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Motion Planning and Trajectory Tracking of Underactuated Three-link Robots

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Abstract: A new method for motion planning and trajectory tracking of underactuated three-link planar robots with a passive rotational third joint is proposed. One fundamental feather is to use the switching of partly stable controllers (PACs) in order to fulfill the control objective. The dynamic model of this kind of underactuated robot system is built based on Lagrange method. Different objective functions are given for motion planning and trajectory tracking. The genetic algorithm (GA) is utilized to get the optimum control actions for a given time-frame with the available set of elemental controllers. Penalty method is utilized when there are constraints and then the constrained optimizations change to be unconstrained ones. Because the proposed method does not make any hypothesis about the degree of freedom, it can be used without modification for arms with a large number of degree of freedom. At last numerical simulations are carried out to illuminate the validity of the proposed method.

1. INTRODUCTION

Underactuated robots are those which have fewer control inputs than the degrees of freedom of the system. As far as underactuated manipulators are concerned, they have one or more joints without actuators, namely these joints are passive or free. Underactuated robots are important from the viewpoint of energy saving, lightweight, and compactness due to fewer actuators.

An extensive amount of research on the kinematics and dynamics of robots has been carried out for regular (full actuated) manipulators. There is an independent generalized force for each degree of freedom that can be applied by a control actuator. But for underactuated robots, the generalized coordinates are not independent and the control objective can be realized only by the dynamic coupling between the active and passive joints (Bergerman, 1995). In most cases, the underactuated manipulators with 2 or more degree of freedoms are a second-order nonholonomic system (Nakamura Y, 1997). That is to say that the systems have accelerations-dependent constraints which are not integrable to obtain velocity or configuration dependent constraints. However control of such kind of system is a complicated task because of the intrinsic characteristics such as complex nonlinear dynamic, nonholonomic behaviours and lack of linearizability exhibited in this kind of nonlinear systems (Bergerman, 1998).

Motion planning and trajectory tracking are the two major research fields of underactuated manipulators. Many valuable conclusions have been gotten in recent years. An oscillatory stabilizing feedback is designed by Nakamura (Nakamura Y, 1997) for rest-to-rest motion task, based on a Poincaré map analysis. Martínez (Hiroshi, 1997) derived dynamic model of underactuated brachiation robot and analysed the nonlinear dynamics and control problem of system. De Luca (1997) showed that the system fails to satisfy the weakest existing sufficient conditions for small time local controllability (STLC), which implies that the design of feasible motion trajectories is an open problem. Smooth feedback is not possible because the drift term tends to zero with the generalized velocities. Arai (1998) obtained position control of planar underactuated 3R manipulator using feedback control method. Wang et al. (2004) presented a stable hierarchical sliding-mode control method of a class of second-order underactuated systems.Arai and Tachi (1991) proposed a method of controlling theoretically and experimentally the position of a two-link underactuated manipulator with a brake at the passive joint by using the coupling characteristics of manipulator dynamics. The iterative state steering technique (De Luca, 1997), consisting of the repeated application of open-loop commands, guarantees the stabilization at a desired configuration. A numerical motion planner and a trajectory controller based on time-scaling have been proposed for the same system (Arai, 1998). The control solutions have been obtained above are limited to case-by-case study only and very complicated when they are used.

Because of the intrinsic characteristics of the second-order nonholonomic system, there are few issues about collision-free motion planning and trajectory tracking of underactuated manipulators. Kevin (2000) proposed a method for collision-free trajectory planning of a 3-dof robot with a passive joint , the problem of planning feasible trajectories in the robot's six-dimensional state space are decoupled into the computationally simpler problems of planning path in the three-dimensional configuration space and time scaling the paths according to the manipulator dynamics. A kind of operational coordinates are defined as a kind of operational coordinate system based on the desired path, then the equation of the motion of the manipulator is described in terms of the path coordinates (Arai, 1991), at last the trajectory tracking to the desired path is realized by the dynamic coupling.

