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Control with guaranteed performance for dual-rate sampled-data systems under stochastic disturbances

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Abstract: The paper considers the digital control of a continuous-time process under stochastic disturbances. The problem consists in finding a controller that guarantees a certain performance of the closed loop, while the disturbances belong only to a given class of disturbances. The digital controller has two parts with different sampling rates, where one rate is a multiple of the other one. The solution is found by applying the parametric transfer function (PTF) concept and the method of guaranteed performance control.

Keywords: Sampled-data systems, Multirate, Stabilizing controllers, Stochastic control, Performance analysis

1. INTRODUCTION

The synthesis of controllers with guaranteed performance has been established as modern direction in control system design. Here the searched controller has to guarantee a certain performance of the closed loop, while the system is excited by stochastic disturbances belonging to a given class. According to continuous LTI systems, the solution method for this problem bases on applying the transfer function concept, and could be find in (Nebylov 2004).

The extension of this approach to single-rate sampled-data systems by applying the parametric transfer function (PTF) concept was done in (Rybinskii and Lampe 2001)-(Rybinskii, Lampe and Rosenwasser 2004). The present paper generalizes the results of (Rybinskii and Lampe 2001)-(Rybinskii et al. 2004) to dual-rate sampled-data systems containing two digital controllers acting on continuous processes. At this we assume that the sampling period of one controller is an integer multiple of the period of the other one, and the higher-rated controller is designed at first. In this case the solution of the control problem with guaranteed performance leads to the design problem for a certain equivalent single-rate system with the period of the slower controller.

The investigation of multirate sampled-data systems is very actual, because in many modern control systems, we have various microcontrollers working with different rates. Making all controllers working with the same sampling rate would be an unnecessary constraint. On the other side, it is well known that multirate sampling needs higher effort in modelling and design of the control system (Ragazzini and Franklin 1958).

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Sampled-data systems of this class are described in literature mostly for the case, where the sampling periods are commensurable, i.e. when we can find a common period for all controllers. So the system becomes periodic and this makes its investigation easier.

Various approaches to analysis and synthesis of multirate systems are presented in (Kranc 1957, Kalman and Bertram 1959, Araki and Yamamoto 1986, Al-Rahmani and Franklin 1990, Colaneri, Scattolini and Schiavoni 1990, Rosenwasser and Lampe 2000). The design problem for sampled-data systems of this class show essential specific properties which do not allow to apply directly the methods developed for single-rate sampled-data systems (see (Chen and Francis 1995)). The papers (Meyer 1990, Ravi, Khargonekar, Minto and Nett 1990) investigate continuous LTI processes with a central multirate digital controller. Here the parameterized set of stabilizing controllers is constructed in a similar way as the well known Youla-Kučera parameterization with sufficient constraints to ensure the causality of the control algorithm. The papers (Berg, Amit and Powell 1988, Lennartson 1988, Meyer 1992, Voulgaris and Bamieh 1993, Qiu and Chen 1994, Colaneri and De Nicolao 1995) provide the solution of the LQR/LQG-problem for multirate systems of general form, while the $\mathcal{H}_\infty$-problem is considered in (Voulgaris and Bamieh 1993, Chen and Qiu 1994, Sågfors, Toivonen and Lennartson 1998).

The present paper deals with the problem of stochastic control with guaranteed performance, when we have to diminish the influence of external stochastic disturbances ensuring stability of the system. At this in contrast to the traditional problem statement, we assume that the spectral density S_g of the excitation $g(t)$ is not exactly known, but it belongs to a given class $\mathcal{S}$ of known characteristics.

