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Design of Model Reference Adaptive Tracking Controllers for Mismatch Perturbed Nonlinear Systems with Input Nonlinearity ⋆

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Abstract: A simple design methodology of model reference adaptive control (MRAC) scheme with perturbation estimation for solving robust tracking problems is proposed in this paper. The plant to be controlled belongs to a class of MIMO dynamic systems with input nonlinearity and mismatched time-varying state delay as well as model uncertainty. The control scheme contains three types of controllers. The first one is a linear feedback optimal controller, which is designed under the condition that no perturbation exists. The second one is an adaptive controller, it is used for adapting the unknown upper bound of perturbation estimation error. The third one is the perturbation estimation mechanism. The property of uniformly ultimately boundness is proved by using Lyapunov stability theorem, and the effects of design parameter on the dynamic performance are also analyzed. An example is demonstrated for showing the feasibility of the proposed control scheme.

1. INTRODUCTION

Reference model has been widely used in the design of control systems (Astrom and Wittenmark [1995], Sastry and Bodson [1989], Ioannou and Sun [1996], Narendra and Annaswamy [1989]). Due to the desired behavior of the controlled system specified by a reference model, one of the advantages of model reference is that the parameters of the controller are adjusted based on an error, which in general is the difference between the outputs of the closed-loop system and the model, so that the tracking precision can be increased. Various adaptive control algorithms using reference model have been developed for counteracting instability and enhancing robustness with respect to model uncertainties and external disturbances. These include Datta and Ioannou [1994], Chou and Cheng [2003], Hua et. al [2006], Miller [2003], and Xie and Li [2006].

The assumption of linear input is that the input of the control system is indeed linearizable. However, due to physical limitation, in practice there do exist nonlinearities in the control input, and their effects cannot be ignored in analysis or controller's design, e.g., saturation, quantization, backlash, deadzone, etc. Time-delay also exists in various dynamic systems due to finite speed of information processing and mechanism of the plant to be controlled. In general the effects of input nonlinearity and time-delay frequently become a source of instability that cannot be ignored during the design of a control system. As a result, the control problems for systems with input nonlinearity and time-delay have received considerable attentions in recent years. Some robust control schemes for uncertain dynamic system with input nonlinearity was proposed in Cheng and Teng [2002], Hsu [1997], Hsu [1998a], Hsu [1998b], Hsu [1998c], Hsu [1998d], Hung and Yan [2007], Shyu et. al [2005], Sun and Hsieh [1995], Sun et. al [1997], Wang et. al [2004], and Yau and Yan [2007].

It is observed that all the robust control schemes mentioned in the second paragraph can only handle matched perturbations. In addition, they also require the information of least upper bounds of perturbations except the method in Cheng and Teng [2002]. It is often noted that these information in general may not be easily obtained in practice because of the complexity of the structure of the system and the perturbations (Zak and Hui [1993]). Therefore, a strategy in which the boundary values on the perturbations can be easily obtained is desired.

Due to the limitations of the control schemes mentioned in the previous paragraph, in this paper we extend the idea of Cheng and Teng [2002] to design a robust model reference adaptive controller for a class of mismatch perturbed MIMO nonlinear systems with input non-linearity and time-delay argument in state, so that the tracking errors can be driven into a small bounded region, whose size, i.e., the tracking precision, can be adjusted through the design parameters. In addition, the uncertainty in the input channel is considered in this paper, and a perturbation estimation mechanism is also embedded in the controller, so that the adaptive gain needs only to overcome the unknown upper bound of perturbation estimation error. The proposed controller is designed without requiring the knowledge of upper bounds of perturbation (except that of the input gain uncertainty), and it has the capability of achieving the property of uniformly ultimately boundness.

2. SYSTEM DESCRIPTIONS AND PROBLEM FORMULATIONS

Consider a class of nonlinear systems whose dynamic equations are governed by

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$$\dot{\mathbf{x}}(t) = [\mathbf{A} + \Delta\mathbf{A}(t,\mathbf{x})]\mathbf{x}(t) + \Delta\mathbf{A}_h(t)\mathbf{x}(t-h(t)) + \big[\mathbf{B} + \Delta\mathbf{B}(t,\mathbf{x})\big]\boldsymbol{\phi}(\mathbf{u}) + \Delta\mathbf{f}\big(t,\mathbf{x}(t),\mathbf{x}(t-h(t))\big), \quad (1)$$
$$\mathbf{x}(t) = \boldsymbol{\theta}(t), \quad -\bar{h} \le t \le 0,$$

where $\mathbf{x}(t) \in R^n$ is the state vector, $\mathbf{u}(t) \in R^m$ is the control input. The constant matrices $\mathbf{A} \in R^{n\times n}$, $\mathbf{B} \in R^{n\times m}$ are known. $\Delta\mathbf{A}(\cdot)$, $\Delta\mathbf{A}_h(\cdot)$, and $\Delta\mathbf{B}(\cdot)$ are unknown real-valued matrix functions representing parameter variations or model uncertainties. The vector $\Delta\mathbf{f}(\cdot)$ denotes the nonlinearity and/or the uncertain extraneous disturbance of the plant. The input nonlinearity, $\boldsymbol{\phi}(\mathbf{u}) \in R^m \to R^m$, is an unknown continuous function. The scalar function $h(t)$ represents the unknown delay argument with the constraint $0 \le h(t) \le \bar{h}$, where $\bar{h}$ is a known constant. The unknown vector $\boldsymbol{\theta}(t)$ is a continuous function and is given for specifying the initial condition. In order to achieve the design of model reference control scheme, a reference model is given by

$$\dot{\mathbf{x}}_m(t) = \mathbf{A}_m\mathbf{x}_m(t) + \mathbf{B}_m\mathbf{r}(t), \qquad \mathbf{x}_m(0) = \mathbf{x}_{m0}, \quad (2)$$

where $\mathbf{x}_m(t) \in R^n$ is the state of the reference model, $\mathbf{r}(t) \in R^r$ denotes piecewise continuous and bounded reference input. The constant matrices $\mathbf{A}_m \in R^{n\times n}$, $\mathbf{B}_m \in R^{n\times r}$ are known, and $\mathbf{A}_m$ is stable.

