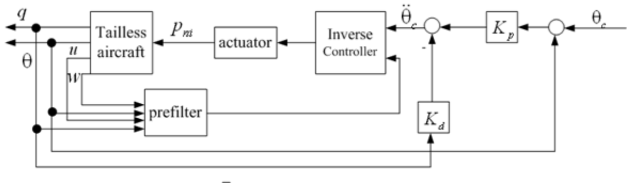

Fig. 3. Dynamic Inversion Controller Structure

To address the robustness requirement we incorporate the effect of uncertainties in (4):

$$\dot{q} = \frac{1}{I_{yy}}(\bar{q}S\bar{c}(C_m + \triangle_m) + 2T_i \sin p_{ni}(X_{cg} + \triangle_{cg})) \quad (25)$$

Substitution of the inverse control law gives:

$$\dot{q} = \ddot{\theta}_c + \frac{1}{I_{yy}}(\bar{q}S\bar{c}\triangle_m - \triangle_{cg}\bar{q}S\bar{c}C_m + \triangle_{cg}\ddot{\theta}_c) \quad (26)$$

In the nominal design condition $\triangle_m$ and $\triangle_{cg}$ are zero. With the inverse controller we obtain eq (22). As we deviate from this condition, the uncertain terms will make the closed loop system nonlinear. The linear control law is not able to achieve the required closed dynamics given in (24). This is illustrated by the nonlinear simulations shown in fig. 4. The performance degradation is substantial when the X_{cg} has 10% and 20% uncertainty.

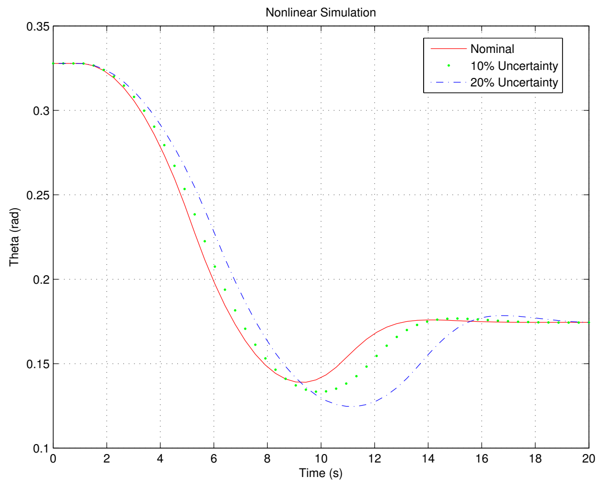

Fig. 4. Response to 10 degrees theta command input

4.2 Outer Loop Design

To overcome the deficiencies encountered in the previous section we address the analysis as well as the synthesis problem within the μ-framework. A synthesis model consisting of the plant model, ideal model, performance weighting, input weighting, and axis axis transformations is formed to design a μ-synthesis controller. The plant model used to formulate the outer loop design problem is the closed inner loop of the central linear model of the tailless airplane.

The outer controller is designed for a straight and level trim condition at an altitude of 5000 ft and nominal velocity of $166m/sec$. It is assumed that only the nozzle angle is used for control. The interconnection structure for synthesis purpose is depicted in fig. 5.

The reference pitch rate response to the nozzle angle input is described by a second order system which is the same transfer function as in (24). The performance weight W_q is constant and set to 100. The input is prefiltered by

$$W_{pf} = \frac{\frac{s}{100} + 1}{\frac{s}{8} + 1} \quad (27)$$

to reduce high frequency excitations. The nozzle has a maximum angle of 20°. Thus the weight W_δ on the nozzle command is chosen as $\frac{1}{20}$. It is assumed that the pitch and pitch rate measurements include a white, zero mean measurement noise. To normalize this input signal, W_n is set to 0.008. The $\triangle$ is the unknown parameters which are normalized with respect to their bounds. The diagonal of the $\triangle$ matrix represent the X_{CG}, C_L, C_D and C_m parameters uncertainties which are shown in table 1.

Parameter	Uncertainty Bounds
X_{CG}	20%
C_{D1}	40%
C_{D2}	50%
C_{L1}	15%
C_{L2}	10%
C_{L3}	15%
C_{m1}	30%
C_{m2}	25%
C_{m3}	20%

