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Decentralized Adaptive Robust Tracking Controllers of Uncertain Large Scale Systems with Time Delays

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Abstract: The problem of decentralized robust tracking and model following is considered for a class of uncertain large scale systems including delayed state perturbations in the interconnections. In this paper, it is assumed that the upper bounds of the delayed state perturbations, uncertainties, and external disturbances are unknown. A modified adaptation law with σ–modification is introduced to estimate such unknown bounds, and on the basis of the updated values of these unknown bounds, a class of decentralized local memoryless state feedback controllers is constructed for robust tracking of dynamical signals. The proposed decentralized adaptive robust tracking controllers can guarantee that the tracking errors between each time–delay subsystem and the corresponding local reference model without time–delay decrease uniformly asymptotically to zero.

1. INTRODUCTION

It is well known that except for significant uncertainties, the time–delay is often encountered in various practical engineering systems to be controlled, such as chemical processes, hydraulic, and rolling mill systems, and the existence of the delay is frequently a source of instability. Therefore, it is important to consider the robust tracking and model following problem of dynamical time–delay systems with significant uncertainties. Such a robust tracking and model following problem has been well discussed, and some approaches for an actual system to tracking dynamical signals of a given reference model have been developed (see, e.g. Shyu and Chen [1995], Oucheriah [1999], Wu [2001] and the references therein).

In the control literature, for dynamical systems with uncertainties and external disturbances, the upper bounds of uncertainties and external disturbances are generally supposed to be known, and such bounds are employed to construct some types of stabilizing state (or output) feedback controllers (see, e.g. Shyu and Chen [1995], Oucheriah [1999], Wu [2001], Hopp and Schmitendorf [1990] and Wu [2000] for the problem of robust tracking and model following). However, in a number of practical control problems, such bounds may be unknown, or be partially known. Specially, in the problem of robust tracking and model following, it is also difficult to evaluate the upper bounds of uncertainties and external disturbances. Therefore, for such a class of uncertain systems whose uncertainty bounds are partially known, some types of adaptive control schemes should be introduced to update these unknown bounds. For such uncertain systems, several types of adaptive robust state feedback controller have been proposed (see, e.g. Brogliato and Trofino Neto [1995], Choi and Kim [1993], Wu [1999], Wu [2000] and Wu [2002]). In particular, in a recent paper Wu [2004], the problem of robust tracking and model following is considered for uncertain time–delay composite dynamical systems. It is assumed in Wu [2004] that the upper bounds of the uncertainties and external disturbances are unknown, an improved adaptation law with σ–modification is proposed to estimate such unknown bounds, and a class of adaptive robust tracking controllers is proposed for robust tracking of dynamical signals. However, because of its complexity, few efforts are made to consider the problem of decentralized robust tracking and model following for uncertain large scale time–delay systems with the unknown bounds of uncertainties and external disturbances.

In this paper, we consider the problem of the decentralized robust tracking and model following for a class of uncertain large scale dynamical systems including delayed state perturbations in the interconnections. We assume that the upper bounds of the delayed state perturbations, uncertainties, and external disturbances are unknown. For such a class of uncertain large scale time–delay systems, we want to develop some decentralized local state feedback controllers for robust tracking of dynamical signals. For this purpose, we first introduce a class of modified adaptation laws with σ–modification to estimate these unknown bounds. Then, by making use of the updated values of the unknown bounds, we construct a class of decentralized adaptive robust tracking controllers. We will shown that by employing the proposed decentralized adaptive robust tracking controllers, one can guarantee that the tracking errors between each time–delay subsystem and the corresponding local reference model without time–delay decrease uniformly asymptotically to zero. That is, it is possible for each time–delay subsystem to track exactly the given local reference system.