Before designing the control system, the following assumptions are assumed to be valid in this paper:

A1. Although the input non-linearity $\boldsymbol{\phi}(\mathbf{u})$ is unknown, it can be divided into the nominal (known) part $\boldsymbol{\phi}_n(\mathbf{u})$, and the variant part (which is unknown) $\Delta\boldsymbol{\phi}(\mathbf{u})$, i.e., $\boldsymbol{\phi}(\mathbf{u}) \triangleq \boldsymbol{\phi}_n(\mathbf{u}) + \Delta\boldsymbol{\phi}(\mathbf{u})$. In addition, there exists a positive real constant γ_1 such that the inequality

$$\gamma_1\mathbf{u}^T\mathbf{u} \le \mathbf{u}^T\boldsymbol{\phi}_n(\mathbf{u})$$

is satisfied (Hsu [1998a]-Hsu [1998c]).

A2. The states of plant are all measurable. The pair $(\mathbf{A},\mathbf{B})$ is completely controllable, and the pair $[(\mathbf{A}_m + \alpha\mathbf{I}), -\gamma_1\mathbf{B}]$ is completely stabilizable, where α is real constant.

Remark 1. The Assumption **A1** allows us to consider systems which may contain the square of the control input function (Sun et. al [1997], Sun and Hsieh [1995]).

Remark 2. The nominal part $\boldsymbol{\phi}_n(\mathbf{u})$ is needed only when designing the perturbation estimator. If the perturbation estimator is not needed in the control scheme, then the inequality

$$\gamma_1\mathbf{u}^T\mathbf{u} \le \mathbf{u}^T\boldsymbol{\phi}(\mathbf{u})$$

is assumed in assumption **A1**. Note that in this case the function $\boldsymbol{\phi}(\textbf{.})$ is unknown.

Now define a tracking error $\mathbf{e}(t)$ as

$$\mathbf{e}(t) \triangleq \mathbf{x}_m(t) - \mathbf{x}(t). \quad (3)$$

Differentiating (3) with respect to time and using (1), (2) one can obtain

$$\dot{\mathbf{e}}(t) = \mathbf{A}_m\mathbf{e}(t) - \mathbf{B}\boldsymbol{\phi}_n(\mathbf{u}) + \Delta\mathbf{p}(\mathbf{x},\mathbf{u},t), \quad (4)$$

where

$$\Delta\mathbf{p}(\mathbf{x},\mathbf{u},t) \triangleq (\mathbf{A}_m - \mathbf{A} - \Delta\mathbf{A})\mathbf{x}(t) + \mathbf{B}_m\mathbf{r}(t) - \Delta\mathbf{A}_h\mathbf{x}(t - h(t)) - \mathbf{B}\Delta\boldsymbol{\phi}(\mathbf{u}) - \Delta\mathbf{B}(t,\mathbf{x})\boldsymbol{\phi}(\mathbf{u}) - \Delta\mathbf{f} \quad (5)$$

stands for the lumped perturbation of the system. According to (5), the following assumption is introduced in order to design a perturbation estimator.

A3. There exist a time-varying vector $\mathbf{w}(\|\mathbf{x}\|_s) \in R^{p+1}$, an unknown positive constant scalar k_u, and an unknown constant vector $\mathbf{k} \in R^{p+1}$ such that the following constraint is satisfied:

$$\|\Delta\mathbf{p}(t,\mathbf{x},\mathbf{u}) - \tilde{\epsilon}(t)\Delta\mathbf{p}_{est}(t)\| \le \mathbf{w}^T(t)\mathbf{k} + k_u\|\mathbf{u}(t)\|_2, \quad (6)$$

where $\mathbf{w}(t) \triangleq [1 \quad \|\mathbf{x}\|_s \quad \|\mathbf{x}\|_s^2 \quad \cdots \quad \|\mathbf{x}\|_s^p]^T$, $\mathbf{k} \triangleq [k_0 \quad k_1 \quad k_2 \quad \cdots \quad k_p]^T$, $\|\mathbf{x}\|_s \triangleq \max_{t-\bar{h}\le\tau\le t}\|\mathbf{x}(\tau)\|$, $k_i, 0 \le i \le p$ are unknown positive constants, $0 \le \tilde{\epsilon}(t) < 1$, and $\Delta\mathbf{p}_{est}(t)$ is the perturbation estimator to be designed in the next section.

The main objective of this paper is, without the information of upper bound of perturbation (the information of the upper bound of k_u is still required, it can be seen clearly in next section), to design a robust tracking controller with perturbation estimator embedded, so that not only the controlled system is stable, but also the tracking error $\mathbf{e}(t)$ of (3) can converge to a small bounded region which can be adjusted through the designed parameters.

3. DESIGN OF ROBUST TRACKING CONTROLLER WITH PERTURBATION ESTIMATION

The proposed controller for perturbed system (1) in this paper is designed as

$$\mathbf{u}(t) = \mathbf{u}_1(\mathbf{e}(t)) + \mathbf{u}_2(\mathbf{e}(t)) + \mathbf{u}_3(\mathbf{e}(t)), \quad (7)$$

