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Data Fusion of Unknown Correlations using Internal Ellipsoidal Approximation

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Abstract: To avoid both the inconsistency of the Kalman filter and the performance conservation of the covariance intersection (CI) in the case of unknown correlations, an internal ellipsoidal approximation (IEA) method is proposed to fuse the local estimations. A numerical example of three-state radar tracking system is presented to illustrate the implementation and effectiveness of the proposed algorithm. From the simulation results in the cases that the sources are (i) independent; (ii) correlated and the cross-covariance are exact known; and (iii) correlated with unknown cross-covariance, it is obvious to see that the IEA method, like CI, circumvents the need for prior knowledge of the correlations but it gets better fusion accuracy than CI.

1. INTRODUCTION

The main idea of data fusion is to obtain an estimate of some unknown state variables, such as position, velocity and attitude, from the available noise-corrupted observations (Li, Zhu, & Han, 2000). The optimal fusion Kalman filter has widely been applied in many fields including guidance, defence, robotics, integrated navigation, industrial process automation, target tracking, and GPS positioning. It has been realized for many years that local estimates (track) have correlated errors (Bar-Shalom, 1981). How to counter this cross correlation, therefore, has been a central topic in data fusion. One problem with the Kalman filtering is that it requires either that the measurements are independent or that the cross-covariance is known (See Smith & Sameer, 2006). Unfortunately, even though under the assumption that the cross-covariance is known, the optimal KF-based approach scales quadratically with the number of updates, which makes it impractical (See Drummond, 1997). A common simplification is to assume the cross-covariance to be zero, i.e. the measurements are independent, though, in this situation, the KF produces nonconservative covariance. This leads to an artificially high confidence value, which can lead to filter divergence (Julier & Uhlmann, 1997). Recently proposed covariance intersection filtering (See Julier & Uhlmann, 1997) is based on convex combination of information matrices, i.e., inverse covariance matrices and the corresponding information states. The algorithm provides a general framework for information fusion with incomplete knowledge about the signal sources since it yields consistent estimates for any degree of cross correlation.

Since covariance intersection filtering requires optimization of a nonlinear cost function and instead of underestimation of the actual covariance matrix as Kalman filter, the covariance intersection method overestimates it, which obviously results in a significant decrease in performance. Therefore, a largest ellipsoid algorithm has been proposed by Benaskeur (2002). The algorithm provided in (Benaskeur, 2002) solved the matrices orientation incompatibility problem in the case of two sensors. Unfortunately, it did not derive the fusion estimate correctly, and the estimation performance may degrade severely as described in section 4.

In this paper, in order to avoid both the inconsistency of the optimal KF-based fusion algorithm and the performance conservation of the covariance intersection method, an Internal Ellipsoid Approximation (IEA) method is proposed to obtain the largest volume ellipsoid within the intersection of estimation covariances. Simulation example of applying to decentralized fusion is also presented to illustrate the effectiveness of the proposed algorithm. The remainder of the paper is organized as follows: the problem discussed and some previous results is stated in section 2; the main results in this paper, data fusion algorithm using internal ellipsoidal approximation are discussed in section 3, which is followed by a simulation example of three-state radar tracking system with two sensors. Conclusions are drawn in section 4, as well as future work is presented.

2. PROBLEM STATEMENT AND PREVIOUS RESULTS

2.1 Problem statement

The system model is described by the following equations:

$$x(k+1) = F(k)x(k) + G(k)\omega(k) \qquad (1)$$

$$y_i(k) = H_i(k)x(k) + v_i(k),\ i=1, 2, \dots, N \qquad (2)$$

where $x(k) \in \mathrm{R}^n$ is the state, $y(k) \in \mathrm{R}^l$ the measurement of sensor i, $F(k) \in \mathrm{R}^{n\times n}$ the state transition matrix, $G(k) \in \mathrm{R}^{n\times m}$ the input matrix of process noise, and $\omega(k) \in \mathrm{R}^m$ white noise with zero mean and covariance matrix of $Q(k) \geq 0$. $v_i(k)$ are observers noises which correlated with $\omega(k)$, in this paper, the correlations are assumed to be not known or incomplete.

