Proceedings of the 17th World Congress
The International Federation of Automatic Control
Seoul, Korea, July 6-11, 2008

Robust H_∞ Controller Design for Aircraft Lateral Dynamics using Multi-objective Optimization and Genetic Algorithms

M Giacomán-Zarzar, R Ramirez-Mendoza*
PJ Fleming, I Griffin, A Molina-Cristóbal*****

** Center for Innovation in Design and Technology, Tecnológico de Monterrey, Monterrey, Nuevo León, México (Tel: 8358-2000;e-mail: manuel.giacoman@exatec.itesm.mx, ricardo.ramirez@itesm.mx.*
*** Department of Automatic Control and Systems Engineering, The University of Sheffield, Sheffield, England, (p.fleming@sheffield.ac.uk, i.griffin@sheffield.ac.uk)*
****Electrical Machines & Drives Group, Department of Electronic and Electrical Engineering, The University of Sheffield, Sheffield, England, (a.molina-cristobal@sheffield.ac.uk)*

Abstract: Two techniques are combined during the design of an optimal controller: Linear Matrix Inequalities (LMIs) and Multi-objective Genetic Algorithms (MOGAs). In this paper the LMI optimization technique is used to obtain a single controller while MOGA is used to convert the controller design into a multi-objective optimization procedure. The combination of these techniques is proposed in this document and is shown to be advantageous against independent application of the aforementioned techniques. It is also presented how the sensitivity and complementary sensitivity functions are shaped with the weighting functions, while restricting the magnitude of the control signals by adding them as a hard objective in the MOGA approach.

1. INTRODUCTION

Through the years, the desire to design better controllers for complex systems has led to the design and implementation of multi-objective controllers. Multi-objective controllers are able to synthesize problems with a mixed time and frequency domain specifications, achieving H_2 and H_∞ performance with pole placement included. For such problems there is not a single optimal solution, rather a Pareto optimal front set, which encompasses a set of equally valid solutions.

To solve the mixed H_2/H_∞ control problem two different approaches have been used; the first one uses the Youla parameterization of the controller (Scherer 1995, Neering et al. 2006) and the second one (Scherer et al, 1997) introduces a dependence in the objectives, making the optimization a single objective problem. Based on the former approach some authors have looked for a way to further optimize their controller; for example, (Feng et. al, 2005) proposed a reliable H_∞ flight tracking controller design in the presence of actuator outage faults and/or surface control impairment; each fault is considered as a vertex to a polytopic uncertain system and the approach is based on multi-objective optimization using parameter-dependent Lyapunov functions. This allows the reduction of conservativeness.

In this paper, two techniques are combined during the design of an optimal controller: Multi-objective Genetic Algorithms (MOGAs), (Fonseca and Fleming 1995) and Linear Matrix Inequalities (LMIs). MOGA is a class of evolutionary algorithms that mimics the natural selection and evolution. MOGA operates on a population of potential solutions randomly selected; each individual is evaluated and ranked according to its fitness (objective function). Highly fit individuals have a high probability of being selected and the mutation and crossover operators are applied over them in order to create a new generation; this assures that good individuals are preserved and bred while less fit individuals are discarded. The procedure continues until the objective or the specified number of generations has been reached (Fonseca and Fleming 1998). Following this idea (Fonseca and Bottura 1999) developed a parallel multi-objective genetic algorithm and applied it to select the Q and R weighting matrices for the linear quadratic regulator (LQR) design; the solution was addressed as a multi-objective optimization problem where the weighting matrices are independent variables and the closed loop stability, design specifications and desired eigenstructure were added as requirements. Also GAs have been utilized to reduce the order of the controller and retain the advantages developed with the traditional methodologies, (Molina-Cristóbal et al, 2006; Kitsios and Pimenides, 2003).

Here, LMIs are used to calculate and obtain a single controller, taking advantage of its systematic procedure; while MOGA has been used to convert the controller design into a multi-objective optimization procedure. This configuration has been advantageous with respect to the independent applications. Some comparisons between using or not MOGA are reported for demonstrating the effectiveness of the proposed technique.