where

$$\mathbf{u}_1(\mathbf{e}(t)) = \gamma_1\mathbf{B}^T\mathbf{P}\mathbf{e}(t), \quad (8)$$
$$\mathbf{u}_2(\mathbf{e}(t)) = \gamma_1\beta_1(\mathbf{e})\mathbf{B}^T\mathbf{P}\mathbf{e}(t), \quad (9)$$
$$\beta_1(\mathbf{e}) = \frac{\|\mathbf{e}^T(t)\mathbf{P}\|_2^2\xi^2(\mathbf{e})\|\mathbf{e}(t)\|_2^2}{\gamma_1\zeta(\mathbf{e})\big[\|\mathbf{e}(t)\|_2^2 + \epsilon(\mathbf{e})\big]\big[\xi(\mathbf{e})\|\mathbf{e}^T(t)\mathbf{P}\|_2 + \epsilon_1\big]} \quad (10)$$
$$\epsilon(\mathbf{e}) = \begin{cases} \dfrac{\gamma_1^2\|\mathbf{e}^T(t)\mathbf{P}\mathbf{B}\|_2^2}{2\xi(\mathbf{e})\|\mathbf{e}^T(t)\mathbf{P}\|_2}\|\mathbf{e}(t)\|_2^2, & \text{if } \|\mathbf{e}(t)\|_2 \ge \theta_0 \\ \theta_0, & \text{otherwise} \end{cases} \quad (11)$$
$$\zeta(\mathbf{e}) = \gamma_1\|\mathbf{e}^T(t)\mathbf{P}\mathbf{B}\|_2^2 - \bar{\gamma}(\mathbf{e})\|\mathbf{e}^T(t)\mathbf{P}\|_2\|\mathbf{B}^T\mathbf{P}\mathbf{e}(t)\|_2 \quad (12)$$
$$\bar{\gamma}(\mathbf{e}) < \frac{\gamma_1\|\mathbf{e}^T(t)\mathbf{P}\mathbf{B}\|_2}{\|\mathbf{e}^T(t)\mathbf{P}\|_2} \quad (13)$$
$$\xi(\mathbf{e}) = \hat{\mathbf{k}}^T(t)\mathbf{w}(t) + \gamma_1\hat{k}_u(t)(1 + \beta_2(\mathbf{e}))\|\mathbf{B}^T\mathbf{P}\mathbf{e}(t)\|_2 \quad (14)$$
$$\frac{d}{dt}\hat{\mathbf{k}}(t) = -2\rho\boldsymbol{\Gamma}\hat{\mathbf{k}}(t) + \mathbf{w}(t)\|\mathbf{e}^T(t)\mathbf{P}\|_2, \quad \hat{\mathbf{k}}(t_0) = \mathbf{0} \quad (15)$$
$$\frac{d}{dt}\hat{k}_u(t) = -2\alpha_u\hat{k}_u(t) + \alpha_u\gamma_1\|\mathbf{e}^T(t)\mathbf{P}\|_2\big(1 + \beta_2(\mathbf{e})\big) \times\|\mathbf{B}^T\mathbf{P}\mathbf{e}(t)\|_2, \qquad \hat{k}_u(t_0) = 0 \quad (16)$$
$$\mathbf{u}_3(\mathbf{e}(t)) = \gamma_1\beta_2(\mathbf{e})\mathbf{B}^T\mathbf{P}\mathbf{e}(t), \quad (17)$$
$$\beta_2(\mathbf{e}) = \begin{cases} \dfrac{1}{\gamma_1^2}\dfrac{\|\mathbf{e}^T(t)\mathbf{P}\|_2\|\Delta\mathbf{p}_{est}(t)\|_2}{\|\mathbf{e}^T(t)\mathbf{P}\mathbf{B}\|_2^2 + \epsilon_2}, & \text{if } \|\mathbf{e}\|_2 \ge \theta_0 \\ 0, & \text{if } \|\mathbf{e}\|_2 < \theta_0, \end{cases} \quad (18)$$

and $\boldsymbol{\Gamma} \in R^{(p+1)\times(p+1)}$ is a diagonal and positive definite matrix. The positive definite and symmetric matrix $\mathbf{P}$ satisfies the following Riccati equation

$$\mathbf{P}(\mathbf{A}_m + \alpha\mathbf{I}) + (\mathbf{A}_m^T + \alpha\mathbf{I})\mathbf{P} - \gamma_1^2\mathbf{P}\mathbf{B}\mathbf{B}^T\mathbf{P} + \mathbf{Q} = \mathbf{0}, \quad (19)$$

where $\mathbf{Q}$ is also a positive definite and symmetric matrix. α_u, ϵ_1, ϵ_2, ρ and θ_0 are positive constants, all these parameters are specified by the designers.

According to Cheng and Teng [2002], it can be verified that under the assumption **A2**, $\mathbf{u}_1(\mathbf{e}(t))$ is an optimal control of (4) if $\boldsymbol{\phi}_n(\mathbf{u}) = \gamma_1\mathbf{u}$ and the lumped perturbation $\Delta\mathbf{p} = \mathbf{0}$. $\mathbf{u}_2(\mathbf{e}(t))$ is the adaptive control part for adapting the unknown upper bound of perturbation estimation error $\Delta\mathbf{p} - \Delta\mathbf{p}_{est}$. $\hat{\mathbf{k}}(t) \triangleq [\hat{k}_0(t) \quad \hat{k}_1(t)\cdots\hat{k}_p(t)]^T \in R^{p+1}$ and $\hat{k}_u(t)$ are the adaptation gains of the unknown vectors $\mathbf{k}$ and $\hat{k}_u(t)$ respectively. $\mathbf{u}_3(\mathbf{e}(t))$ is the perturbation estimation mechanism.

In order to design the perturbation estimator $\Delta\mathbf{p}_{est}(t)$ embedded in $\mathbf{u}_3(t)$, we adopt the method proposed in Cheng et. al [2001]. First construct a nominal system as

$$\dot{\mathbf{e}}_n(t) = \mathbf{A}_m\mathbf{e}_n(t) - \mathbf{B}\boldsymbol{\phi}_n(\mathbf{u}). \tag{20}$$

Subtracting (20) from (4) one can obtain $\frac{d}{dt}[\mathbf{e}-\mathbf{e}_n] = \mathbf{A}_m[\mathbf{e}-\mathbf{e}_n] + \Delta\mathbf{p}(\mathbf{x},\mathbf{u},t)$, it also means that

$$\Delta\mathbf{p}(\mathbf{x},\mathbf{u},t) = \frac{d}{dt}[\mathbf{e}-\mathbf{e}_n] - \mathbf{A}_m[\mathbf{e}-\mathbf{e}_n]. \tag{21}$$

Equation (21) suggests that the perturbation estimator can be realized by a filter with transfer function as

$$\Delta\mathbf{p}_{est}(s) \triangleq \begin{cases} \left(\frac{s}{1+\delta s}\mathbf{I} - \mathbf{A}_m\right)\big[\mathbf{e}(s) - \mathbf{e}_n(s)\big], & \|\mathbf{e}\|_2 \ge \theta_0 \\ 0, & \text{otherwise} \end{cases} \tag{22}$$

where δ is a small designed positive real number.

Now the following theorem shows that the proposed control scheme can achieve the property of uniformly ultimately boundness.