Based on above work, motion planning and trajectory tracking of underactuated robots are mainly discussed in this study. The algorithms are also effective when there are obstacles in configuration space. The rest of the paper are organized as follows: In section II the dynamic model of underactuated 3R robots is built, motion planning and trajectory tracking of underactuated robots are discussed in section III and numerical simulations are carried out in section IV Finally some discussions and conclusions are made in section V.

2. DYNAMIC EQUATION

Consider a planar 3R underactuated manipulator as shown in Fig.1. Assume that the manipulator moves in the horizontal plane and the gravity force does not work. All joints are rotational ones and the third joint is free.

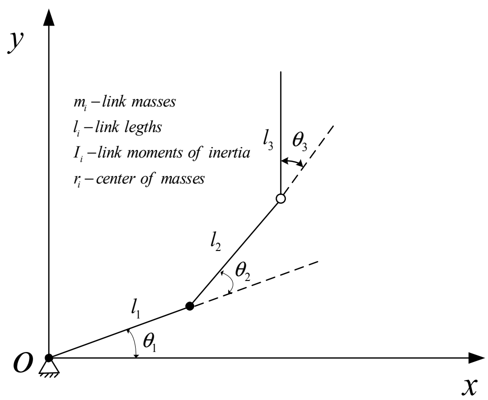

Fig. 1 Planar 3R underactuated manipulator

Let $q_i[i=1,2,3]$ be the each joint angles and $q=[\theta_1,\theta_2,\theta_3]^T$ be the generalized coordinates. Using Lagrange method, the dynamic model of planar 3R underactuated can be obtained as follows:

$$M(\theta)\ddot{\theta}+h(\theta,\dot{\theta})=\tau \tag{1}$$

Where $\dot{\theta}$ and $\ddot{\theta}$ are the joint angular velocities and joint angular accelerations respectively. $M(\theta)\in R^{3\times3}$ is the inertia matrix which is symmetric and positive definite by a suitable choice of manipulator parameters. $h(\theta,\dot{\theta})\in R^{3\times1}$ denotes the element of Coriolis, Centrifugal and viscous friction vector. $\tau\in R^{3\times1}$ indicates the generalized forces matrix. The dynamic equation (1) can be rewritten as:

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix}+\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}=\begin{bmatrix} \tau_1 \\ \tau_2 \\ 0 \end{bmatrix} \tag{2}$$

Each parameter in (2) is given by:

$$\begin{aligned}
m_{11} &= a_1+a_2+a_3+2b_1\cos(q_2)+2b_2\cos(q_2) \\
&\quad +2b_3\cos(q_3)+2b_4\cos(q_2+q_3) \\
m_{12} &= a_2+a_3+(b_1+b_2)\cos(q_2)+2b_3\cos(q_3) \\
&\quad +b_4\cos(q_2+q_3);\, m_{21}=m_{12} \\
m_{22} &= a_2+a_3+b_3\cos(q_3) \\
m_{13} &= a_3+b_3\cos(q_3)+b_4\cos(q_2+q_3) \\
m_{31} &= m_{13};\, m_{22}=a_2+a_3+2b_3\cos(q_3) \\
m_{23} &= a_3+b_3\cos(q_3);\, m_{32}=m_{23};\, m_{33}=a_3 \\
h_1 &= -b_4(\dot{q}_2+\dot{q}_3)(2\dot{q}_1+\dot{q}_2+\dot{q}_3)\sin(q_2+q_3)- \\
&\quad (2\dot{q}_1+\dot{q}_2)\dot{q}_2(b_1+b_2)\sin(q2)-b_3(2\dot{q}_1+2\dot{q}_2 \\
&\quad +\dot{q}_3)a_3\sin(q_3) \\
h_2 &= \dot{q}_1^{\,2}\{(b_1+b_2)\sin(q_2)+b_4\sin(q_2+q_3)\}- \\
&\quad 2b_3\dot{q}_1\dot{q}_3\sin(q_3)-2b_3\dot{q}_2\dot{q}_3\sin(q_3)-b_3\dot{q}_3^{\,2}\sin(q_3) \\
h_3 &= \dot{q}_1^{\,2}(b_3\sin(q_3)+b_4\sin(q_2+q_3)) \\
&\quad +2\dot{q}_1\dot{q}_2b_3\sin(q_3)+b_3\dot{q}_2^{\,2}\sin(q_3)
\end{aligned}$$