The paper is organized as follows: Section 2 introduces the aircraft lateral equations of motion. Section 3 describes the actual properties of the aircraft used (Boeing 747-200) and specifies the design requirements. Section 4 shows the methodology to design a multi-objective robust controller. Section 5 illustrates the results obtained when MOGA and LMIs are applied in conjunction for the controller design and Section 6 summarizes the results.

2. THE AIRCRAFT MODEL

Based on the small disturbance theory, consider the following linear time-invariant system which describes the lateral dynamics of an aircraft:

$$\begin{aligned} \dot{x} &= Ax + Bu + Fw \\ y &= Cx + Du \end{aligned} \qquad (1)$$

where $x \in \Re^n$, $u \in \Re^m$, $y \in \Re^m$ and $w \in \Re^q$ represent the state, input, output and disturbance vectors respectively and $\Re$ denotes the set of real numbers. The matrices A, B, C, D and F are calculated from the lateral directional derivatives, which depend on the aerodynamic coefficients.

$$A = \begin{bmatrix} \frac{Y_\beta}{u_0} & \frac{Y_p}{u_0} & -\left(1-\frac{Y_r}{u_0}\right) & \frac{g\cos\theta_0}{u_0} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & \frac{Y_{\delta_r}}{u_0} \\ L_{\delta_a} & L_{\delta_r} \\ N_{\delta_a} & N_{\delta_r} \\ 0 & 0 \end{bmatrix} \qquad C = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$F = \begin{bmatrix} -\frac{Y_\beta}{u_0} & -\frac{Y_p}{u_0} & -\frac{Y_r}{u_0} \\ -L_\beta & -L_p & -L_r \\ -N_\beta & -N_p & -N_r \\ 0 & 0 & 0 \end{bmatrix} \qquad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

and

$x = [\ \beta\ \rho\ r\ \phi\ ]^T$

$u = [\ \delta_a\ \delta_r]^T$

$w = [\ \beta_w\ \rho_w\ r_w\ ]^T$

$y = [\ \phi\ \beta\ ]^T$

where

β = sideslip angle

ρ = roll rate

r = yaw rate

ϕ = roll angle

δ_a = aileron deflection

δ_r = rudder deflection

$\Delta\beta_w$ = change in sideslip angle due to horizontal wind perturbations

$\Delta\rho_w$ = roll rate perturbations due to vertical winds

Δr_w = yaw rate changes due to wind disturbances

The lateral dynamics simulates 3 modes of the aircraft: the roll, sideslip and Dutch roll mode. For each one there is a performance specification that must be fulfilled in order to find a controller that is in accordance with the pilot's opinion about the controllability and manoeuvrability of the aircraft.

3. DESIGN SPECIFICATIONS AND PLANT DESCRIPTION

In this example, the lateral dynamics parameters are based on a Boeing 747-200 aircraft and the control is designed to achieve the handling qualities specified in Nelson (1998) and the following requirements, taking into account the presence of disturbances and uncertainties.

The controller must be robust to 25% of multiplicative uncertainty and to parametric uncertainty.

Decoupling of roll and sideslip angle.

The overshoot due to a step change in the command reference should be minimized. Target < 5%.

Any kind of disturbance should be rejected as much as possible.

3.1 Plant Description

The plant has two imaginary poles (-0.159+0.954i, -0.159-0.954i), two real poles (-0.3669, 0.058) and one zero at -82.618; it is state controllable, observable and output controllable. The difference in the magnitude of the singular values (48.925 dB – (-16.651 dB) = 65.576 dB) states that some control problems will arise since a large range of gains must be satisfied; also, the small singular value, make the system sensitive to disturbances in the low frequency range, while the condition number corroborates that the system is ill-conditioned.