Theorem 1. Consider the dynamic system (1) and the reference model (2) with the proposed controller (7). If all the aforementioned Assumptions **A1-A3** are satisfied, and

$$k_u \le \bar{k}_u \le \bar{\gamma}(\mathbf{e}) < \frac{\gamma_1\|\mathbf{e}^T(t)\mathbf{PB}\|_2}{\|\mathbf{e}^T(t)\mathbf{p}\|_2}, \tag{23}$$

where $\bar{k}_u$ is the least upper bound of k_u, then the tracking error $\mathbf{e}(t)$ and the adaptive gains $\hat{\mathbf{k}}(t)$, $\hat{k}_u(t)$ are uniformly ultimately bounded. In addition, the signals $\beta_1(\mathbf{e}), \epsilon(\mathbf{e}), \xi(\mathbf{e}), \beta_2(\mathbf{e})$, $\mathbf{x}(t)$, $\beta_3(\mathbf{e})$, $\mathbf{u}(t)$, and $\Delta\mathbf{p}_{est}(t)$ are all bounded.

Proof: First define a Lyapunov function candidate as

$$V(\mathbf{e},\tilde{\mathbf{k}},\tilde{k}_u) = \frac{1}{2}\mathbf{e}^T(t)\mathbf{P}\mathbf{e}(t) + \frac{1}{2}\tilde{\mathbf{k}}(t)^T\boldsymbol{\Gamma}^{-1}\tilde{\mathbf{k}}(t) + \frac{1}{2\alpha_u}\tilde{k}_u^2(t),$$

where $\tilde{\mathbf{k}}(t) \triangleq \hat{\mathbf{k}}(t) - \mathbf{k}$ and $\tilde{k}_u(t) \triangleq \hat{k}_u(t) - k_u$ are the adaptive errors. By noting that $\dot{\tilde{\mathbf{k}}}(t) = \dot{\hat{\mathbf{k}}}(t)$, $\dot{\tilde{k}}_u(t) = \dot{\hat{k}}_u(t)$, and using (4), one can obtain

$$\begin{aligned}\frac{d}{dt}V = {} & \mathbf{e}^T\big[\mathbf{PA}_m + \mathbf{A}_m^T\mathbf{P}\big]\mathbf{e}/2 - \mathbf{e}^T\mathbf{PB}\boldsymbol{\phi}_n(\mathbf{u}) + \mathbf{e}^T\mathbf{P}\Delta\mathbf{p}(\mathbf{x},\mathbf{u},t) \\ & + \tilde{\mathbf{k}}^T(t)\boldsymbol{\Gamma}^{-1}\dot{\tilde{\mathbf{k}}}(t) + \tilde{k}_u(t)\dot{\tilde{k}}_u(t)/\alpha_u.\end{aligned} \tag{24}$$

According to (7), (8), (9), and (17), it can be seen that

$$\mathbf{u} = \mathbf{u}_1 + \mathbf{u}_2 + \mathbf{u}_3 = \big[\gamma_1 + \gamma_1\beta_1(\mathbf{e}) + \gamma_1\beta_2(\mathbf{e})\big]\mathbf{B}^T\mathbf{Pe}. \tag{25}$$

Based on (7) and the assumption **A1**, one can easily identify that

$$\gamma_1\mathbf{u}^T\mathbf{u} = \gamma_1\big[\gamma_1 + \gamma_1\beta_1(\mathbf{e}) + \gamma_1\beta_2(\mathbf{e})\big]^2\mathbf{e}^T\mathbf{PBB}^T\mathbf{Pe} \le \mathbf{u}^T\boldsymbol{\phi}_n(\mathbf{u}). \tag{26}$$

Using the similar technique as shown in Hsu [1998a] - Hsu [1998c], from (7), (25), and (26) one can verify that

$$\begin{aligned}& -\mathbf{e}^T\mathbf{PB}\boldsymbol{\phi}_n(\mathbf{u}) \\ & = -\big[\gamma_1 + \gamma_1\beta_1(\mathbf{e}) + \gamma_1\beta_2(\mathbf{e})\big]^{-1}\mathbf{u}^T\boldsymbol{\phi}_n(\mathbf{u}) \\ & \le -\big[\gamma_1 + \gamma_1\beta_1(\mathbf{e}) + \gamma_1\beta_2(\mathbf{e})\big]^{-1}\gamma_1\mathbf{u}^T\mathbf{u} \\ & = -\gamma_1^2\mathbf{e}^T\mathbf{PBB}^T\mathbf{Pe} - \gamma_1^2\beta_1(\mathbf{e})\|\mathbf{e}^T\mathbf{PB}\|_2^2 - \gamma_1^2\beta_2(\mathbf{e})\|\mathbf{e}^T\mathbf{PB}\|_2^2.\end{aligned}$$

According to the previous inequality, from (24) one can obtain

$$\begin{aligned}\frac{d}{dt}V \le {} & \mathbf{e}^T\big[\mathbf{PA}_m + \mathbf{A}_m^T\mathbf{P}\big]\mathbf{e}/2 - \big[\gamma_1^2\mathbf{e}^T\mathbf{PBB}^T\mathbf{Pe} + \gamma_1^2\beta_1(\mathbf{e}) \\ & \times\|\mathbf{e}^T\mathbf{PB}\|_2^2 + \gamma_1^2\beta_2(\mathbf{e})\|\mathbf{e}^T\mathbf{PB}\|_2^2\big] + \mathbf{e}^T\mathbf{P}\Delta\mathbf{p}(\mathbf{x},\mathbf{u},t) \\ & + \tilde{\mathbf{k}}^T(t)\boldsymbol{\Gamma}^{-1}\dot{\tilde{\mathbf{k}}}(t) + \tilde{k}_u(t)\dot{\tilde{k}}_u(t)/\alpha_u \\ \le {} & -\alpha\mathbf{e}^T\mathbf{Pe} - \gamma_1^2\|\mathbf{e}^T\mathbf{PB}\|_2^2/2 - \gamma_1^2\beta_1(\mathbf{e})\|\mathbf{e}^T\mathbf{PB}\|_2^2 \\ & - \gamma_1^2\beta_2(\mathbf{e})\|\mathbf{e}^T\mathbf{PB}\|_2^2 + \|\mathbf{e}^T\mathbf{P}\|_2\,\|\Delta\mathbf{p}(\mathbf{x},\mathbf{u},t)\|_2 \\ & + \tilde{\mathbf{k}}^T(t)\boldsymbol{\Gamma}^{-1}\dot{\tilde{\mathbf{k}}}(t) + \tilde{k}_u(t)\dot{\tilde{k}}_u(t)/\alpha_u.\end{aligned} \tag{27}$$