Where $a_i[i=1,2,3], b_j[j=1,2,3,4]$ are the constants and with the following expressions:

$$\begin{aligned}
a_1 &= m_1r_1^2+m_2l_1^2+m_3l_1^2+I_1 \\
a_2 &= m_2r_2^2+m_3l_2^2+I_2;\, a_3=m_3r_3^2+I_3; \\
b_1 &= m_2l_1r_2;\, b_2=m_3l_1l_2;\, b_3=m_3l_2r_3;\; b_4=m_3l_1r_3
\end{aligned}$$

The zero external torque in the last row of (2) represents the dynamic constraint on the system. The passive joint variable q_3 appear in the inertia matrix and the gravitational terms are absent in (2). As a result of the necessary and sufficient condition (Oriolo, 1991) for the partial integrability is not satisfied. Hence the system is a second-order nonholonomic system.

3. MOTION PLANNING & TRAJECTORY TRACKING

The objective of motion planning is to find a feasible motion from the initial state to the desire state. Here partly stable controllers are adopted.

3.1 Partly Stable Controller

The equation (1) can be rearranged as follows in order to get the desired second-order derivatives:

$$\ddot{q}=M(q)^{-1}\{\tau-h(q,\dot{q})\} \tag{3}$$

Expanding (3) gives expression of each control variable:

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix}=M(q)^{-1}\left\{\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix}-\begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}\right\} \tag{4}$$

From (4) we can devise partly stable controllers for underactuated system. For the n degree of freedom underactuated mechanical system with m actuators, C_n^m partly stable controllers (PSCs) can be designed totally. For

$3RR\bar{R}$ robot with one passive joint, 3 partly stable controllers can be obtained:
Control law 1:

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \end{bmatrix} = \begin{bmatrix} \ddot{q}_{1d} + k_{v1}(\dot{q}_{1d} - \dot{q}_1) + k_{p1}(q_{1d} - q_1) \\ \ddot{q}_{2d} + k_{v2}(\dot{q}_{2d} - \dot{q}_2) + k_{p2}(q_{2d} - q_2) \\ -m_{33}^{-1}(h_3 + m_{31}\ddot{q}_1 + m_{32}\ddot{q}_2) \end{bmatrix} \tag{5}$$

Control law 2:

$$\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_3 \\ \ddot{q}_2 \end{bmatrix} = \begin{bmatrix} \ddot{q}_{1d} + k_{v1}(\dot{q}_{1d} - \dot{q}_1) + k_{p1}(q_{1d} - q_1) \\ \ddot{q}_{3d} + k_{v3}(\dot{q}_{3d} - \dot{q}_3) + k_{p3}(q_{3d} - q_3) \\ -m_{32}^{-1}(h_3 + m_{31}\ddot{q}_1 + m_{33}\ddot{q}_3) \end{bmatrix} \tag{6}$$

Control law 3:

$$\begin{bmatrix} \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{q}_1 \end{bmatrix} = \begin{bmatrix} \ddot{q}_{2d} + k_{v2}(\dot{q}_{2d} - \dot{q}_2) + k_{p2}(q_{2d} - q_2) \\ \ddot{q}_{3d} + k_{v3}(\dot{q}_{3d} - \dot{q}_3) + k_{p3}(q_{3d} - q_3) \\ -m_{31}^{-1}(h_3 + m_{32}\ddot{q}_2 + m_{33}\ddot{q}_3) \end{bmatrix} \tag{7}$$

Where q_{id} , $\dot{q}_{id}$ and $\ddot{q}_{id}(i = 1 \cdots 3)$ are the desired joint angles, joint angular velocities and joint angular accelerations respectively. $k_{vi} > 0$, $k_{pi} > 0 (i = 1 \cdots 3)$ are the derivative and position gain coefficients.