The controller is designed for the nominal plant, i.e. the aircraft flying at high cruise velocity, at an altitude of 40,000 ft and a Mach number equals to 0.9. The uncertainties will be handled as output multiplicative uncertainty; while the disturbances presented are simulated by passing white noise through Dryden filters and then added to the states derivatives, as illustrated in Figure 1. These filters were obtained according to the military specifications MIL-F-8785C by selecting the severe turbulence intensity curve.

$$H_p = \frac{0.1933}{3.466s + 12.05} \qquad H_\beta = \frac{0.051s + 0.01468}{4.034s^2 + 4.017s + 1}$$

$$H_r = \frac{0.051s^2 + 0.01468s}{0.8652s^3 + 4.896s^2 + 4.232s + 1}$$

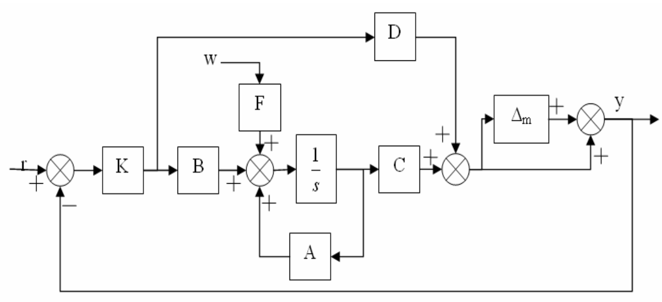


Fig. 1. Plant model with uncertainty and turbulence.

A mixed H_2/H_∞ Controller is proposed to solve the control problem as explained in the next section.

4. MULTI-OBJECTIVE ROBUST CONTROL

In many real applications, the standard controllers cannot capture all design specifications and restriction. For example the LQG problem designs in a natural way a controller with the ability to reject random noise disturbances, but it lacks of a formal procedure to guarantee the LQG robustness. Similarly, the H_∞ synthesis controller focuses on closed-loop stability leaving aside the possibility to place the closed-loop poles in strategic regions of the left half-plane. Since the pole locations have a direct influence on the time response and transient behaviour of the system it is highly desirable to impose some constraints on the closed-loop dynamics (Doyle, 1996).

On the other hand, the mixed H_2/H_∞ approach provides a defined and well-structured methodology for handling multivariable systems and it allows us to obtain a robust controller which is ideal for handling most of the variations and assumptions presented in the aircraft model. Additionally, the H_2 approach permits the specification of the required performance for the aircraft.

4.1 Multi-objective optimization

In the mixed H_2/H_∞ optimization method the H_2 and H_∞ constraints are solved and specified with Linear Matrix Inequalities (LMIs); this permits a search for a convex solution restricted to the time- and frequency-domain specifications.

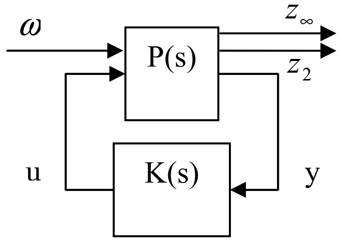


Fig. 2. Multi-objective Plant Configuration

If the plant equation 1 is rearranged into the generalized plant configuration as shown in Figure 2, the following formulation is obtained.

$$\begin{aligned} \dot{x} &= Ax + B_1\omega + B_2u \\ z_\infty &= C_\infty x + D_{\infty 1}\omega + D_{\infty 2}u \\ z_2 &= C_2x + D_{21}\omega + D_{22}u \\ y &= C_y x + D_{y1}\omega + D_{y2}u \end{aligned} \qquad (2)$$

where

ω represents the exogenous inputs.

u is the output of the controller.

z_∞ is the output associated with the H_∞ performance.

z_2 is the output associated with the H_2 performance.

y are the measured signals.