Using (18) and (6), one can simplify the two terms in (27) as

$$\begin{aligned}& -\gamma_1^2\beta_2(\mathbf{e})\|\mathbf{e}^T\mathbf{PB}\|_2^2 + \|\mathbf{e}^T\mathbf{P}\|_2\,\|\Delta\mathbf{p}(\mathbf{x},\mathbf{u},t)\|_2 \\ & = \|\mathbf{e}^T\mathbf{P}\|_2 \times \left[\|\Delta\mathbf{p}(\mathbf{x},\mathbf{u},t)\|_2 - \frac{\|\mathbf{e}^T\mathbf{PB}\|_2^2}{\|\mathbf{e}^T\mathbf{PB}\|_2^2 + \epsilon_2}\|\Delta\mathbf{p}_{est}\|_2\right] \\ & \le \|\mathbf{e}^T\mathbf{P}\|_2 \times \big[\mathbf{w}^T(t)\mathbf{k} + k_u\|\mathbf{u}(t)\|_2\big].\end{aligned}$$

According to the previous inequality, one can further derive (27) by using (7), (15), and (16) as

$$\begin{aligned}\frac{d}{dt}V \le {} & -\alpha\mathbf{e}^T\mathbf{Pe} - \gamma_1^2\|\mathbf{e}^T\mathbf{PB}\|_2^2/2 - \gamma_1^2\beta_1(\mathbf{e})\|\mathbf{e}^T\mathbf{PB}\|_2^2 \\ & + \gamma_1\|\mathbf{e}^T\mathbf{P}\|_2 k_u\beta_1(\mathbf{e})\|\mathbf{B}^T\mathbf{Pe}\|_2 + \tilde{\mathbf{k}}^T(t)\big(-2\rho\hat{\mathbf{k}}(t) \\ & + \mathbf{w}(t)\|\mathbf{e}^T\mathbf{P}\|_2\big) + 2\rho\mathbf{k}^T\hat{\mathbf{k}}(t) + \hat{k}_u(t)\big[-2\hat{k}_u(t) \\ & + \gamma_1\|\mathbf{e}^T\mathbf{P}\|_2(1+\beta_2(\mathbf{e}))\|\mathbf{B}^T\mathbf{Pe}\|_2\big] + 2k_u\hat{k}_u(t)\end{aligned} \tag{28}$$

Now using (13), (14), and (23), one can further simplify (28) as

$$\begin{aligned}\frac{d}{dt}V \le {} & -\alpha\mathbf{e}^T\mathbf{Pe} - \gamma_1^2\|\mathbf{e}^T\mathbf{PB}\|_2^2/2 \\ & - \gamma_1\beta_1(\mathbf{e})\left(\gamma_1\|\mathbf{e}^T\mathbf{PB}\|_2^2 - \bar{\gamma}(t)\|\mathbf{e}^T\mathbf{P}\|_2\|\mathbf{B}^T\mathbf{Pe}\|_2\right) \\ & - 2\rho\hat{\mathbf{k}}^T(t)\hat{\mathbf{k}}(t) + 2\rho\mathbf{k}^T\hat{\mathbf{k}}(t) - 2\hat{k}_u^2(t) + 2k_u\hat{k}_u(t) \\ & + \left(\hat{\mathbf{k}}^T\mathbf{w} + \gamma_1\hat{k}_u(t)(1+\beta_2(\mathbf{e}))\|\mathbf{B}^T\mathbf{Pe}(t)\|_2\right)\|\mathbf{e}^T\mathbf{P}\|_2 \\ = {} & -\alpha\mathbf{e}^T\mathbf{Pe} - \gamma_1^2\|\mathbf{e}^T\mathbf{PB}\|_2^2/2 - \gamma_1\zeta(\mathbf{e})\beta_1(\mathbf{e}) \\ & - 2\rho\hat{\mathbf{k}}^T(t)\hat{\mathbf{k}}(t) + 2\rho\mathbf{k}^T\hat{\mathbf{k}}(t) - 2\hat{k}_u^2(t) + 2k_u\hat{k}_u(t) \\ & + \xi(\mathbf{e})\|\mathbf{e}^T\mathbf{P}\|_2.\end{aligned} \tag{29}$$

Since it is known that $0 \le ab/(a+b) \le b$, $\forall\, a,b > 0$, by using (10) and (11), one can simplify three terms in (29) under the condition $\|\mathbf{e}\|_2 \ge \theta_0$ as

$$\begin{aligned}& -\gamma_1^2\|\mathbf{e}^T\mathbf{PB}\|_2^2/2 - \gamma_1\zeta(\mathbf{e})\beta_1(\mathbf{e}) + \xi(\mathbf{e})\|\mathbf{e}^T\mathbf{P}\|_2 \qquad (30) \\ & = -\frac{1}{2}\gamma_1^2\|\mathbf{e}^T\mathbf{PB}\|_2^2 - \frac{\|\mathbf{e}^T\mathbf{P}\|_2^2}{\|\mathbf{e}\|_2^2 + \epsilon(\mathbf{e})} \times \frac{\xi^2(\mathbf{e})\|\mathbf{e}\|_2^2}{\xi(\mathbf{e})\|\mathbf{e}^T\mathbf{P}\|_2 + \epsilon_1} \\ & + \frac{\|\mathbf{e}\|_2^2 + \epsilon(\mathbf{e})}{\|\mathbf{e}\|_2^2 + \epsilon(\mathbf{e})}\xi(\mathbf{e})\|\mathbf{e}^T\mathbf{P}\|_2\end{aligned}$$