It is supposed that the performance of the system is characterized by a certain cost function $J(S_g, C)$, depending on the spectral density S_g and the controller C. In the paper at hand, we use as cost function a weighted sum of the expected mean variance of the output signal under stationary stochastic excitation at the system input. Then the optimization problem can symbolically be written in the form

$$C = \operatorname*{arginf}_{C \in \mathcal{C}} \sup_{S_g \in \mathcal{S}} J(S_g, C) . \tag{1}$$

Assume C_* be a solution of (1), i.e.

$$J_* = \sup_{S_g \in \mathcal{S}} J(S_g, C_*) .$$

Then it can be guaranteed that for all excitations satisfying $S_g \in \mathcal{S}$, the cost function does not exceed J_*. Therefore, the corresponding control algorithm realizing this solution of problem (1), will be called *performance guaranteing control*.

For the calculation of the cost function $J(S, C)$, we will use the parametric transfer function (PTF) approach and the parametric frequency response (PFR), which are able to consider the properties of multirate sampled data systems in continuous time (Rosenwasser and Lampe 2000, Rosenwasser and Lampe 2006).

The paper is organized as follows. Section 2 provides the statement of the optimization problem for performance guaranteing control. In section 3 we derive a close expression for the cost function using expressions for the PTF of the system. Closed expressions for the PTFs of dual-rate systems are presented in section 4. They open possibilities for practical calculations. In section 5 is shown that under the taken suppositions, the stabilization problem for dual-rate systems can be reduced to the stabilization problem for a corresponding single-rate system, for which the parameterized set of stabilizing controllers is constructed having the period of the system. Section 6 addresses special aspects of the numerical search of the optimal solution and section 7 presents an illustrative numerical example.

2. PROBLEM

Consider the dual-rate sampled-data system with the structure shown in Fig. 1. Solid and dashed lines in Fig. 1 indicate analogue and digital signals, respectively. The system contains analogue elements with rational transfer functions $F_1(s)$, $F_2(s)$ and $P(s)$ as well as two digital blocks (in the dashed boxes), working with different sampling periods $T_1 \neq T_2$. However, there exists an integer $N > 1$ such that one of the relations $T_1 = NT_2$ or $T_2 = NT_1$ is fulfilled. For concreteness, let us assume

$$T_1 = NT_2 .$$

Therefore, the whole system is periodic with the period $T = T_1$.

The digital control algorithm is given by a linear difference equation associated with the discrete transfer function (see (Rosenwasser and Lampe 2000))

$$C_i(s) = \frac{\sum_{r=0}^{q_i} \beta_{ir} e^{-rT_i s}}{\sum_{r=0}^{q_i} \alpha_{ir} e^{-rT_i s}} , \qquad i = 1, 2 , \tag{2}$$

where $q_i\,(i = 1, 2)$ are nonnegative integers and β_{ir}, α_{ir} are constant coefficients, hereby $\alpha_{i0} \neq 0$. Hereinafter, the number q_i is called the order of the controller C_i. The blocks with the transfer functions $H_i(s)\,(i = 1, 2)$ model the hold devices, converting the discrete-time signals into the analogue inputs to the continuous-time processes.

Furthermore, we assume that all blocks apart from $C_1(s)$, including the transfer function of fast-rate controller $C_2(s)$, are given. The excitation signal $g(t)$ is acting on the input of the continuous process $P(s)$. As output of the system, entering into the performance, we consider the signals $y(t)$ and $u(t)$.

Let $g(t)$ be a stationary centered stochastic excitation with known spectral density. When the system is stable, then the centered signals $y(t)$ and $u(t)$ in the stationary mode can be characterized by its variances

$$v_y(t) = \mathsf{E}\left\{y^2(t)\right\} , \quad v_u(t) = \mathsf{E}\left\{u^2(t)\right\} ,$$

where $\mathsf{E}\{\cdot\}$ symbolizes the mathematical expectation. Since the system is T-periodic, the variances are also time-periodic:

$$v_y(t) = v_y(t + T) , \quad v_u(t) = v_u(t + T) .$$

For the cost function, we apply

$$J = \bar{v}_y + \rho^2\, \bar{v}_u . \tag{3}$$

This quantity characterize the dynamics of the system in continuous time. Herein, ρ^2 is a nonnegative real weighting coefficient, and

$$\bar{v}_y = \frac{1}{T}\int_0^T v_y(t)\,\mathrm{d}t , \quad \bar{v}_u = \frac{1}{T}\int_0^T v_u(t)\,\mathrm{d}t$$

are the *mean variances* of the signals $y(t)$ and $u(t)$, respectively.