2. PROBLEM FORMULATION

We consider a large scale time-delay system S composed of N interconnected subsystems $S_i, i = 1, 2, \cdots, N$, described by the following differential–difference equations:

$$\frac{dx_i(t)}{dt} = \Big[A_i + \Delta A_i(\upsilon_i, t) \Big] x_i(t) + \Big[B_i + \Delta B_i(\xi_i, t) \Big] u_i(t) \tag{1a}$$

$$y_i(t) = C_i x_i(t) \tag{1b}$$

where $t \in R^+$ is the time, $x_i(t) \in R^{n_i}$ is the current value of the state, $u_i(t) \in R^{m_i}$ is the input (or control) vector, and $y_i(t) \in R^{l_i}$ is the output vector. Each dynamical subsystem is interconnected as

$$u_i(t) = \sum_{j=1}^{N} A_{ij}(\zeta_i, t) x_j(t - h_{ij}) + w_i(\nu_i, t) \tag{2}$$

where $i = 1, 2, \ldots, N$.

In (1) and (2), for each $i \in \{1, 2, \ldots, N\}$, A_i, B_i, C_i are known constant matrices of appropriate dimensions. In particular, the matrix $A_{ij}(\cdot)$ stands for the extent of interconnection between S_i and S_j, and are assumed to be continuous in all their arguments; $\Delta A_i(\cdot)$, $\Delta B_i(\cdot)$ represent the uncertainties of the systems, $w_i(\cdot)$ is the external disturbance vector, and are also assumed to be continuous in all their arguments. Moreover, the uncertain parameters $(\upsilon_i,\, \xi_i,\, \zeta_i,\, \nu_i) \in \Psi_i \subset R^{L_i},\ i \in \{1, 2, \ldots, N\}$, are Lebesgue measurable and take values in a compact bounding set Ω_i ; the time delays $h_{ij},\ i, j = 1, 2, \ldots, N$, are assumed to be any positive constants which are not required to be known for the system designer. In this paper, $x(\cdot) \in R^n$ denotes $\big[\, x_1^\top(\cdot) \cdots x_N^\top(\cdot) \,\big]^\top$, where $n = n_1 + \cdots + n_N$.

The initial condition for each subsystem with time delays is given by

$$x_i(t) = \chi_i(t), \qquad t \in [t_0 - \bar{h}_i,\ t_0] \tag{3}$$

where $\chi_i(t)$ is a continuous function on $[t_0 - \bar{h}_i,\ t_0]$, and $\bar{h}_i$ is defined as follows.

$$\bar{h}_i := \max\{\, \bar{h}_{ij},\ j = 1, 2, \ldots, N \,\}$$

For this class of input–interconnected large scale dynamical systems including delayed state perturbations in the interconnections, we introduce a decentralized local memoryless state feedback controller $\bar{u}_i(t)$ given by

$$\bar{u}_i(t) = p_i(x_i(t),\, t), \qquad i = 1, 2, \ldots, N \tag{4}$$

for each subsystem which modifies (2) to

$$u_i(t) = \bar{u}_i(t) + \sum_{j=1}^{N} A_{ij}(\zeta_i, t) x_j(t - h_{ij}) + w_i(\nu_i, t) \tag{5}$$

where $i = 1, 2, \ldots, N$, $p_i(\cdot) : R^{n_i} \times R^+ \to R^{m_i}$ is a continuous function which will be proposed later.

On the other hand, for each $i \in \{1, 2, \cdots, N\}$, the reference signal $\hat{y}_i(t)$, which should be followed by the output $y_i(t)$ of each subsystem S_i, is assumed to be the output of a reference model $\hat{S}_i$ described by the differential equation of the form:

$$\frac{d\hat{x}_i(t)}{dt} = \hat{A}_i \hat{x}_i(t) + \hat{B}_i r_i(t) \tag{6a}$$

$$\hat{y}_i(t) = \hat{C}_i \hat{x}_i(t) \tag{6b}$$

where $\hat{x}_i(t) \in R^{\hat{n}_i}$ is the state vector of the reference model, $\hat{y}_i(t) \in R^{\hat{l}_i}$ is the output vector of the reference model, $r_i(t) \in R^{\hat{m}_i}$ is the input vector of the reference model, and $\hat{A}_i$, $\hat{B}_i$, $\hat{C}_i$ are known constant matrices of appropriate dimensions. Here, $\hat{y}_i(t)$ has the same dimension as $y_i(t)$, i.e. $\hat{l}_i = l_i$. Furthermore, we require that the model state must be bounded, i.e. for each reference model $\hat{S}_i$, there exists a finite positive constant M_i such that for all $t \geq t_0$, $\|\hat{x}_i(t)\| \leq M_i$. In addition, the input vector of each reference model is assumed to be bounded, i.e. $\|r_i(t)\| \leq \bar{r}_i$, where $\bar{r}_i$ is any positive constant.