$$= -\frac{1}{2}\gamma_1^2\|\mathbf{e}^T\mathbf{PB}\|_2^2 + \frac{\|\mathbf{e}\|_2^2}{\|\mathbf{e}\|_2^2+\epsilon(\mathbf{e})} \times \frac{\epsilon_1\xi(\mathbf{e})\|\mathbf{e}^T\mathbf{P}\|_2}{\xi(\mathbf{e})\|\mathbf{e}^T\mathbf{P}\|_2+\epsilon_1}$$
$$+\frac{\|\mathbf{e}\|_2^2\epsilon(\mathbf{e})}{\|\mathbf{e}\|_2^2+\epsilon(\mathbf{e})} \times \frac{\xi(\mathbf{e})\|\mathbf{e}^T\mathbf{P}\|_2}{\|\mathbf{e}\|_2^2}$$
$$\leq -\frac{1}{2}\gamma_1^2\|\mathbf{e}^T\mathbf{PB}\|_2^2 + \frac{\|\mathbf{e}\|_2^2\epsilon_1}{\|\mathbf{e}\|_2^2+\epsilon(\mathbf{e})} + \epsilon(\mathbf{e}) \times \frac{\xi(\mathbf{e})\|\mathbf{e}^T\mathbf{P}\|_2}{\|\mathbf{e}\|_2^2}$$
$$\leq -\frac{1}{2}\gamma_1^2\|\mathbf{e}^T\mathbf{PB}\|_2^2 + \epsilon_1 + \frac{\gamma_1^2\|\mathbf{e}^T\mathbf{PB}\|_2^2\,\|\mathbf{e}\|_2^2}{2\xi(\mathbf{e})\|\mathbf{e}^T\mathbf{P}\|_2} \times \frac{\xi(\mathbf{e})\|\mathbf{e}^T\mathbf{P}\|_2}{\|\mathbf{e}\|_2^2}$$
$$= \epsilon_1.$$

Hence according to the result of (30), when $\|\mathbf{e}(t)\|_2 \geq \theta_0$, one can again further simplify (29) as

$$\frac{d}{dt}V \leq -\alpha\mathbf{e}^T\mathbf{Pe} - 2\rho[\tilde{\mathbf{k}}(t)+\mathbf{k}]^T[\tilde{\mathbf{k}}(t)+\mathbf{k}] + 2\rho\mathbf{k}^T\hat{\mathbf{k}}(t)$$
$$-2[\tilde{k}_u(t)+k_u]^2 + 2k_u\hat{k}_u(t) + \epsilon_1$$
$$= \left[-\alpha\mathbf{e}^T\mathbf{Pe} - 2\rho\tilde{\mathbf{k}}^T(t)\tilde{\mathbf{k}}(t) - 2\tilde{k}_u^2(t)\right] - 2\rho\hat{\mathbf{k}}^T(t)\mathbf{k}$$
$$+2\rho\mathbf{k}^T\mathbf{k} - 2\hat{k}_u(t)k_u + 2k_u^2 + \epsilon_1$$
$$\leq -2\nu\left[\mathbf{e}^T\mathbf{Pe}/2\right] - 2\nu\left[\tilde{\mathbf{k}}^T(t)\mathbf{\Gamma}^{-1}\tilde{\mathbf{k}}(t)/2\right]$$
$$-2\nu\left[\tilde{k}_u^2(t)/(2\alpha_u)\right] + 2\rho\mathbf{k}^T\mathbf{k} + 2k_u^2 + \epsilon_1$$
$$= -2\nu V + 2\rho\mathbf{k}^T\mathbf{k} + 2k_u^2 + \epsilon_1,$$

where $\nu \triangleq min\{\alpha,\ 2\alpha_u,\ 2\rho\min(diag(\mathbf{\Gamma}))\}$. Now define $\boldsymbol{\omega} \triangleq [\mathbf{e}^T\ \ \tilde{\mathbf{k}}^T\ \ \tilde{k}_u]^T$, then one can see that

$$\lambda_0\|\boldsymbol{\omega}\|_2^2 \leq V(\boldsymbol{\omega}) \leq \lambda_2\|\boldsymbol{\omega}\|_2^2, \qquad \dot{V}(\boldsymbol{\omega}) \leq -\kappa\|\boldsymbol{\omega}\|_2^2$$

are satisfied for all

$$\|\boldsymbol{\omega}\|_2 \geq \sqrt{\frac{2\rho\mathbf{k}^T\mathbf{k} + 2k_u^2 + \epsilon_1}{2\nu\lambda_0 - \kappa}}. \tag{31}$$

where $\lambda_0 \triangleq \frac{1}{2}min\{\lambda_{min}(\mathbf{P}), \lambda_{min}(\mathbf{\Gamma}^{-1}), \alpha_u^{-1}\}$, $\lambda_2 \triangleq max\{\lambda_{max}(\mathbf{P}), \lambda_{max}(\mathbf{\Gamma}^{-1}), \alpha_u^{-1}\}$, and $0 < \kappa < 2\nu\lambda_0$.

Therefore, according to Theorem 3.3.3 in Ioannou and Sun [1996] (page 111), when $\|\mathbf{e}(t)\|_2 \geq \theta_0$, the solutions of the dynamic equations (4), (15), and (16) are uniformly ultimately bounded, i.e., $\mathbf{e}(t)$, $\tilde{\mathbf{k}}(t)$, and $\tilde{k}_u(t)$ (and hence $\hat{\mathbf{k}}(t)$, $\hat{k}_u(t)$) are uniformly ultimately bounded. According to (16), one can see that the boundedness of $\hat{k}_u(t)$ and $\mathbf{e}(t)$ implies the boundedness of $\beta_2(\mathbf{e})$ since $\alpha_u > 0$, and the boundedness of $\hat{k}_u(t)$, $\beta_2(\mathbf{e})$, $\hat{\mathbf{k}}(t)$, $\bar{\gamma}(\mathbf{e})$, and $\mathbf{e}(t)$ implies the boundedness of $\mathbf{x}(t)$, $\xi(\mathbf{e})$, and $\zeta(\mathbf{e})$ in accordance with (3), (14) and (12) respectively. Hence, from (11), (10), and (18) it is seen that $\epsilon(\mathbf{e})$, $\beta_1(\mathbf{e})$, $\Delta\mathbf{p}_{est}(t)$ are bounded. One can also conclude from (8), (9) and (17) that the control input function $\mathbf{u}(t)$ is bounded.