The classical stochastic optimization problem consists in finding a controller $C_1(s)$, which ensures the stability of the system and minimizes cost function (3). The cost function can be presented in the form $J(S_g, C_1)$, where $S_g(\nu)$ is the spectral density of the signal $g(t)$ as a function of the frequency ν. However, in many cases the spectral density $S_g(\nu)$ is not exactly known, and therefore, the calculation of cost function (3) is impossible.

Assume that the spectral density S_g belongs to a certain class $\mathcal{S}$, for which we can find upper estimate of the mean variances M_y and M_u, so that we have

$$\bar{v}_y \leq M_y, \quad \bar{v}_u \leq M_u, \quad \forall S_g(\nu) \in \mathcal{S} .$$

Then we can choose as cost function the upper estimate

$$\widetilde{J}(\mathcal{S}, C_1) = M_y + \rho^2 M_u , \tag{4}$$

which only depends on the properties of the class $\mathcal{S}$ and the chosen $C_1(s)$.

Obviously, for any controller $C_1(s)$

$$J(S_g, C_1) \leq \widetilde{J}(\mathcal{S}, C_1) , \quad \forall S_g(\nu) \in \mathcal{S} .$$

If $\widetilde{J}(\mathcal{S}, C_1) \leq J_0$, where J_0 is an admissible limit value of the cost function, then this controller is *performance*

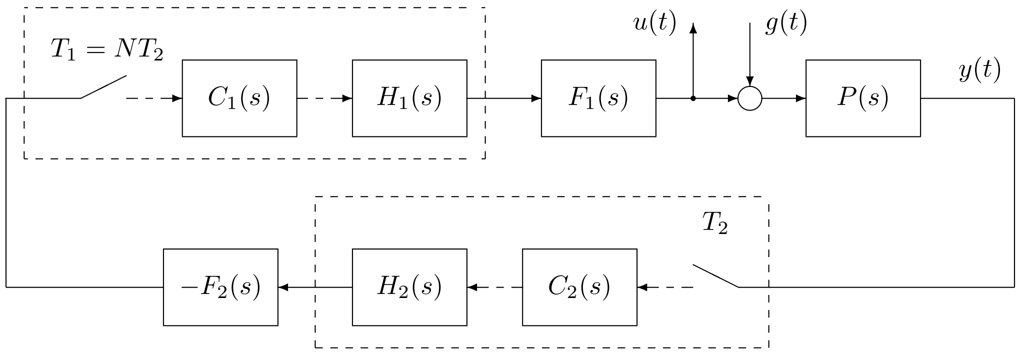

Figure 1. Structure of the dual-rate sampled-data control system

guaranteeing, i.e. it ensures the demanded properties of the system for all excitations of the given class.

Thus the problem of optimal guaranteed performance controller design can be formulated as follows (Rybinskii et al. 2004):

Optimal control with guaranteed performance: Let all elements of the system in Fig. 1, except for $C_1(s)$, and an upper bound for the admissible controller order q_1 be given. Moreover, assume that the class of excitations is described by the envelope spectral density $\bar{S}_g(\nu)$, such that

$$\mathcal{S}: \; S_g(\nu) \leq \bar{S}_g(\nu), \; \forall\, \nu\,. \tag{5}$$

Then, find the transfer function of a controller $C_{1*}(s)$ of order not higher than q_1, which stabilizes the system and minimizes the criterion (4).

3. CONSTRUCTION OF THE COST FUNCTION

For the calcualtion of the cost function (3), we apply the parametric transfer function (PTF) method (Rosenwasser and Lampe 2000, Rosenwasser and Lampe 2006), which allows an exact description of the behavior of linear periodic sampled-data systems in continuous time.