As pointed out in Hopp and Schmitendorf [1990], not all models of the form given in (6) can be tracked by a corresponding subsystem given in (1) with a feedback controller. Here, the requirement for the developed decentralized local controller to track the model described by (6) is the existence of the matrices $G_i \in R^{n_i \times \hat{n}_i}$, $H_i \in R^{m_i \times \hat{n}_i}$, $F_i \in R^{m_i \times \hat{m}_i}$, such that for each $i \in \{1, 2, \ldots, N\}$, the following matrix algebraic equation holds.

$$\begin{bmatrix} A_i & B_i & 0 \\ 0 & 0 & B_i \\ C_i & 0 & 0 \end{bmatrix} \begin{bmatrix} G_i \\ H_i \\ F_i \end{bmatrix} = \begin{bmatrix} G_i \hat{A}_i \\ G_i \hat{B}_i \\ \hat{C}_i \end{bmatrix} \tag{7}$$

For each $i \in \{1, 2, \ldots, N\}$, if a solution cannot be found to satisfy this algebraic matrix equation, a different local model or output matrix C_i must be chosen. In particular, the approach to finding the solution of the algebraic matrix equation similar to (7), where B_i is not included, is also discussed in detail in Shyu and Chen [1995], Hopp and Schmitendorf [1990].

Now, the question is how to synthesize a decentralized local state feedback controller $\bar{u}_i(t)$ such that the output $y_i(t)$ of each time–delay subsystem follows the output $\hat{y}_i(t)$ of the corresponding local reference model without time–delay.

Remark 2.1. For the model following problem of uncertain composite dynamical systems, some robust state (or output) feedback tracking controllers are presented in the control literature (see, e.g. Hopp and Schmitendorf [1990], Wu [2000] for uncertain systems without time–delay, and Shyu and Chen [1995], Oucheriah [1999], Wu [2001] for uncertain time–delay systems, and the references therein). In particular, in a recent paper Shigemaru and Wu [2001], the model following problem of uncertain large scale interconnected systems has been discussed. However, few efforts are made to consider the problem of decentralized robust tracking and model following for uncertain large scale systems with time–delay, because of its complexity. In this paper, we will consider the problem of robust tracking and model following for a class of large scale dynamical systems with delayed state perturbations, uncertainties, and external disturbances, and want to propose decentralized robust tracking controller.

Before proposing our decentralized robust tracking controllers, we introduce for system (1) the following standard assumptions.

Assumption 2.1. The pairs (A_i, B_i), $i = 1, 2, \cdots, N$, given in system (1) are completely controllable.

Assumption 2.2. For each $i \in \{1, 2, \ldots, N\}$, there exist some continuous and bounded matrix functions $N_i(\cdot)$, $E_i(\cdot)$ of appropriate dimensions such that

$$\Delta A_i(\upsilon_i, t) = B_i N_i(\upsilon_i, t)$$
$$\Delta B_i(\xi_i, t) = B_i E_i(\xi_i, t)$$

Remark 2.2. It is obvious that Assumption 2.2 defines the matching condition about the uncertainties of the isolated subsystems, and is a rather standard assumption for robust control problem. It is well known that these matching conditions restrict the structure of each subsystem by stipulating that all uncertainties and interconnections should fall into the range space of the control vector B_i. However, this fact is true for a large class of systems, particularly mechanical systems.