Let the closed-loop equations be

$$\begin{aligned} \dot{x}_{cl} &= A_{cl}x_{cl} + B_{cl}\omega \\ z_\infty &= C_{cl1}x_{cl} + D_{cl1}\omega \\ z_2 &= C_{cl2}x_{cl} + D_{cl2}\omega \end{aligned} \qquad (3)$$

then, the objectives can be presented as:

$$\begin{pmatrix} A_{cl}x_\infty + x_\infty A_{cl}^T & B_{cl} & x_\infty C_{cl1}^T \\ B_{cl}^T & -I & D_{cl1}^T \\ C_{cl1}x_\infty & D_{cl1} & -\gamma^2 I \end{pmatrix} < 0 \qquad x_\infty > 0$$

$$\begin{pmatrix} A_{cl}x_2 + x_2 A_{cl}^T & B_{cl} \\ B_{cl}^T & -I \end{pmatrix} < 0 \qquad \begin{pmatrix} Q & C_{cl2}x_2 \\ x_2 C_{cl2}^T & x_2 \end{pmatrix} > 0$$

$$trace(Q) < \nu^2$$

The H_2 and the H_∞-norm are objectives that mutually compete; therefore a controller is sought by solving equation 4, restricted to the former LMIs constraints.

$$\text{minimize } \|W_1S\|_2 \text{ subject to } \|W_2T\|_\infty < \gamma_\infty \qquad (4)$$

4.2 Multi-Objective Genetic Algorithm

MOGA is used to assist and further optimize the robust controller obtained in section 4.1. The procedure followed is illustrated in Figure 3: The population is randomly initialized, normally 100 individuals are selected; each individual contains in its phenotype (real representation) the values of the proposed weighting functions. For each individual, a controller is obtained following the procedure stated in section 4.1, the objectives are evaluated and ranked using Pareto-optimal ranking procedure and the fitness is assigned to the individuals; also fitness sharing is applied to avoid the population to drift to an arbitrary region (Fonseca and Fleming, 1995). The individuals are selected using stochastic universal sampling; the ones that are best fitted have a higher opportunity of being selected. Then, the individuals are reproduced applying the crossover operator and some of them are mutated in order to reduce the probability to leave any region without been explored. The new generation is formed and the procedure repeats until the final generation is reached.

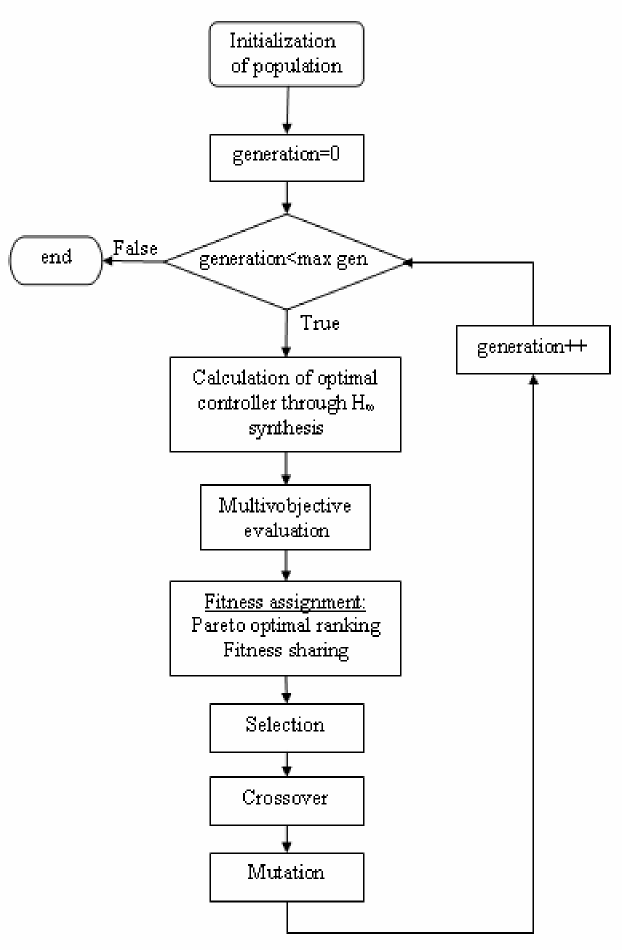


Fig. 3. Flow chart of the H∞ optimization process assisted with MOGA.