The I/O behavior of the investigated system is characterized by the PTFs $W_y(s,t)$ and $W_u(s,t)$ from the input $g(t)$ to the outputs $y(t)$ and $u(t)$, respectively. Suppose the input signal $g(t)$ to have the spectral density $S_g(\nu)$. As was shown in (Rosenwasser and Lampe 2000), the variances $v_y(t)$ and $v_u(t)$ could be find by the expressions

$$v_y(t) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} A_y^2(\nu,t)\, S_g(\nu)\, \mathrm{d}\nu\,, \tag{6}$$

$$v_u(t) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} A_u^2(\nu,t)\, S_g(\nu)\, \mathrm{d}\nu\,, \tag{7}$$

where $A_y(\nu,t)$ and $A_u(\nu,t)$ denote the parametric amplitude frequency responses (Rosenwasser and Lampe 2000):

$$A_y(\nu,t) = |W_y(\mathrm{j}\nu,t)|\,, \quad A_u(\nu,t) = |W_u(\mathrm{j}\nu,t)|\,,$$

and $\mathrm{j} = \sqrt{-1}$. For the mean variance of the weighted outputs over the interval $[0,T]$, we obtain

$$J(S_g, C_1) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} A^2(\nu)\, S_g(\nu)\, \mathrm{d}\nu\,,$$

where

$$A^2(\nu) = \frac{1}{T} \int\limits_0^T \left[A_y^2(\nu,t) + \rho^2 A_u^2(\nu,t)\right] \mathrm{d}t\,. \tag{8}$$

Owing to the fact that $A^2(\nu)$ and $S_g(\nu)$ are nonnegative by construction, for all spectral densities of the class (5), we find the estimate $J(S_g, C_1) \leq \widetilde{J}(\mathcal{S}, C_1)$, where

$$\widetilde{J}(\mathcal{S}, C_1) = \frac{1}{2\pi} \int\limits_{-\infty}^{\infty} A^2(\nu)\, \bar{S}_g(\nu)\, \mathrm{d}\nu\,. \tag{9}$$

Below for concrete explanation, we consider the class of excitations given by an envelope spectral density. Nevertheless, the treated approach is directly extendable, e.g. to another case, when the variance of the signal $g(t)$ and those of several derivatives are known (Nebylov 2004).

4. PTF OF MULTIRATE SYSTEMS

The parametric transfer function is an universal I/O characteristic of a periodic system, and its properties are very close to the properties of the ordinary transfer function of LTI systems (Rosenwasser and Lampe 2000, Rosenwasser and Lampe 2006). An important role for the construction of the PTF is imputed to the *d*isplaced pulse frequency response (DPFR), which is defined for the quantity $X(s)$ by

$$\Phi_X(T,s,t) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X\left(s + k\frac{2\pi \mathrm{j}}{T}\right) \mathrm{e}^{k\frac{2\pi \mathrm{j}}{T}t}\,.$$

As was shown in (Rosenwasser and Lampe 2000), for all t and integers $N > 1$ the following equation holds:

$$\Phi_X(T,s,t) = \sum_{i=0}^{N-1} \Phi_X(NT, s, t - iT)\,.$$

On basis of this relation, the so-called *polyphase decomposition* is possible, which allows to present a system with period T as a parallel connection of N systems with period NT.

Using the general approach, developed in (Rosenwasser and Lampe 2000, Rosenwasser and Lampe 2006), we can show that

$$W_y(s,t) = P(s)\left(1 - \frac{\Phi_{PF_1H_1}(T,s,t)\,\Psi(s)}{\Lambda(s)}\right), \quad (10)$$

$$W_u(s,t) = -P(s)\frac{\Phi_{F_1H_1}(T,s,t)\,\Psi(s)}{\Lambda(s)}, \quad (11)$$

where

$$\Psi(s) = C_1(s)C_2(s)\Phi_{F_2H_2}(T_2,s,0),$$
$$\Lambda(s) = 1 + C_1(s)\,D(s),$$

and

$$D(s) = \sum_{i=0}^{N-1} \Phi_{PF_1H_1}(T,s,iT_2)\,\Phi_{C_2F_2H_2}(T,s,-iT_2). \quad (12)$$

Closed formulae for the calculation of the DPFR arising in (10) and (11), are provided in (Rosenwasser and Lampe 2000).