For convenience, we now introduce the following notations which represent the bounds of the delayed state perturbations, uncertainties, and external disturbances.

$$\rho_i(t) := \max_{\upsilon_i} \Big\| N_i(\upsilon_i, t) \Big\|$$

$$\kappa_i(t) := \max_{\xi_i} \Big\| E_i(\xi_i, t) \Big\|$$

$$\mu_i(t) := \min_{\xi_i} \left[\frac{1}{2} \lambda_{\min}\big(E_i(\xi_i, t) + E_i^\top(\xi_i, t)\big) \right]$$

$$\tilde{w}_i(t) := \max_{\nu_i} \Big\| w_i(\nu_i, t) \Big\|$$

$$\rho_{ij}(t) := \max_{\zeta_i} \Big\| A_{ij}(\zeta_i, t) \Big\|, \quad j = 1, 2, \ldots, N$$

where $i \in \{1, 2, \ldots, N\}$, $\|\cdot\|$ is the spectral norm of a matrix, and $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of the matrix, respectively. Here, the functions $\rho_i(t)$, $\kappa_i(t)$, $\mu_i(t)$, $\tilde{w}_i(t)$, $\rho_{ij}(t)$ are assumed to be *unknown*. Moreover, the uncertain $\rho_i(t)$, $\kappa_i(t)$, $\mu_i(t)$, $\tilde{w}_i(t)$, $\rho_{ij}(t)$ are also assumed, without loss of generality, to be uniformly continuous and bounded for any $t \in R^+$.

By employing the notations given above, we also introduce for uncertain large scale system (1) the following standard assumption.

Assumption 2.3. For every $t \geq t_0$ and any $i \in \{1, 2, \ldots, N\}$, the unknown function $\mu_i(t) > -1$.

Remark 2.3. It is worth pointing out that for the uncertain large scale interconnected system described by (1) and (5), Assumption 2.3 is standard. It is well known that the assumption mentioned in Assumption 2.3 is a necessary condition for robust stability of uncertain dynamical systems (see, e.g., Shyu and Chen [1995], Oucheriah [1999], Wu [2000], Wu [2002] and the references relative to robust stabilization of uncertain systems).

Remark 2.4. It is well known that in a number of practical control problems, the bounds $\rho_i(t)$, $\kappa_i(t)$, $\mu_i(t)$, $\tilde{w}_i(t)$, $\rho_{ij}(t)$, may be unknown, or it is difficult to evaluate them. Therefore, some updating laws to such unknown bounds must be introduced to construct adaptive robust controllers. In this paper, we will propose a class of decentralized local memoryless adaptive robust tracking controllers which can guarantee that tracking errors between each time–delay subsystem and the local reference model without time–delay decrease asymptotically to zero.

On the other hand, it follows from Assumption 2.1 that for any given positive definite matrix $Q_i \in R^{n_i \times n_i}$, there exists an unique positive definite matrix $P_i \in R^{n_i \times n_i}$ as the solution of the algebraic Riccati equation of the form

$$A_i^\top P_i + P_i A_i - \eta_i P_i B_i B_i^\top P_i \; = \; -Q_i \tag{8}$$

where η_i is any given positive constant.

3. MAIN RESULTS

In this section, we propose a class of decentralized local memoryless state feedback controllers which can guarantee that the output $y_i(t)$ of each subsystem follows the output $\hat{y}_i(t)$ of the corresponding local reference model and the tracking error decreases asymptotically to zero. For this, let the tracking error between each subsystem and the local reference model be defined as

$$e_i(t) \; = \; y_i(t) - \hat{y}_i(t), \quad\quad i \in \{1, 2, \ldots, N\} \tag{9}$$

then the decentralized local state feedback tracking control laws can be constructed as

$$\bar{u}_i(t) \; = \; H_i \hat{x}_i(t) + F_i r_i(t) + \tilde{p}_i(t) \tag{10}$$

where $H_i \in R^{m_i \times \hat{n}_i}$ and $F_i \in R^{m_i \times \hat{m}_i}$ are assumed to satisfy (7), and $\tilde{p}_i(t)$ is auxiliary control function which will be given later.

Here, we first define for each subsystem a new state vector $z_i(t)$, called the auxiliary state, as follows:

$$z_i(t) \; := \; x_i(t) - G_i \hat{x}_i(t), \quad\quad i \in \{1, 2, \ldots, N\} \tag{11}$$

where $G_i \in R^{n_i \times \hat{n}_i}$ is still assumed to satisfy the algebraic equation described by (7).