Define the error terms $e_i = q_{id} - q_i$ and $\dot{e}_i = \dot{q}_{id} - \dot{q}_i$. Error functions of controlled variables for (5) can be gotten:
Control law 1:

$$\ddot{e}_1 + k_{v1}\dot{e}_1 + k_{p1}e_1 = 0 \tag{8a}$$

$$\ddot{e}_2 + k_{v2}\dot{e}_2 + k_{p2}e_2 = 0 \tag{8b}$$

Equations (8) are differential equations with constant coefficients. The controlled variable q_1 and q_2 can be converge to zero in finite time if the gain coefficients k_{vi} and k_{pi} are chosen property, but q_3 can not be controlled in (8). This can be compensated in (6) and (7). So the control objective can be achieved by the proper switching of control laws. This process is based on the evolutionary computation for searching the best combination of PSCs from a set of elemental controllers.

3.2 Best switching sequence searching

Genetic Algorithms [11] are adaptive heuristic search algorithm premised on the evolutionary ideas of natural selection and genetic. The basic concept of GA is to simulate processes in natural system necessary for evolution, specifically those that follow the principles of survival of the fittest first laid down by Charles Darwin. As such they represent an intelligent exploitation of a random search within a defined search space to solve a problem.

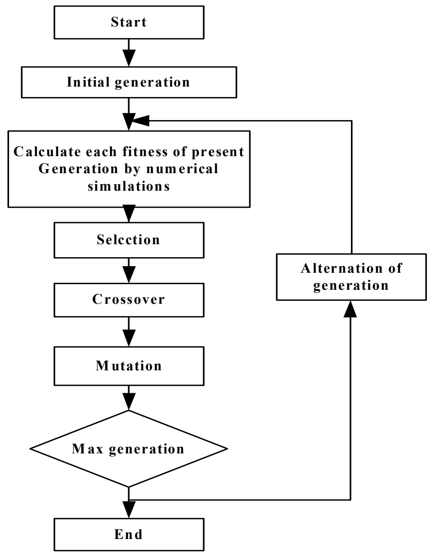

Fig. 2 GA process

Fig.2 describes the GA process when it is used. Whereas Genetic Algorithms include a variety of operators (i.e. selection, crossover and mutation), the basics of Genetic Algorithm can be described as follows: Given some initial population and the terminal generation, proceed as follows:

(1) Sort the population from the best to worst according to given cost function;
(2) Selection rules: select the individuals, called parents that contribute to the population at the next generation;
(3) Crossover rules: combine two parents to form children for the next generation;
(4) Mutation rules; apply random changes to individual parents to form children;
(5) Terminal condition judgment: if satisfied the algorithm terminated, otherwise return to (1).

3.3 Generating an Initial population

To solve the general problem with optimum switching of available PSCs, first define the total time span T . The genes of a chromosome are represented as controller indices. For $3RR\bar{R}$ underactuated robot, control law 1 to control law 3 can be expressed as the binary code 1, 2 and 3. So each individual can be coded as shown in Fig.3. Then the initial population is generated by a random set of M "individual", here M is the population size.

1	2	3	2	1	3	$\cdots$	1	2

$t_1 \quad t_2 \quad t_3 \quad t_4 \cdots\cdots \quad \cdots \quad \cdots \qquad T$

Fig.3 Coding of genes with control indices

3.4 Fitness Function of Motion Planning

The fitness function is the driving force behind the GA. The evaluation function is called from the GA to determine the fitness of each solution string generated during the search process. In this paper, the fitness function is defined as follows when there are no obstacles in workspace:

$$\min \quad E=\sum_{i=1}^{N} e_i^T w e_i \tag{9}$$

Where $e_i=[q_1-q_{1d},\cdots q_n-q_{nd} \quad \dot{q}_1-\dot{q}_{1d},\cdots \dot{q}_n-\dot{q}_{nd}]^T$, N is the final discrete time instant. Matrix $w=diag\left[w_1,w_2,\cdots w_{2n}\right]$ denotes the weights of controlled error $(w_1,\cdots w_{2n}>0)$. When there are obstacles in workspace the fitness function is state:

$$\begin{aligned} \min \quad & E=\sum_{i=1}^{N} e_i^T w e_i \\ s.t \quad & d>0 \end{aligned} \tag{10}$$

Where d is the minimum distance from the obstacle to the manipulators (If the obstacles intersect with one of the manipulators then the minimum distance between the manipulators and the obstacles is negative. i.e. d<0). The meanings of other parameters are the same as that in (9).