5. STABILIZATION

As in the single-rate case (Rosenwasser and Lampe 2000), the poles of the PTF $W_y(s,t)$ and $W_u(s,t)$ determine the dynamics of the transient processes and the stability. When non-controllable and non-observable poles are absent, the system of Fig. 1 is stable, if and only if all roots of the equation

$$\Lambda(s) = 0 \quad (13)$$

posses negative real part.

It can be shown that the function $D(s)$ in (12) comes out as a rational function of the variable $\zeta = \mathrm{e}^{-sT}$. Moreover, the transfer function of the unknown controller $C_1(s)$ has this property too. Therefore, in equation (13) we can pass to the variable ζ:

$$1 + C_1(\zeta)\,D(\zeta) = 0, \quad (14)$$

where $C_1(\zeta) = C_1(s)|_{\mathrm{e}^{-sT}=\zeta}$ and $D(\zeta) = D(s)|_{\mathrm{e}^{-sT}=\zeta}$. Let us write the rational functions in the form

$$C_1(\zeta) = \frac{\beta_1(\zeta)}{\alpha_1(\zeta)}, \quad D(\zeta) = \frac{b(\zeta)}{a(\zeta)},$$

where $(\beta_1(\zeta), \alpha_1(\zeta))$ and $(b(\zeta), a(\zeta))$ are pairs of coprime polynomials. If the system is non-pathological, then the product $C_1(\zeta)D(\zeta)$ is irreducible and the set of roots of (14) coincides with the set of roots of the polynomial

$$\Delta(\zeta) = \beta_1(\zeta)b(\zeta) + \alpha_1(\zeta)a(\zeta). \quad (15)$$

With account of $\zeta = \mathrm{e}^{-sT}$, it is easy to show that for the stability of the system all roots of the *characteristic polynomial* $\Delta(\zeta)$ are located outside of the unit circle. We will say, that this polynomial is *stable*.

Expression (15) formally coincides with the characteristic polynomial of a single-rate system containing a process with the discrete model $D(\zeta)$. Therefore, for solving the stabilization problem, we can apply in this case all results from (Rosenwasser and Lampe 2000).

For the solution of the original problem we need to find polynomials $\beta_1(\zeta)$ and $\alpha_1(\zeta)$ of degree not higher than q_1, for which the polynomial $\Delta(\zeta)$ in (15) has only roots outside the unit circle and cost function (9) takes its minimum.

6. NUMERICAL OPTIMIZATION

Since the envelope spectral density $\bar{S}_g(\nu)$ is not supposed to be a rational function of ν, for the search of the optimal controller, we will apply numerical search algorithms.

Let $p = \max\{\deg b(\zeta), \deg a(\zeta)\}$. Depending on the desired controller degree q_1, we have to consider two cases:

Case 1. If $q_1 < p - 1$, a stabilizing controller does not always exist, and the parameter vector for the optimization has the $2q_1 + 1$ unknown coefficients of the polynomials $\beta_1(\zeta)$ and $\alpha_1(\zeta)$ (without loss of generality we set $\alpha_{10} = 1$, because the controller should become realizable). For every test controller the stability of the polynomial $\Delta(\zeta)$ (15) must be ensured.