From (7) and (11) we can obtain the relationship between the tracking error $e_i(t)$ and the auxiliary state vector $z_i(t)$ as follows.

$$e_i(t) \; = \; C_i z_i(t), \quad\quad i \in \{1, 2, \ldots, N\} \tag{12}$$

For each subsystem, applying (10) to (1a) and (5) yields an auxiliary time–delay subsystem $\hat{S}_i$, $i \in \{1, 2, \ldots, N\}$, of the form:

$$\begin{aligned} \frac{dz_i(t)}{dt} \; &= \; \Big[A_i + \Delta A_i(\upsilon_i, t) \Big] z_i(t) \\ &+ \Big[B_i + \Delta B_i(\xi_i, t) \Big] \tilde{p}_i(t) + g_i(\upsilon_i, \xi_i, \zeta_i, \nu_i, r_i, \hat{x}_i, t) \end{aligned}$$

$$+\Big[B_i+\Delta B_i(\xi_i,t)\Big]\sum_{j=1}^{N}A_{ij}(\zeta_i,t)z_j(t-h_{ij}) \tag{13}$$

where

$$g_i(\,\cdot\,):=\Big[\;\Delta A_iG_i+\Delta B_iH_i\;\Big]\hat{x}_i(t)+\Delta B_iF_ir_i(t)$$
$$+\Big[B_i+\Delta B_i\Big]\Big(\sum_{j=1}^{N}A_{ij}G_j\hat{x}_j(t-h_{ij})+w_i\Big) \tag{14}$$

Then, by making use of the matching condition (see Assumption 2.2), (14) can be readily reduced to

$$g_i(\upsilon_i,\xi_i,\zeta_i,\nu_i,r_i,\hat{x}_i,t)=B_if_i(\upsilon_i,\xi_i,\zeta_i,\nu_i,r_i,\hat{x}_i,t) \tag{15}$$

where

$$f_i(\,\cdot\,):=\Big[\;N_iG_i+E_iH_i\;\Big]\hat{x}_i(t)+E_iF_ir_i(t)$$
$$+\Big[\;I_i+E_i\;\Big]\Big(\sum_{j=1}^{N}A_{ij}G_j\hat{x}_j(t-h_{ij})+w_i\Big) \tag{16}$$

Furthermore, we introduce for (16) the following notation.

$$\begin{aligned}\beta_i(t)\;:=\;\max\Big\{&\big\|f_i(\upsilon_i,\xi_i,\zeta_i,\nu_i,r_i,\hat{x}_i,t)\big\|\;:\\&(\upsilon_i,\xi_i,\zeta_i,\nu_i)\in\Psi_i,\;\big\|r_i(t)\big\|\le\bar{r}_i,\\&\big\|\hat{x}_i(t)\big\|\le M_i,\;t\in R^+\Big\}\end{aligned}$$

Here, without loss of generality, the uncertain function $\beta_i(t),\;\; i\in\{1,2,\ldots,N\}$, is still assumed to be uniformly continuous and bounded for any $t\in R^+$.

In this paper, since the bounds $\rho_i(t)$, $\rho_{ij}(t)$, $\kappa_i(t)$, $\mu_i(t)$, $\beta_i(t)$ have been assumed to be continuous and bounded for any $t\in R^+$, we can suppose that there exist some positive constants ρ_i^*, ρ_{ij}^*, κ_i^*, μ_i^*, β_i^*, which are defined by

$$\begin{aligned}\rho_i^*&:=\max\{\;\rho_i(t)\;:\;\;t\in R^+\;\}\\\rho_{ij}^*&:=\max\{\;\rho_{ij}(t)\;:\;\;t\in R^+\;\}\\\kappa_i^*&:=\max\{\;\kappa_i(t)\;:\;\;t\in R^+\;\}\\\mu_i^*&:=\min\{\;\mu_i(t)\;:\;\;t\in R^+\;\}\;>\;-1\\\beta_i^*&:=\max\{\;\beta_i(t)\;:\;\;t\in R^+\;\}\end{aligned}$$

Here, it is worth pointing out that the constants ρ_i^*, ρ_{ij}^*, κ_i^*, μ_i^*, β_i^*, are still unknown. Therefore, such unknown bounds can not be directly employed to construct the decentralized robust tracking controllers.