When $\|\mathbf{e}(t)\|_2 < \theta_0$, from (3) and (2) it is known that $\mathbf{x}(t)$ is bounded since $\mathbf{x}_m(t)$ is bounded, and from (22) $\Delta\mathbf{p}_{est}(t)$ is also bounded. According to (8), (18), (12), (15), and (16), one can easily see that $\mathbf{u}_1(\mathbf{e}(t))$, $\beta_2(\mathbf{e})$, $\zeta(\mathbf{e})$, $\hat{\mathbf{k}}(t)$, $\hat{k}_u(t)$ are all bounded respectively. It also implies that $\xi(\mathbf{e})$ and $\mathbf{u}_3(\mathbf{e})$ are bounded in accordance with (14) and (17). Hence according to (10) and (11) one can conclude that $\beta_1(\mathbf{e})$ and $\epsilon(\mathbf{e})$ are bounded, i.e., $\mathbf{u}_2(\mathbf{e})$ is bounded. The foregoing analysis shows that all the signals in the proposed control scheme are bounded. □

There are at least six parameters θ_0, ρ, ν, ϵ_1, ϵ_2, and λ_0 which can be adjusted by the designer to improve the dynamic performance of the controlled system. In general, smaller θ_0 can improve tracking accuracy directly, however, it might also enhance the chattering phenomenon of the control inputs. (31) indicates that the values of ρ, ϵ_1, ν and λ_0 will also affect the control precision. The smaller the value of ρ or ϵ_1(or the larger the value of ν or λ_0) is used, the higher the tracking precision one can obtain. It is also observed that reducing the value of ρ will decrease the adaptive speed of $\hat{\mathbf{k}}(t)$. On the other hand, one can know from (18) that reducing the value of ϵ_2 will in general increase the magnitude of $\mathbf{u}_3$.

Remark 3. According to Theorem 1, the value of $\bar{\gamma}(\mathbf{e})$ has to satisfy (23). This reveals that the upper bound of k_u, i.e., $\bar{k}_u$, should be known. Note also in the case of $\mathbf{e}(t) = \mathbf{0}$, from (8), (9), and (17), one can see that $\mathbf{u}(t) = \mathbf{0}$ no matter what the value of $\bar{\gamma}(t)$ is, and the system is still stable in accordance with Theorem 1.

Remark 4. If the perturbation estimation is not needed, then one can simply let $\Delta\mathbf{p}_{est}(t) = \mathbf{0}$. From (17) and (18) it implies that $\mathbf{u}_3 = \mathbf{0}$. In this case it is easy to show that all the signals in the controlled system are still bounded by following the similar procedure as proved in Theorem 1.

4. SUMMARY OF DESIGN PROCEDURE

Based on the analysis and design presented in the previous section, the design procedure of the proposed control scheme is summarized as follows:

Step 1:
Choose an appropriate constant $\alpha > 0$ and matrix $\mathbf{Q} = \mathbf{Q}^T > 0$, then solve the Riccati equation (19) to find the matrix $\mathbf{P}$.

Step 2:
Assign each value of θ_0, ρ, $\mathbf{\Gamma}$, α_u, ϵ_1, ϵ_2, then design the proposed controller in accordance with (7) to (18).

Step 3:
Use (22) to design the perturbation estimator.

5. EXAMPLE

In this section an example is given for demonstrating the feasibility of the proposed control scheme presented in Section 3. The dynamic equations of this example are the same as those in (1) with

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -2 & 3 \\ 0 & -4 & 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

In order to demonstrate the robustness of the proposed control scheme, and to perform the computer simulation, it is assumed that

$$\Delta\mathbf{A} = \begin{bmatrix} 0.1x_2x_3 & 0 & 0 \\ 0 & 0.2 & 0.4\cos(t) \\ 0 & 0.1\sin(t) & 0.3 \end{bmatrix},$$

$$\Delta\mathbf{A}_h = \begin{bmatrix} 0 & 0 & -0.2\cos(t) \\ 2 & (1-0.2\cos(t)) & 0.1 \\ 2 & 0.2 & -0.1\sin(0.5t) \end{bmatrix},$$

$$\Delta\mathbf{B} = \begin{bmatrix} 0 & -0.1x_2x_3 \\ -0.1\cos(2t) & 0 \\ 0 & -(0.2+0.1\sin(t)) \end{bmatrix},$$

$$\Delta\mathbf{f} = \begin{bmatrix} 0.1x_2^2x_3\sin(t) + 0.08u_1 \\ \cos(t)x_3(t-h(t)) + 0.5u_2\sin(t) \\ \sin^2(t)x_2^2(t-h(t)) + 0.5u_2 \end{bmatrix},$$

$$\boldsymbol{\phi}(\mathbf{u})=\begin{bmatrix} [0.2\cos(u_2)+\exp(|\cos(u_1+u_2)|)]u_1 \\ [1+0.3\sin(u_1+u_2)+0.2\exp(1+\sin(u_2))]u_2 \end{bmatrix},$$

$$h(t)=0.2+0.1\cos(t).$$

Note that in this example we let $\boldsymbol{\phi}_n(\mathbf{u}) = [u_1 \quad u_2]^T$, and $\gamma_1 = 0.5$ will satisfy the assumption **A1**.

The reference model is given by

$$\begin{bmatrix}\dot{x}_{m1}\\ \dot{x}_{m2}\\ \dot{x}_{m3}\end{bmatrix}=\begin{bmatrix}-1 & 1 & 0\\ 0 & -2 & 3\\ 0 & -5 & -2\end{bmatrix}\begin{bmatrix}x_{m1}\\ x_{m2}\\ x_{m3}\end{bmatrix}+\begin{bmatrix}0 & 0\\ 1 & 1\\ 0 & 1\end{bmatrix}\begin{bmatrix}r_1(t)\\ r_2(t)\end{bmatrix},$$

and the reference signals are $r_1(t) = 0.2\cos(2t)$ and $r_2(t) = 0.4\sin(t)$

The objective of control is to use the proposed control technique presented in Section 3 to design a model reference adaptive controller with perturbation estimation so that the states $x_1(t)$, $x_2(t)$, and $x_3(t)$ can track the desired reference signals $x_{m1}(t)$, $x_{m2}(t)$, and $x_{m3}(t)$ respectively. The following is the detailed design procedure of the proposed control scheme.