We consider the problem that two information sources, x_{0_1} and x_{0_2}, are to be fused together to yield the output x_0 and the fusion covariance P_0. The only available information about the two sources are the statistical representation of $\hat{x}_{0_1}$ and $\hat{x}_{0_2}$, and the estimate covariance

$$\hat{P}_1 = E\{\tilde{x}_{0_1}\tilde{x}^T_{0_1}\} \text{ and } \hat{P}_2 = E\{\tilde{x}_{0_2}\tilde{x}^T_{0_2}\} \tag{3}$$

The correlation between the two sources, stand by cross-covariance

$$\hat{P}_{12} = E\{\tilde{x}_{0_1}\tilde{x}^T_{0_2}\} \tag{4}$$

are unknown or incomplete, where the local estimation errors, $\tilde{x}_{0_1}$ and $\tilde{x}_{0_2}$, are defined by

$$\tilde{x}_{0_1} = \hat{x}_{0_1} - x_{0_1} \text{ and } \tilde{x}_{0_2} = \hat{x}_{0_2} - x_{0_2}$$

2.2 Previous results

Several fusion methods compute a linear combination of the estimates x_{0_1} and x_{0_2}, and analytically determine the covariance of the result. This leads to the optimal result when knowledge is available about the actual system statistics. Unfortunately, the knowledge is usually unachievable due to uncertainty in the cross-correlation. The Kalman filter, for instance, linearly combines estimates $\{x_{0_1}, P_1\}$ and $\{x_{0_2}, P_2\}$ into the fused estimates $\{x_0, P_0\}$ according to

$$x_0 = W_1 x_{0_1} + W_2 x_{0_2} \tag{5}$$

$$P_0 = W_1 P_1 W_1^T + W_1 P_{12} W_2^T + W_2 P_{21} W_1^T + W_2 P_2 W_2^T \tag{6}$$

Weights W_1 and W_2 minimize the trace of P_0. Consistency of estimates $\{x_{0_1}, P_1\}$ and $\{x_{0_2}, P_2\}$ are enough to ensure consistency in Eq. (6) when $P_{12} = P_{21} = 0$ (Jazwinski, 1970). In this case, $P_0^{-1} = P_1^{-1} + P_2^{-1}$ and the gains in Eq. (6) are $W_1 = P_0 P_1^{-1}$ and $W_2 = P_0 P_2^{-1}$ which correspond to the derivation of the Kalman gain in the Kalman filter (Chen et al., 2002; Uhlmann, 2003). Consistency in Eq. (6) is not assured when the actual correlation $P_{12} \neq 0$. In this case, the overconfident estimation such as the Kalman filter method can lead to fusion divergence. What's more, in the case that the correlations are unknown or incomplete, the optimal data fusion Klaman filters cannot be applied directly.

As estimating the cross-covariance is computationally expensive, Covariance Intersection applies a convex combination of means and covariances in the inverse covariance space circumventing the need for knowledge of the cross-correlation. CI-based fusion for two information sources is given by (Julier & Uhlmann, 1997)

$$P_0^{-1} = \omega P_1^{-1} + (1-\omega) P_2^{-1} \tag{7}$$

$$P_0^{-1} x_0 = \omega P_1^{-1} x_{0_1} + (1-\omega) P_2^{-1} x_{0_2} \tag{8}$$

Parameter ω is used to minimize a fixed measure of fusion covariance size, say the trace or determinant of the fused covariance P_0.

Hurley (2002) gave an information theoretic proof of the CI technique and pointed out that CI is capable of fusing any probability density function, not just Gaussian distributions. But CI is pessimistic with the ellipse being larger than it needs to be; therefore, the largest ellipsoid algorithm (Benaskeur, 2002) avoids this by creating the largest ellipse that will fit within the intersection of the covariances (See Fig. 2 in Benaskeur, 2002). Largest ellipsoid leads to tighter estimates than CI method since it underestimates the covariance rather than overestimating it, though this is less of an underestimate than the KF, so filter divergence is still avoid (Smith & Sameer, 2006).

3. FUSION USING INTERNAL ELLIPSOIDAL APPROXIMATION

The algorithm provided in (Benaskeur, 2002) solved the matrices orientation incompatibility problem in the case of two sensors, and the inscribed largest volume ellipsoid within the intersection of two ellipsoids can be computed. Unfortunately, it did not derive the computation of estimated fusion correctly, and the estimation performance may degrade severely, which motivates the fusion algorithm using internal ellipsoidal approximation that can be formulated in the following Algorithm.