Withot loss of generality, we also introduce the following definitions:

$$\psi_i^*:=\frac{1}{1+\mu_i^*}\Bigg(1+\eta_i^{-1}\alpha_i\,(\rho_i^*)^2+\sum_{j=1}^{N}\eta_i^{-1}\alpha_i\,\big((1+\kappa_i^*)\,\rho_{ij}^*\big)^2\Bigg) \tag{17a}$$

$$\phi_i^*:=\frac{\beta_i^*}{1+\mu_i^*} \tag{17b}$$

where η_i and α_i are any positive constants. In particular, it is obvious from (17) that for any $i\in\{1,2,\ldots,N\}$, ψ^* and ϕ^* are unknown positive constants.

Now, we give the auxiliary control function $\tilde{p}_i(t)$, $i=1,2,\ldots,N$, as follows.

$$\tilde{p}_i(t)\;=\;p_{i1}(z_i(t),t)+p_{i2}(z_i(t),t) \tag{18a}$$

where $p_{i1}(\cdot)$ and $p_{i2}(\cdot)$ are given by the following functions:

$$p_{i1}(z_i(t),t)=-\frac{1}{2}\,\eta_i\hat{\psi}_i(t)B_i^\top P_iz_i(t) \tag{18b}$$

$$p_{i2}(z_i(t),t)=-\frac{\hat{\phi}_i^2(t)B_i^\top P_iz_i(t)}{\big\|B_i^\top P_iz_i(t)\big\|\,\hat{\phi}_i(t)\;+\;\sigma_i(t)} \tag{18c}$$

and where $\sigma_i(t)\in R^+$ is any positive uniform continuous and bounded function which satisfies $\lim_{t\to\infty}\int_{t_0}^{t}\sigma_i(\tau)d\tau\le\bar{\sigma}_i<\infty$, where $\bar{\sigma}_i$ is any positive constant. Here for any $i\in\{1,2,\ldots,N\}$, $P_i\in R^{n_i\times n_i}$ is the solution of the Riccati equation described by (8).

In particular, for any $i\in\{1,2,\ldots,N\}$, $\hat{\psi}_i(\cdot)$ and $\hat{\phi}_i(\cdot)$ in (18) are, respectively, the estimates of the unknown ψ_i^* and ϕ_i^*, which are updated by the following adaptive laws:

$$\frac{d\hat{\psi}_i(t)}{dt}=-\gamma_{1i}\sigma_i(t)\hat{\psi}_i(t)+\gamma_{1i}\eta_i\,\big\|B_i^\top P_iz_i(t)\big\|^2 \tag{19a}$$

$$\frac{d\hat{\phi}_i(t)}{dt}=-\gamma_{2i}\sigma_i(t)\hat{\phi}_i(t)+\gamma_{2i}\,\big\|B_i^\top P_iz_i(t)\big\| \tag{19b}$$

where for any $i\in\{1,2,\ldots,N\}$, γ_{1i}, γ_{2i} are any positive constants, and $\hat{\psi}_i(t_0)$, $\hat{\phi}_i(t_0)$ are finite.

For each auxiliary subsystem, applying (18) to (13) yields the following closed–loop auxiliary time–delay subsystem:

$$\begin{aligned}\frac{dz_i(t)}{dt}\;&=\;\Big[A_i-\frac{1}{2}\eta_i\hat{\psi}_i(t)B_iB_i^\top P_i\Big]\,z_i(t)\\&+\Big[\Delta A_i(\upsilon_i,t)-\frac{1}{2}\eta_i\hat{\psi}_i(t)\Delta B_i(\xi_i,t)B_i^\top P_i\Big]\,z_i(t)\\&+\Big[B_i+\Delta B_i(\xi_i,t)\Big]p_{i2}(z_i(t),t)\\&+\Big[B_i+\Delta B_i(\xi_i,t)\Big]\sum_{j=1}^{N}A_{ij}(\zeta_i,t)z_j(t-h_{ij})\\&+g_i(\upsilon_i,\xi_i,\zeta_i,\nu_i,r_i,\hat{x}_i,t)\end{aligned} \tag{20}$$