Case 2. If $q_1 \geq p - 1$, the optimization must be taken over the set of stabilizing controllers in the following way. Suppose that we have selected a certain stable polynomial Δ, such that

$$\deg \Delta(\zeta) \leq p + q_1.$$

Then equation (15) is solvable for polynomials $\beta_1(\zeta)$ and $\alpha_1(\zeta)$ of degree not higher than q_1, where the set of all admissible solutions can be parameterized in the form

$$\beta_1(\zeta) = \beta_1^0(\zeta) + \xi(\zeta)\,a(\zeta),$$
$$\alpha_1(\zeta) = \alpha_1^0(\zeta) - \xi(\zeta)\,b(\zeta),$$

where $(\beta_1^0(\zeta), \alpha_1^0(\zeta))$ is the solution of minimal degree, and $\xi(\zeta)$ is zero or any polynomial with degree not higher then $q_1 - p$ (Kučera 1979, Polyakov, Rosenwasser and Lampe 2005). Therefore, the vector of unknown coefficients includes the roots of the polynomial $\Delta(\zeta)$ (or more suitable, their reverse quantities) and the coefficients of the polynomial $\xi(\zeta)$. Thus the optimization runs directly over the set of stabilizing controllers and an additional stability test is not necessary.

For the search of the optimal solution numerical procedures of direct search can be used, e.g. genetic algorithms (Man, Tang and Kwong 1999).

7. EXAMPLE

Consider the system from Fig. 1, where $P(s)$ is a plant, $F_2(s)$ is a measuring device, and $F_1(s)$ is an actuator. We have to control the motion variable $y(t)$ using the control $u(t)$. Assume that

$$P(s) = \frac{0.1}{s(s-3)}, \quad F_1(s) = 1, \quad F_2(s) = \frac{0.1}{s+1}.$$

Moreover, let $T_1 = 0.6$sec, and $T_2 = 0.2$sec, i.e. $N = 3$. Notice that the considered process has one unstable pole and one pole at the stability border. Assume that the system does not make use of preprocessing for the measuring data, i.e. $C_2(s) = 1$. Furthermore, we use zero-order hold elements with the transfer functions

$$H_i(s) = \frac{1 - e^{-sT_i}}{s}, \quad i = 1, 2.$$

The class of admissible excitation $\mathcal{S}$ is determined by the envelope spectral density (Rybinskii et al. 2004)

$$\bar{S}_g(\nu) = \frac{7.57}{\nu^4 + 2.49\nu^2 + 1.85}. \tag{16}$$

So any stochastic excitation $g(t)$ with spectral density

$$S_g(\nu) \leq \bar{S}_g(\nu) \quad \forall \nu$$

is admissible. In this case for $\xi(\zeta) = 0$ ($q_1 = 2$), the controller with guaranteed performance, providing for $\rho^2 = 0.1$ a minimal estimate (9), has the transfer function

$$C_{1*}(s) = \frac{413.5 - 638\, e^{-sT} + 225.6\, e^{-2sT}}{0.554 + 2.065\, e^{-sT} + e^{-2sT}}, \tag{17}$$

yielding the values

$$M_y = 2.73 \cdot 10^2, \quad M_u = 2.31 \cdot 10^3 \Rightarrow J = 5.04 \cdot 10^2. \tag{18}$$

So we have shown that for any excitation $g(t)$ satisfying $S_g(\nu) \in \mathcal{S}$, the weighted mean variances of the signals $y(t)$ and $u(t)$ will not exceed the value (18).

In order to demonstrate the benefit from applying the above derived methods for the investigation of dual-rate systems, for comparison we consider the design problem for the controller $C_1(s)$ under the assumption of a simplified single-rate model, when in the system of Fig. 1 the sampler with T_2 and the hold $H_2(s)$ are absent. But all other conditions of the considered example remain valid.