Table. 1. Parameter Uncertainty Bounds

Unmodeled dynamics uncertainty at the input is another important type of uncertainty, when this is applied to tailless aircraft, the following weighting function is used

$$W_{in} = \frac{s+1}{s+100} \quad (28)$$

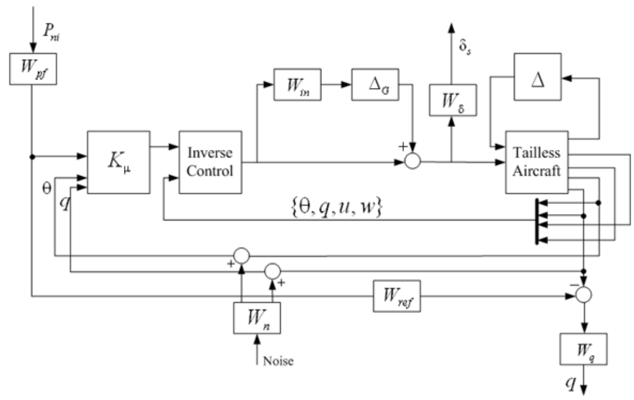

Fig. 5. Interconnection structure for μ-synthesis

4.3 Controller Reduction

A μ controller was designed using Matlab Robust Control ToolBox. The design process converged after 5 D-K iterations and achieved a μ of 1.2503 with a 18-th order controller. For implementation, the order of this controller was reduced using the optimal Hankel norm approximation technique to order 6. The Hankel norm of controller is depicted in fig. 6, and the fig. 7 shows a comparison plot of the original μ controller and reduced one.

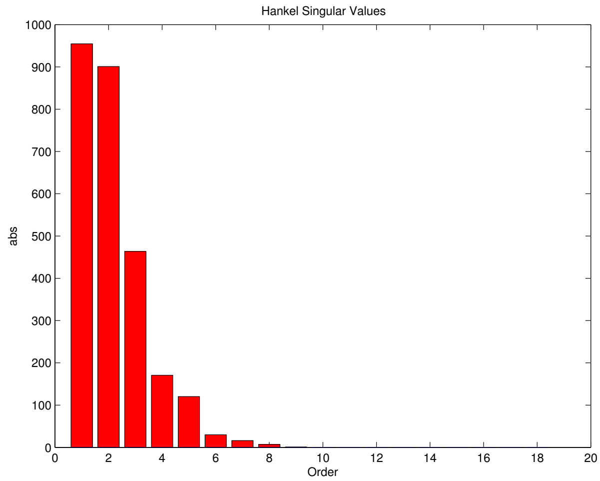

Fig. 6. Hankel singular values of μ controller

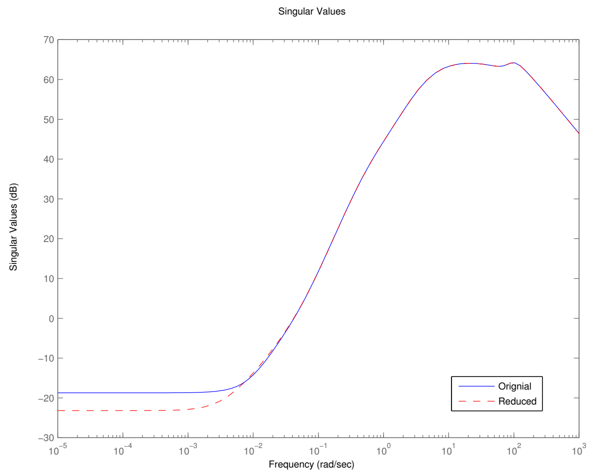

Fig. 7. Comparison of the original controller and reduced controller

5. STABILITY AND PERFORMANCE ANALYSIS

The two loops controller, inner loop dynamic inversion and outer loop μ-synthesis, are inserted into the nonlinear tailless aircraft simulation and performed well as shown in fig. 8. The desired θ command is a doublet pulse which has the value 1 from 1 to 4 second and value -1 from 4 to 7 second. The nonlinear response is nearly identical for the nominal and worst-case nonlinear closed-loop system underlying the controllers.

The robust controller has sightly worse performance for the nominal tailless aircraft when compared to only dynamic inversion (upper fig. 9). It maintains this performance consistently for all perturbed models(worst case gain near 1.2).

The final value of μ obtained is shown as a function of frequency in Fig. 10. The maximum value for μ for the complex analysis is 1.01. One notices that there is minimal performance degradation due to modeling the uncertainties (see table 1) as complex in the μ synthesis procedure. Fig. 11 indicates the value of μ associated with system stability, this value of μ must be less than one in order to insure that the system is stable for all uncertainties included in the design model. As can be seen in Fig. 11 this stability criterion is satisfied.

Fig. 12 is the nozzle angle output with robust controller underlying uncertainties. Through Monte Carlo simulation, it is clear that the nozzle angle output is not saturated.