4.3 Proposed Methodology

For multivariable systems the selection of the weighting functions is not straightforward, since they shape the closed-loop and the relation between the parameters variation and the effect on the response is not very clear. Also, in many cases the minimization of the H_∞ norm leads to the degradation of the H_2 norm or demands a high magnitude from the control signals. In order to tackle the problem, MOGA is used as a master for the control design which calls the slave Matlab function "hinfmix" (Gahinet et al., 1995) for the selection of the controller.

In order to optimize the design of a multivariable controller the following steps have been implemented: 1) Analyze the plant; verify that it is controllable and observable. 2) Select the structure of the weighting functions. (W_1 and W_2 as first order filters). 3) Calculate the controller following the procedure in section 4.1. 4) Optimize the performance of the controller using MOGA as explained in section 4.2 and selecting the H_∞ norm, H_2 norm, overshoot, and rudder and aileron manipulations as the objectives. 5) Evaluate the results during each generation and select the fittest individuals. 6) From the Pareto Optimal Front obtained, choose the individual that best fit the handling qualities and requirements of the aircraft.

5. CONTROL IMPLEMENTATION AND SIMULATION RESULTS

The mixed H_2/H_∞ can be designed by shaping the closed-loop transfer functions, i.e., the performance and robustness specifications are selected as desire gain responses for the closed loop transfer functions, as illustrated in Figure 4.

$$W_{11} = \frac{\frac{s}{M_1} + \omega_{b1}}{s + \omega_{b1} e_{ss1}} \qquad W_{12} = \frac{\frac{s}{M_2} + \omega_{b2}}{s + \omega_{b2} e_{ss2}}$$

$$W_1 = \begin{bmatrix} W_{11} & 0 \\ 0 & W_{12} \end{bmatrix} \qquad W_2 = \frac{\tau s + r_0}{\frac{\tau s}{r_\infty} + 1}$$

where e_{ss} and M set the upper bound of $\boldsymbol{S}$ at low and high frequencies and the asymptote crosses one at ω_b.

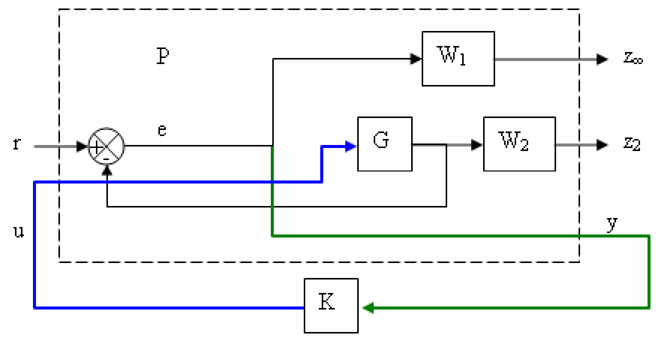
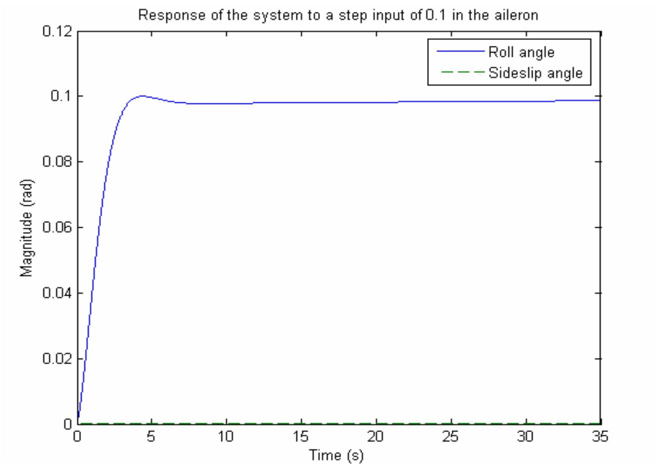


Fig. 4. Mixed H_2/H_∞ control problem design

A robust controller can be easily obtained with the MATLAB function "hinfmix" (Gahinet et al., 1995); however, due to the large number of trials that have to be made to find W_1 and W_2 that satisfy the time-domain and frequency response, MOGA was used to optimize the search. Five objectives were set in MOGA: the H_∞ norm, H_2 norm, overshoot, and rudder and aileron manipulations; the control signals were selected as hard constraints and the others as soft constraints; this allows setting a maximum limit in the control signals. (Another way that could have been done is to set the restrictions in Simulink). Also 10 variables (M_1, ω_{b1}, e_{ss1}, M_2, ω_{b2}, e_{ss2}, τ, r_0, r_∞, and γ_∞) were manipulated by MOGA during each iteration.