First, let us introduce the definition of regular generalized ellipsoids as below.

Definition 1. Ellipsoid $\varepsilon(x_0, P_0)$ in R^n with centre x_0 and shape matrix P_0 is the set

$$\varepsilon(x_0, P_0) = \{x \in R^n \mid (x - x_0)^T P_0^{-1} (x - x_0) \leq 1\} \tag{9}$$

where $P_0 > 0$ might be standing for the covariance matrix of the estimation or fusion error.

Since the estimate covariances of the two information sources x_{0_1} and x_{0_2} can be intuitively interpreted as two ellipsoids denoted as $\varepsilon(0, P_1)$, $\varepsilon(0, P_2)$, the intersection region represents an upper limit for the actual error covariance matrix, the design of proposed algorithm is based on the internal approximation of the intersection region of the covariance matrices.

Algorithm 1(Internal Ellipsoidal Approximation Fusion).

Step 1. Introduce two numbers β_1 and β_2, which are invariant with respect to affine coordinate transformations of the space and play an important role in the computation of weights of the fusion.

$$\beta_1 = \min_{<x,P_2^{-1}x>=1} < x, P_1^{-1}x >= \min_{x^T P_2^{-1}x=1} x^T P_1^{-1}x \tag{10}$$

$$\beta_2 = \min_{<x,P_1^{-1}x>=1} < x, P_2^{-1}x >= \min_{x^T P_1^{-1}x=1} x^T P_2^{-1}x \tag{11}$$

Note β_1 and β_2 describe the position relationship of covariance ellipsoids $\varepsilon(0,P_1)$, $\varepsilon(0,P_2)$ with respect to each other, where 0 represents the origin of the *n*-dimension plane:

(1) If $\beta_1 \geq 1, \beta_2 \leq 1$, then $\varepsilon(0,P_1) \subseteq \varepsilon(0,P_2)$

(2) If $\beta_1 \leq 1, \beta_2 \geq 1$, then $\varepsilon(0,P_1) \supseteq \varepsilon(0,P_2)$

(3) If $\beta_1 < 1, \beta_2 < 1$, then $\varepsilon(0,P_1) \cap \varepsilon(0,P_2) \neq \phi$,

$\varepsilon(0,P_1) \not\subset \varepsilon(0,P_2)$, $\varepsilon(0,P_2) \not\subset \varepsilon(0,P_1)$

Remark 1. The optimization problem in (10) and (11) is a Quadratic Programming (QP) problem with quadratic constraints, which can be solved using the function *fmincon* in MATLAB. It is shown that independently of the dimension of the space, the problem of finding these two numbers can be reduced to a one-dimensional optimization problem on an interval (Vazhentsev, 2000; Kurzhanski, 1991). Here we use the Lagrange multiplier approach to solve this optimization problem. Take (10) as an example, our goal is to find β_1 under the equation restriction $x^T P_2^{-1} x = 1$, to minimize the performance index $J = x^T P_1^{-1} x$. Applying Lagrange multiplier method, we introduce an auxiliary function

$$F = J + \lambda\left(x^T P_2^{-1} x - 1\right) \tag{12}$$

where λ is a scalar weight considering that the performance index J is a scalar. Setting $\partial F / \partial x = 0$ and with some manipulations yield

$$[P_2 P_1^{-1} + \lambda I]x = 0 \tag{13}$$

Considering the restriction $x^T P_2^{-1} x = 1$, we have the Lagrange multiplier λ and the minimizing points *x*, as the eigenvalues and the normalized eigenvector of the matrix $-P_2 P_1^{-1}$ (using P_2^{-1} weighted norm), respectively.