3.5 Fitness Function of Trajectory Tracking

Trajectory tracking of second or high order nonholonomic systems is a challenge area and few issues can be found in this field. The purpose of this part is to devise a universal method of geometry path trajectory tracking for systems with high-order nonholonomic constraints in Cartesian space. The control objective is to move the underactuated manipulators from initial state to desired state along a specified path. Assume that tracking error between the factual path and the desired trajectory is *min_d*, and then the objective function can be obtained as follows:

$$\begin{aligned} \min \quad & E=\sum_{i=1}^{N} e_i^T w e_i \\ s.t \quad & \left|\min\_d\right| \le \varepsilon \end{aligned} \tag{11}$$

Where $\varepsilon>0$ denotes the infinitesimal real value and the meanings of other parameters are the same as that in (9).

3.6 Constrained Function optimization

Objective function (10) and (11) are constrained optimizations which have been studied for many years. Evolutionary computation techniques have received considerable attention regarding their robustness in solving complex optimization problems involving no differentiable and discontinuous nonlinearity and high dimension. There are three major methods to deal with the constraint optimization problems: rejecting methods, repairing methods and penalty methods. In this paper the penalty methods are adopted, and then the fitness function (10) turns to be unconstraint optimization:

$$Fitness1=\begin{cases} E & if \;\; d>0 \\ E+\lambda\times d & if \;\; d\le 0 \end{cases} \tag{12}$$

Where $\lambda\in R^1$ is the common penalty parameter. Under this conversation, the overall objective function when there are obstacles is (12), which serves as a fitness function in evolutionary algorithm. Similar treatment can be done on (11) and gives:

$$Fitness2=\begin{cases} E\,, & if \;\; \left|\min\_d\right|\le\varepsilon \\ E+\lambda_1\times\left|\min\_d\right|, & if \;\; \left|\min\_d\right|>\varepsilon \end{cases} \tag{13}$$

Where $\lambda_1\in R^1$ denotes the penalty parameter. Now the constraint optimizations turn to be unconstraint ones and the best solutions can be obtained by evolutionary algorithms.

4. NUMERICAL SIMULATION

Numerical simulations are carried out here to test the performance of the proposed method. The whole simulation process contains two parts which adopt different fitness functions.

4.1 Motion Planning

To test the legality of (9) and (12), the same initial state and desire state are adopted. The dynamic parameters of the underactuated robot in Fig.1 are listed in Tab.1. The size of a population is 100. The maximum number of generations is 300. The obstacle is a circle which the center is located at (0.5, 0.5) with a radius of 0.06m. The gains are selected as $[k_{v1}\ k_{v2}\ k_{v3}\ k_{p1}\ k_{p2}\ k_{p3}]=[2\ 4\ 2\ 4\ 8\ 14]$, the weight matrix $w=diag([10^4\ \ 10^4\ \ 10^4\ \ 100\ \ 100\ \ 100])$ and the penalty operator $\lambda=4000$.