MOGA was run with 50 individuals and for 100 generations. The response of the system to a step change of 5.72° (0.1 rad) in the aileron command is presented in figure 5. The response has an overshoot less than 1% and a steady-state error of 1.5%



Fig, 5. Response of the system to a step change of 5.72° (0.1 rad).

Incorporating an integrator in the weighting function W_1 often leads to a singularity; the solution proposed by (Skogestad et al, 1996) is to add a pseudo integrator with the form $\frac{1}{s+\varepsilon}$, as performed in Figure 4. However, the controller still presents some difficulties when disturbances are presented and it is not possible to achieve a zero steady-state error. This is seen graphically in the open-loop response of GK, when it has a flat curve at low frequencies. A more efficient approach consists of removing the pseudo integrator from W_1 and moving it backward in the main loop as presented in Figure 6; this defines a new controller $K = \tilde{K}/s$. The second step is to specify $W_2=a_1s+b_1$ as a nonproper filter with derivative action (Scherer, 1997).

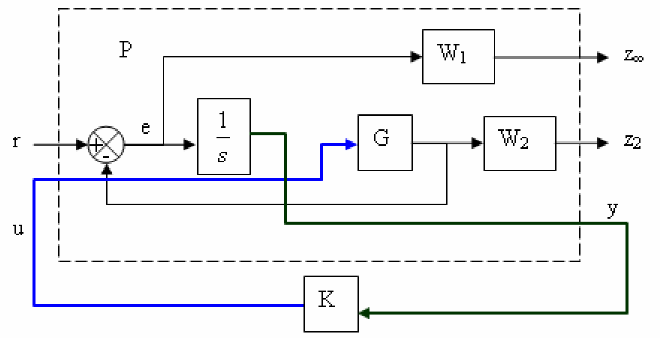


Fig. 6. Modified control structure.

In order to obtain a controller, the same methodology was applied as before, with the difference that the steady-state error was also added as an objective with soft constraint. The results for a step change in the aileron command and with the presence of disturbances (turbulence simulated by passing white noise through Dryden filters) are illustrated in Figure 7.

The system presents zero steady-state error and 1.5% of overshoot. Figure 8 compares the behaviour of the system in the presence of 20% of uncertainty at low-frequency, 100% at ω=50 rad/s and 200% at high-frequency.

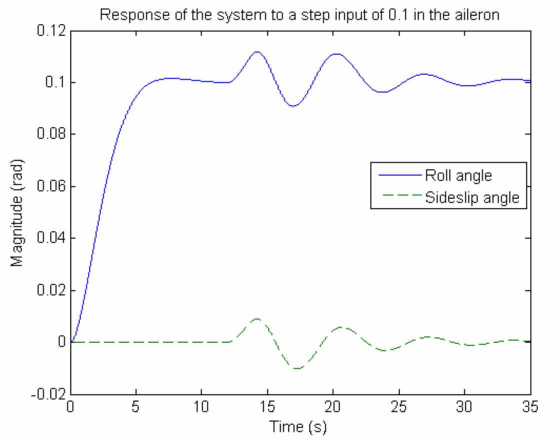


Fig. 7. Response of the system to a step change of 5.72° (0.1 rad) and turbulence input between 12 and 14 seconds.