Then for the design of the discrete controller with guaranteed performance $C_1(s)$, the design methods with guaranteed performance for single-rate systems are applicable, which are described in (Rybinskii et al. 2004). As a result, we obtain

$$C_{1*alt}(s) = \frac{602.7 - 933.5\, e^{-sT} + 330.8\, e^{-2sT}}{1.463 + 2.962\, e^{-sT} + e^{-2sT}}. \tag{19}$$

The simulation results of the system in Fig. 1 with the calculated controllers $C_{1*}(s)$ and $C_{1*alt}(s)$ are presented in Fig. 2 and 3, where respectively the signals $y(t)$ and $u(t)$ are shown under the condition of disturbances with 'worst' spectral density (16). Moreover, these pictures contain the limits of the standard deviation, which are held from the estimate of the variance (18) by the formula

$$\sigma_y = \sqrt{M_y}, \quad \sigma_u = \sqrt{M_u}.$$

As can be seen from these pictures, the behavior of the system with controller C_{1*alt} is essentially worse than that of the system with controller C_{1*}. The values of both signals $y(t)$ and $u(t)$ substantially leave the admissible region (18), when controller C_{1*alt} is used.

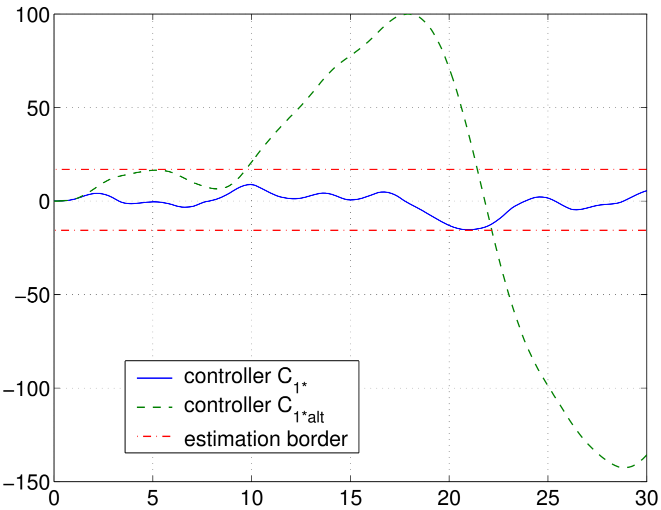

Figure 2. Output signal $y(t)$ of the closed loop

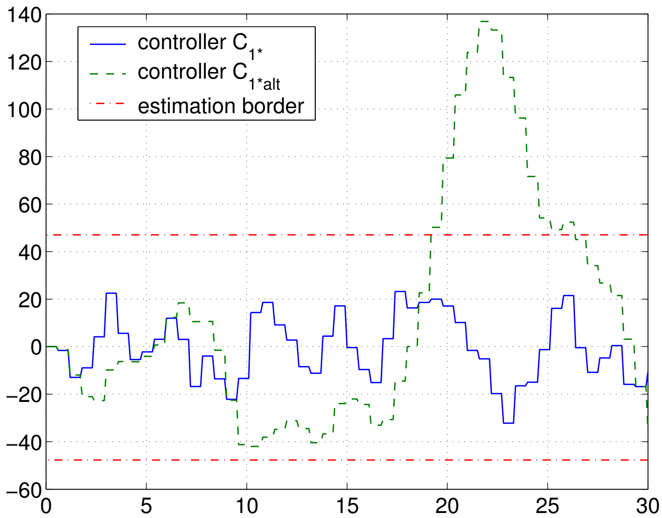

Figure 3. Control signal $u(t)$ of the closed loop

CONCLUSIONS

The paper presents an approach to optimization of dual-rate sampled-data systems containing two digital controllers working with different rates, that allows to guarantee the performance of the control system for all stochastic excitations of a given class. As cost function, the weighted sum of the mean variances of the output signals is used, when a stationary stochastic signal is acting on the input of the system. For the calculation of the cost function, the concepts of the parametric transfer function and the parametric frequency response have been applied. They allow to take into account all values of the continuous signals, not only the values at the sampling instants.

The results have been achieved under the assumption that the controller C_2 possesses the smaller sampling period. However, the case $T_2 = NT_1$ can be considered analogously without essential modifications.

The proposed methods allow to design a controller with guaranteed performance, where the sampling period coincides with the period of the whole system. In case of an unknown fast-rate controller, additional difficulties arise from the causality constraints to the solution (Meyer 1990, Chen and Qiu 1994). The extension of the provided method to this case will be a topic of our future work.

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