Tab.1 Setting parameters of simulations

Conditions	Setting values
Simulation time	10[s]
Sample interval	0.01[s]
Mass of each link	m_1=m_2=m_3=0.3[kg]
Length of each link	l_1=l_2= l_3=0.3[m]
Distance between center	l_{c1}=0.15[m]
of Gravity and each joint	l_{c2}= l_{c3}=0.15[m]
Initial state	[0 0 0 0 0 0]
Desired state	[1 1 1 0 0 0]

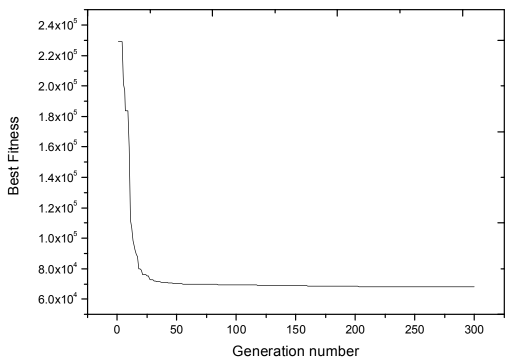

Fig.4 Evolutionary history of GA

Fig.4 is the evolutionary history of GA when there are obstacles in workspace. It can be shown from Fig.4 that the fitness values are met the desired values within an acceptable level of generations.

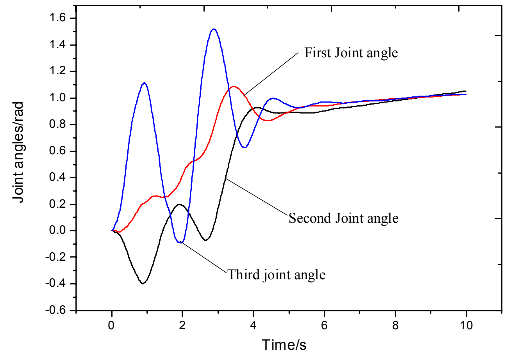

Fig.5 Joint angles

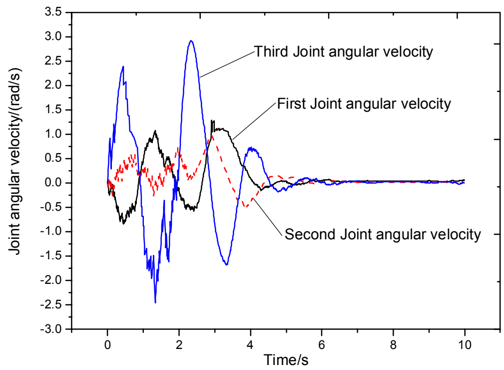

Fig.6 Joint angular velocity

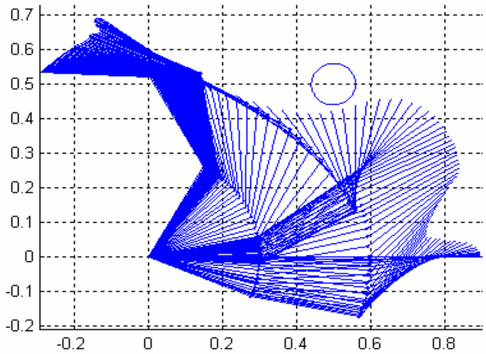

Fig.7 Rest-to-rest planning: with obstacles in workspace

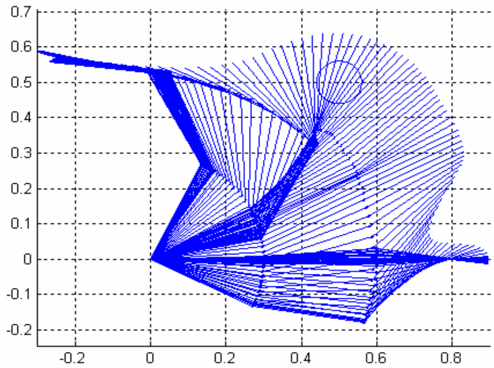

Fig.8 Rest-to-rest planning: without obstacles in workspace

Fig.5 to Fig.7 are the simulation results when there are obstacles. Fig.5 and Fig.6 show the values of joint angles and joint angular velocities respectively. It is obvious in Fig.5 and Fig.6 that the system turns to be stable at last. The final joint angles are (1.042, 1.022, 1.025) which are very close to the desired value. Fig.7 is the motion diagram of underactuated system when there are obstacles and it is obviously that the linkages steer clear of the obstacle successfully. Fig.8 is the optimum results using (9) when there are no obstacles in workspace and the simulation parameters are the same with that of Fig.7. It can be seen that the proposed method is effective for both cases when there are obstacles and no obstacles in workspace.