1. Solve the Riccati Equation

According to (19), the matrix $\mathbf{P}$ is solved with $\gamma_1 = 0.5, \alpha = 3$, $\mathbf{Q} = \mathbf{I}$. By using the software MATLAB, one can obtain

$$\mathbf{P}=\begin{bmatrix}364.0746 & 29.8318 & 40.4481\\ 29.8318 & 10.1308 & 2.1756\\ 40.4481 & 2.1756 & 8.9009\end{bmatrix}.$$

2. Design of controller

The control efforts of the proposed control scheme is designed as (7) to (18), where we set the design parameters as $(\gamma_1, \rho, \epsilon_1, \epsilon_2, \delta, \theta_0, \alpha_u, \bar{\kappa}) = (0.5, 10, 20, 40, 0.02, 1, 20, 0.5)$ and

$$\boldsymbol{\Gamma}=\begin{bmatrix}0.4 & 0 & 0\\ 0 & 0.4 & 0\\ 0 & 0 & 0.2\end{bmatrix}. \qquad (32)$$

Note that $\nu = 3$, $\lambda_0 = 0.025$.

3. Design of perturbation estimator

The perturbation estimator is designed in accordance with (22).

The results of simulation (with step time 0.1m sec, and initial condition $\boldsymbol{\theta}(t) = [0.2e^{-t}, -0.3e^{-t}, -0.3e^{-t}]^T$, $-\bar{h} \le t \le 0$, $\bar{h} = 0.3$, $\hat{\mathbf{k}}(0) = [0, 0, 0]^T$) are shown form Fig. 1 to Fig. 4. Fig. 1 shows the tracking errors e_1, e_2, and e_3, which are all driven into a small bounded region respectively. Fig. 2 displays the two control input functions u_1 and u_2 respectively, note that there is no chattering phenomenon in this case. The adaptive gains $\hat{\mathbf{k}}$ and $\hat{k}_u$ are illustrated in Fig. 3 and Fig. 4 respectively, which are all bounded as expected.

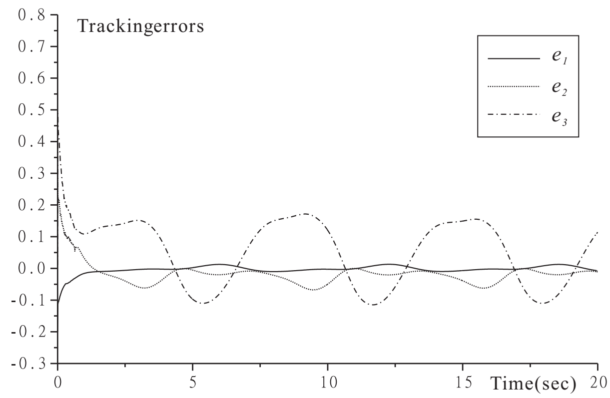

Fig. 1. Tracking error **e**.

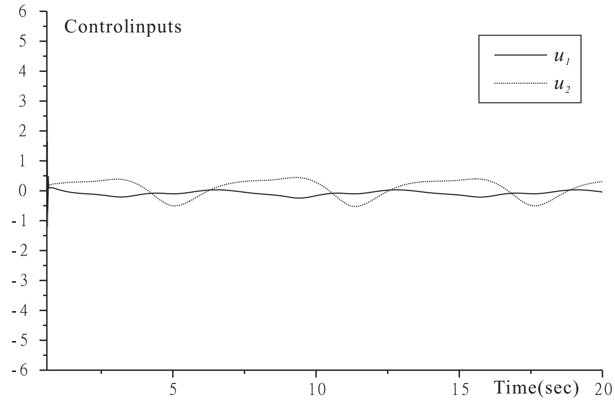

Fig. 2. Control input **u**.

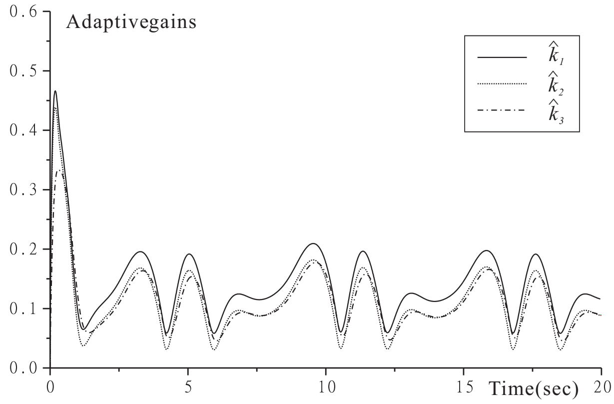

Fig. 3. Adaptive gain $\hat{\mathbf{k}}$.

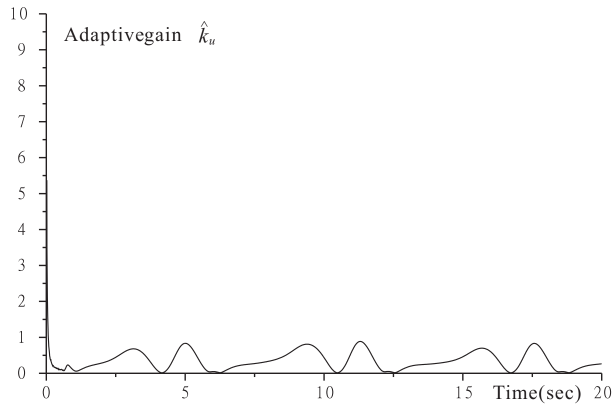

Fig. 4. Adaptive gain $\hat{k}_u$.

6. CONCLUSIONS

In this paper a model reference adaptive controller with perturbation estimation embedded is successfully proposed for a class of perturbed multi-input systems with mismatched time-varying state delay and input nonlinearity. Since there is a perturbation estimator embedded in the proposed control scheme, not only the information of the upper bound of perturbation as well as perturbation estimation error is not required, but also the control energy in general will smaller than the case without using perturbation estimator. The control precision can be increased by decreasing the value of θ_0 directly, however, it might increase the chattering phenomenon in control input, and might also increase the magnitudes of $\mathbf{u}_1$ and $\mathbf{u}_2$.

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