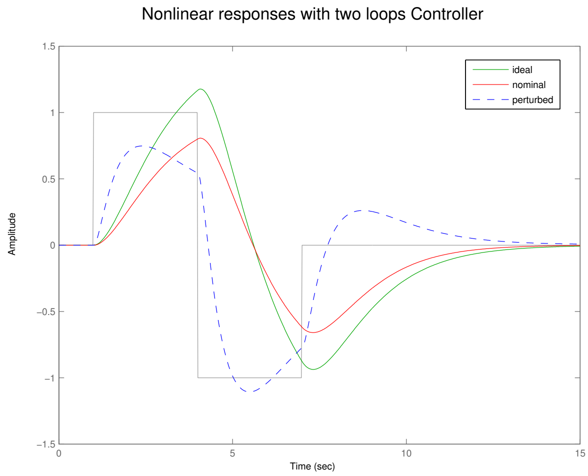

Fig. 8. Nonlinear simulation with two loops controller

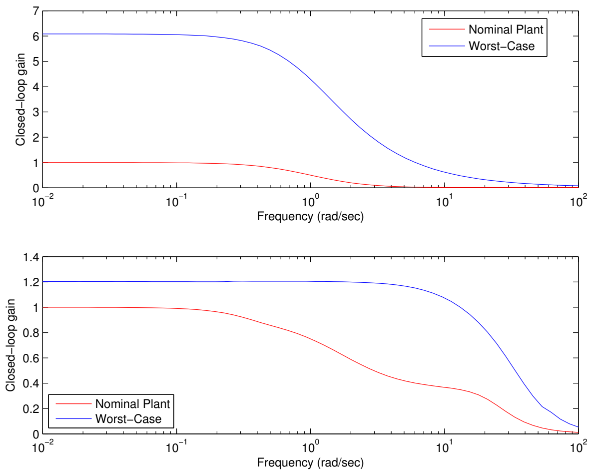

Fig. 9. Robustness compare to dynamic inversion

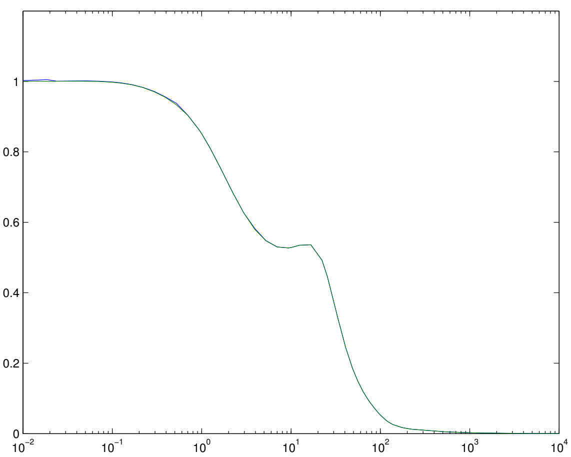

Fig. 10. Complex μ analysis for robust performance of the robust controller

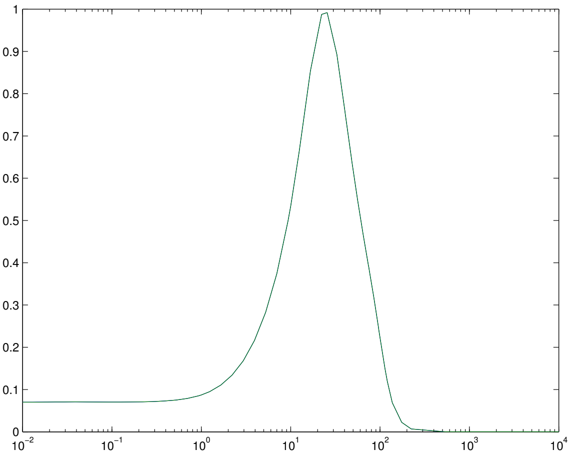

Fig. 11. Complex μ analysis for stability of the robust controller

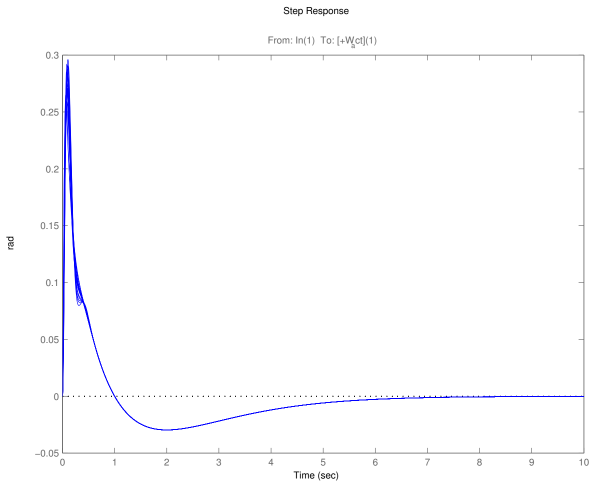

Fig. 12. Actuator output

6. CONCLUSION

The tailless aircraft are regarded as a developing way of the new type aircrafts because of their special advantages. A control law for a tailless aircraft using dynamic inversion and μ-synthesis design technique was presented. The inner loop controller was designed using dynamic inversion to equalize to aircraft dynamic in the flight envelope. And using the dynamic inversion can avoid the complex gain-scheduling. The outer loop was found using μ-synthesis. The high order controller was reduced using Hankel norm, and the reduced controller achieve almost performance and stability like the original controller. The design objective of the outer loop controller to satisfy the stability, and robustness was achieved underlying uncertainties, and the actuator did not have saturation with the controller.

ACKNOWLEDGEMENTS

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