Step 2. The fused estimate can be derived by

$$x_0 = (\omega_1 P_1^{-1} + \omega_2 P_2^{-1})^{-1} (\omega_1 P_1^{-1} x_{0_1} + \omega_2 P_2^{-1} x_{0_2}) \tag{14}$$

where the weight coefficients ω_1 and ω_2 are

$$\omega_1 = \frac{1 - \min(1,\beta_2)}{1 - \min(1,\beta_1)\cdot\min(1,\beta_2)}$$
$$\omega_2 = \frac{1 - \min(1,\beta_1)}{1 - \min(1,\beta_1)\cdot\min(1,\beta_2)} \tag{15}$$

Step 3. Compute the fused covariance by solving the following equation (Vazhentsev, 2000).

$$P_0 = (1 - x_{0_1}^T P_1^{-1} x_{0_1} - x_{0_2}^T P_2^{-1} x_{0_2} + x_0^T P_0^{-1} x_0) \cdot$$
$$(\omega_1 P_1^{-1} + \omega_2 P_2^{-1})^{-1} \tag{16}$$

Step 4. In the next sampling period, repeat the step 1-step 3, according the updated $\{x_{0_1}, P_1\}$ and $\{x_{0_2}, P_2\}$.

Remark 2. The fusion covariance P_0 can be solved the following Linear Matrix Inequalities (LMIs) (Boyd & Ghaoui, 1994; Ben-Tal & Nemirovski, 2001)

$$P_0 = \arg\min \ \log\det \ P_0^{-1} \tag{17}$$

$$\text{s.t.} \begin{bmatrix} -P_i^2 & 0 & P_0 \\ 0 & \kappa_i - 1 & 0 \\ P_0 & 0 & -\kappa_i I \end{bmatrix} \leq 0, \ (i=1,2)$$

where κ_i are nonnegative and $\det \ P_0^{-1}$ denotes the determinant of the matrix P_0^{-1}. The fusion covariance P_0 can also be computed from the matrix orientation problems as (Benaskeur, 2002), although the fused estimate there is deduced incorrectly.

Remark 3. The consistency of the algorithm using IEA is granted by the Fig. 2 in (Benaskeur, 2002) graphically.

Corollary 1. Two special cases are

(1) If $\beta_1 \geq 1$, $\beta_2 \leq 1$, then $\omega_1 = 1, \omega_2 = 0$, we have $\varepsilon(x_0, P_0) = \varepsilon(x_{0_1}, P_1)$;

(2) If $\beta_1 \leq 1$, $\beta_2 \geq 1$, then $\omega_1 = 0, \omega_2 = 1$, we have $\varepsilon(x_0, P_0) = \varepsilon(x_{0_2}, P_2)$.

These are the same results as in Step 1 and can be derived readily from (12)-(16).

4. SIMULATION RESULTS

Consider the three-state radar tracking system with two sensors (Anderson & Moore, 1979; Julier & Uhlmann, 1997)

$$x(t+1)=\begin{bmatrix}1 & T & T^2/2\\ 0 & 1 & T\\ 0 & 0 & 1\end{bmatrix}x(t)+\begin{bmatrix}T^3/6\\ T^2/2\\ 1\end{bmatrix}\omega(t)$$

$$y_i(t)=H_i x(t)+\upsilon_i(t),\ i=1,2$$

$$\upsilon_i(t)=\alpha_i\omega(t)+\xi_i(t)$$

where the sampling period T=0.1s, H_1=[1, 0, 0], and H_2=[0, 1, 0]. The state $x(t)=\left[s(t)\ \ \dot{s}(t)\ \ \ddot{s}(t)\right]^T$, where $s(t)$, $\dot{s}(t)$, and $\ddot{s}(t)$ are the position, velocity and acceleration respectively of the target at time tT, respectively. $\upsilon_i(t)$, i=1, 2 are the measurement noises of the two sensors, which are correlated with Gaussian white noise $\omega(t)$ with zero mean and variance σ_ω^2. $\xi_i(t)$ are Gaussian white noises with mean zeros and variance matrices $\sigma_{\xi_i}^2$, and are independent of $\omega(t)$.

In the simulation, we setting $\sigma_\omega^2=15$, $\sigma_{\xi_1}^2=5$, $\sigma_{\xi_2}^2=8$. The initial state $x(0)$=[10, 1, 0], and $P_0=0.1I_3$, where I_3 stands for 3-dimentional identity matrix. As comparison, the optimal fusion Kalman filter (Bar-Shalom, 1981; Sun & Deng, 2004), the CI fusion method, and the Largest Ellipsoid (LE) algorithm (Benaskeur, 2002) are included in the simulation, which is performed in three-fold. The Mean Square Error (MSE), computed by $\mathrm{MSE}=\dfrac{1}{M}\sum_{i=1}^{M}[\tilde{x}_k(i)^T\tilde{x}_k(i)]$ where M is the number of Monte Carlo runs and $\tilde{x}_k(i)$ the estimation error of the fused estimate in the ith run, is adopted as the performance criterion.