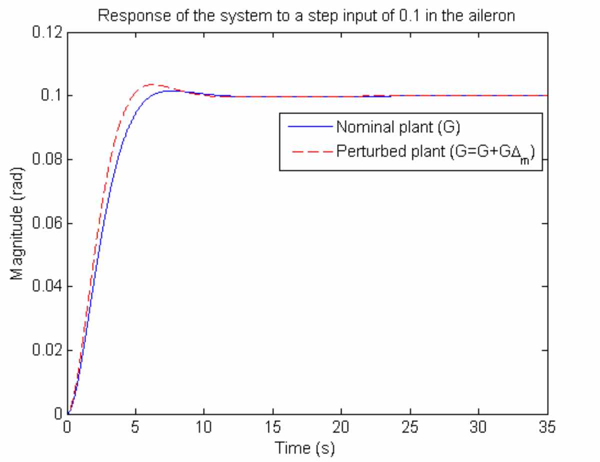


Fig. 8. Comparison between the nominal plant with and without uncertainties.

Achieving an optimal solution for multi-objective systems requires quite an effort. From Figure 9 it is clearly seen that the H_2 norm is in conflict with the H_∞ norm, and also improving the percentage of overshoot leads to the degradation of the capacity to reject disturbances.

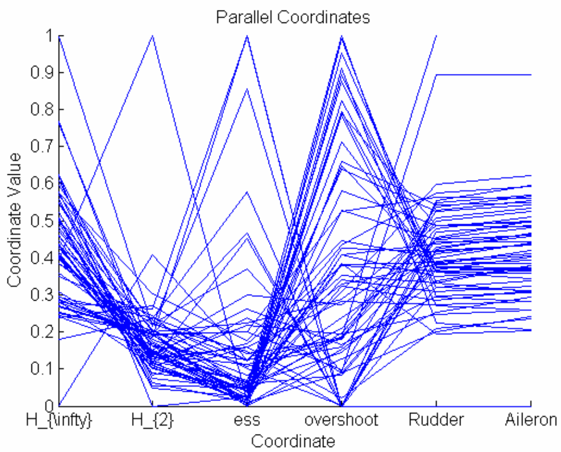


Fig. 9.Parallel coord show the trade off between objectives.

If Figure 9 is analyzed through the generations, it can be seen that at the beginning not all of the objectives compete with each other, but as long as the improvements evolve, the difficulty to obtain an optimal solution increases; i.e. some point is reached in which no further improvements can be obtained and even more, a small improvement in one objective can lead to a high degradation of one or more objectives.

Molina-Cristóbal et al. (2006) solve the multi-objective problem using LMIs and compare it with MOGA, achieving better results with the second approach. However they remarked on two important aspects: LMI lacks full flexibility while the search space in MOGA is not always known or too large for MIMO systems. When combining both methodologies we take advantage of the benefits while their weaknesses are complemented. If the H_2 and H_∞ are set as independent objectives tighter bounds can be found with a better performance, see results in Figure 10.

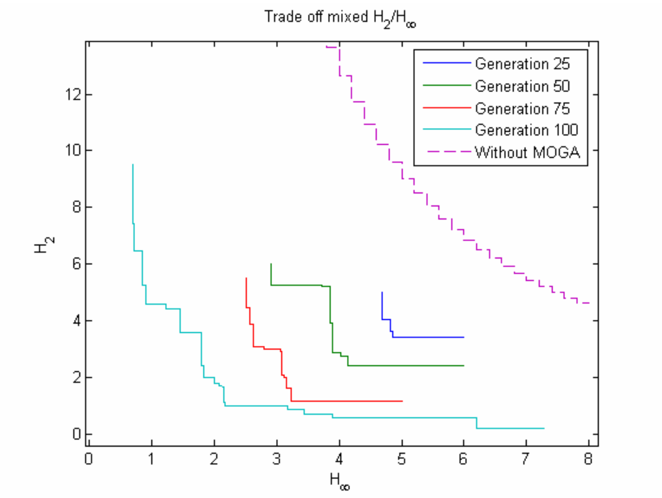


Fig. 10. Evolution of the trade off mixed H_2/H_∞ achieved with MOGA, and the comparison with the manual procedure.