4.2 Trajectory Tracking

The dynamic parameters of the underactuated robot are listed in Tab.1. The only difference is that the initial joint angles are [-0.4 0.5 -0.4] and the desired joint angles are [0.4 -0.5 0.4]. A line segment trajectory from the initial point to the desired point is tracked by the switching of PSCs. The best switching sequence is obtained by optimization of (13) using GA method. Let gain coefficients k_{vi}, k_{pi} and matrix w with the same value as that in part 4.1. The penalty operator $\lambda_1 = 800$.

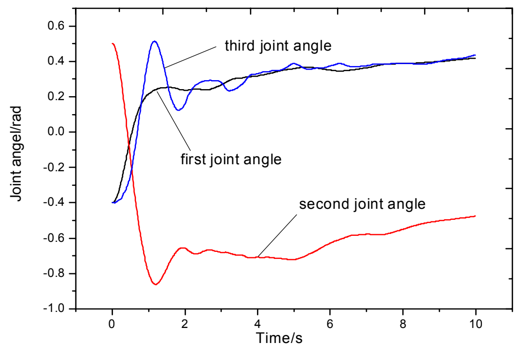

Fig.9 Joint angles

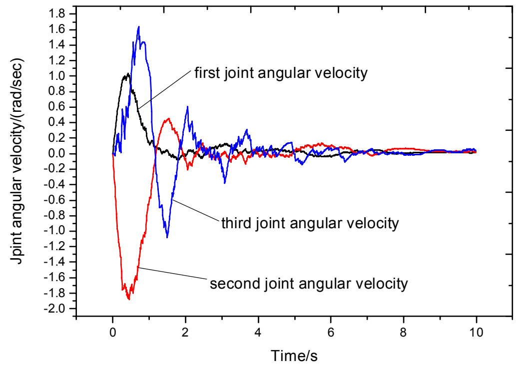

Fig.10 Joint angular velocities

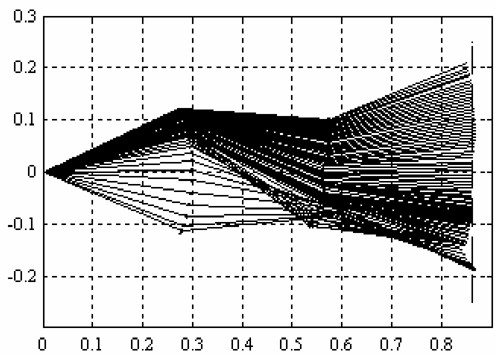

Fig.11 Trajectory tracking of a line segment

Fig.9 and Fig.10 show the responses of joint angles and joint angular velocities respectively. The final joint angles are [0.41 -0.48 0.42] and the relative error is less than 3%. Fig.11 is the motion diagram of the underactuated robots. The maximum distance from the endpoint of manipulators to the desired path is 0.019 (*m)*, which means that the method is effective in trajectory tracking of underactuated robots.

5. CONCLUSIONS

Motion planning and trajectory tracking of underactuated robots are investigated. A universal method is proposed to solve this kind of problem. The partly stable controllers are derived and the goals are fulfilled by the switching of the partly stable controllers. Penalty method is utilized when there are constraints and then the constrained optimizations change to be unconstrained ones. At last the best solutions are obtained by Genetic algorithm.

One of the major advantages of the proposed method is that the rigorous linearization or deformations of the original nonlinear system in the whole process are not considered. Another advantage of the proposed method is that it does not make any hypothesis about the degree of freedom and so it can be used without modification for arms with a large number of degrees of freedom.

The trajectory planner is effective in motion planning and trajectory tracking of underactuated robots. The major distinction is the diverse of the fitness functions. However there are still improvements of the method: First considering the influence of friction for passive joints which makes it more closely as reality, and then investigating other effective algorithms in dealing with the constrained optimization problems are our future research.

ACKNOWLEDGMENT

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