4.1 The sensors are independent

Setting $\alpha_1=0$ and $\alpha_2=0$, 200 sampling steps are taken in this case. The results presented in Table 1 have been obtained via 100 Monte Carlo runs.

Table 1. MSE in the case of independent sources

	Optimal	CI	LE	IEA
Position	5.7647	9.3999	1.2017e+003	7.1284
Velocity	2.3473	4.6089	4.4701e+004	3.3239
Acceleration	2.5197	2.9775	786.1805	2.5442
Ratio	1.0	1.5977	4.3915e+003	1.2224

From Table 1, it is easy to see that in the case of independent sources, the optimal fusion Kalman filtering has the highest accuracy, while the proposed algorithm gets the results very close to the optimal fusion. As described before, the CI has some conservation to some extent, but the LE is non-convergent, due to the incorrect computation of the centre of the largest volume inscribed ellipsoid. The ratio of the cumulated sum of the MSE of position, velocity, and acceleration is also listed in table 1, and table 2-3 as well.

4.2 The correlations of the sources are exact known

In the case of known correlations, we set $\alpha_1=1.5$, and $\alpha_2=3.0$. The results over 100 Monte Carlo runs are presented in Table 2. It is obvious that the proposed algorithm is nearly optimal; the difference between them is only 3.34%. The difference between CI and the optimal fusion algorithm is about 13.8%, which is much larger than 3.34% in the case of known correlations, due to the little conservation of CI. It is also illustrated in Table 2 that the correct correlations do not significantly improve the fusion accuracy (Scala & Farina, 2000; Smith & Sameer, 2006).

Table 2. MSE in the case of known correlations

	Optimal	CI	LE	IEA
Position	13.9324	18.2349	2.1782e+004	16.6028
Velocity	12.2251	13.0761	1.3438e+005	12.1807
Acceleration	7.0191	6.4538	4.5819e+003	5.4997
Ratio	1.0	1.1383	4.8450e+003	1.0334

4.3 The correlations are unknown

When the correlations are infeasible or uncertain, the optimal fusion Kalman filtering cannot be used directly owing to the optimality is obtained using the cross-covariance, a common simplification is to assume the cross-covariance to be zero, i.e. assuming the sources are independent. On the contrary, CI, LE and the proposed IEA algorithm circumvent the need for knowledge of the cross-covariances, and the proposed IEA algorithm gets much better fusion accuracy than the optimal and CI method, which can be seen from Table 3.

Table 3. MSE performance with unknown correlations

	Optimal	CI	LE	IEA
Position	14.7856	16.7418	752.5704	16.1701
Velocity	16.0640	11.4302	3.6567e+003	11.5083
Acceleration	11.7611	5.2704	123.2628	5.0498
Ratio	1.0	0.7848	106.3717	0.7681

5. CONCLUSIONS AND FUTURE WORK

To avoid both the inconsistency of the Kalman filter and the performance conservation of the covariance intersection method in the case of unknown correlations, an internal ellipsoidal approximation method is proposed to fuse the local estimation in the fusion centre. A numerical example of three-state radar tracking system with two sensors is provided to illustrate the practicability and effectiveness of the proposed algorithm. From the simulation results in the cases that the sources are (ⅰ) independent; (ⅱ) correlated and the cross-covariance are exact known; and (ⅲ) correlated but the cross-covariance are uncertain or unknown, it is obvious to see that the IEA method, like CI, circumvents the need for prior knowledge of the correlations but it gets better fusion accuracy than CI. When the correlations are exact known, the proposed algorithm is nearly optimal with less computation burden, especially with the increase of the sensors' number due to the optimal fusion Kalman filter scales quadratically with the number of updates, making it impractical.

Next the focuses will be concentrated on how to extend the results to more than 3 sensors cases and some explicit formulations are need to make the algorithm more applicable in the fusion settings.

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