6. CONCLUSION

Many advantages are realised when MOGA assists the mixed H_2/H_∞ design. As known from the H_∞ methodology, the selection of W_1, W_2 and W_3 which penalized the sensitivity function (S), the complementary sensitivity function (T) and the control signals (KS) is difficult to implement; therefore it is recommended to combine at most two of these weighting matrices for each controller. In this case we have selected W_1 and W_2 to specify the desire robustness to uncertainty and time response specifications, while the limit on the control signals was established as an objective within MOGA.

When MOGA is used alone for multi-objective problems it is quite difficult to specify the correct range for the variables to select through the algorithm; moreover the complexity increases as the number of variables increases. In the case of our MIMO system example, the number of variables could be more than 120. On the other hand, LMIs provide a systematic and easy to calculate procedure, but with the complexity to select the appropriate weighting functions. Through this work it is shown that the combination of the two methodologies results in a powerful strategy for the design of robust controllers. The well-structured H_∞ procedure is introduced in the MOGA algorithm and the result is a multi-objective optimization of the desired parameters while assuring internal stability of the controller. Further research will be needed to deal with the theoretical proof of the relevance of the proposed approach.

REFERENCES

Doyle, John (1996). Robust and Optimal Control. *Proceeding of the 35th Conference on Decision and Control*. **Vol. 2** Pages 1595-1598.

Feng, Le, J. Wang, E., Poh, F, Liao, (2005). Reliable H_∞ Aircraft Flight Controller Design against Faults with State/Output Feedback. *American Control Conference*. **Vol. 4**, pages 2664-2669.

Fonseca C.M. and P.J. Fleming (1995). Multiobjective Genetic Algorithms Made Easy: Selection, Sharing and Mating Restriction. *Genetic Algorithms in Engineering Systems: Innovations and Applications*. **Conference Publication No. 414.** Pages 45-52.

Fonseca C.M. and P.J. Fleming (1998). Multiobjective Optimization and Multiple Constraint Handling with Evolutionary Algorithms-Part I: A Unified Formulation. *IEEE Transactions on Systems, Man and Cybernetics-Part A: Systems and Humans*. **Vol. 28, No. 1,** pages 26-37.

Fonseca Neto, J.V. and C.P. Bottura, (1999). Parallel Genetic Algorithm Fitness Function Team for Eigenstructure Assignment via LQR Designs. **Vol 2,** pages 1035-1042.

Gahinet, P., A. Nemirovski, A. Laub and M. Chilali (1995). *LMI Control Toolbox for Use with Matlab*. The Mathworks Inc.

Kitsios, I. and T. Pimenides (2003). Structured-specified robust-multivariable-controller design for practical applications using genetic algorithms. *IEE Proceedings Control Theory Applications*. **Vol .150, No. 3,** pages 317-323.

Molina-Cristóbal, A., I. A. Griffin, P.J. Fleming, and D. H. Owens (2006). Linear Matrix Inequalities and Evolutionary Optimization in Multiobjective Control. *International Journal of Systems Science*. **Vol. 37, No. 8,** pages 513-522.

Nelson, Robert (1998). *Flight Stability and Automatic Control*. McGraw Hill.

Neering, J., R. Drai, M. Bordier, N. Maïzi (2006). Multiobjective Robust Control via Youla Parametrization. *Proceedings of the 2006 IEEE International Conference on Control Applications*. Munich, Germany. Pages 1450-1456.

Scherer, Carsten (1995). Multiobjective H_2/H_∞ Control. *IEEE Transactiond on Automatic Control*. **Vol. 40, No. 6**, pages 1054-1062.

Scherer, C., P. Gahinet, M. Chilali (1997). Multiobjective Output-Feedback Control via LMI Optimization. *IEEE Transactions on Automatic Control*, **Vol. 42, No. 7**, pages 896-911.

Skogestad, S. and I. Postlethwaite (1996). Multivariable Feedback Control, Analysis and Design. Wiley.