On the other hand, letting $\tilde{\psi}_i(t)=\hat{\psi}_i(t)-\psi_i^*$ and $\tilde{\phi}_i(t)=\hat{\phi}_i(t)-\phi_i^*$ we can rewrite (19) as the following error system:

$$\begin{aligned}\frac{d\tilde{\psi}_i(t)}{dt}&=-\gamma_{1i}\sigma_i(t)\tilde{\psi}_i(t)+\gamma_{1i}\eta_i\,\big\|B_i^\top P_iz_i(t)\big\|^2\\&\quad-\gamma_{1i}\sigma_i(t)\psi^*\end{aligned} \tag{21a}$$

$$\begin{aligned}\frac{d\tilde{\phi}_i(t)}{dt}&=-\gamma_{2i}\sigma_i(t)\tilde{\phi}_i(t)+\gamma_{2i}\,\big\|B_i^\top P_iz_i(t)\big\|\\&\quad-\gamma_{2i}\sigma_i(t)\phi^*\end{aligned} \tag{21b}$$

Here, we define $\tilde{\psi}(t):=\big[\;\tilde{\psi}_1(t)\;\tilde{\psi}_2(t)\;\cdots\;\tilde{\psi}_{\scriptscriptstyle N}(t)\;\big]^\top$ and $\tilde{\phi}(t):=\big[\;\tilde{\phi}_1(t)\;\tilde{\phi}_2(t)\;\cdots\;\tilde{\phi}_{\scriptscriptstyle N}(t)\;\big]^\top$.

In the following, by $(z, \tilde{\psi}, \tilde{\phi})(t)$ we denote a solution of the closed–loop auxiliary time–delay system and the error system. Then, we can obtain the following theorem.

Theorem 3.1. Consider the adaptive closed–loop auxiliary time–delay system described by (20) and (21) with (18), which satisfies *Assumptions 2.1* to *2.3*. Then, the solutions $(z, \tilde{\psi}, \tilde{\phi})\ (t;\ t_0, z(t_0), \tilde{\psi}(t_0), \tilde{\phi}(t_0))$ of the closed-loop auxiliary time–delay system described by (20) and the error system described by (21) are uniform bounded and

$$\lim_{t\to\infty} z(t;\ t_0, z(t_0)) \ = \ 0 \tag{22}$$

Proof: For the adaptive closed–loop auxiliary time-delay system described by (20) and (21), we first define a Lyapunov–Krasovskii functional candidate as follows.

$$\begin{aligned} V(z,\tilde{\psi},\tilde{\phi}) = & \sum_{i=1}^{N} z_i^\top(t) P_i z_i(t) + \frac{1}{2}\tilde{\psi}^\top(t)\big(I+\mu^*\big)\Gamma_1^{-1}\tilde{\psi}(t) \\ & + \tilde{\phi}^\top(t)\big(I+\mu^*\big)\Gamma_2^{-1}\tilde{\phi}(t) \\ & + \sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i^{-1} \int_{t-h_{ij}}^{t} z_j^\top(\tau) z_j(\tau) d\tau \end{aligned} \tag{23}$$

where for each $i \in \{1, 2, \ldots, N\}$, P_i is the solution to (8), α_i is any positive constant, and $(I + \mu^*) \in R^{N\times N}$, $\Gamma_1^{-1} \in R^{N\times N}$, $\Gamma_2^{-1} \in R^{N\times N}$ are positive definite matrices which are defined by

$$\begin{aligned} \big(I+\mu^*\big) &:= \mathrm{diag}\left\{\big(1+\mu_1^*\big),\ \ldots,\ \big(1+\mu_N^*\big)\right\} \\ \Gamma_1^{-1} &:= \mathrm{diag}\left\{\gamma_{11}^{-1},\ \gamma_{12}^{-1},\ \ldots,\ \gamma_{1_N}^{-1}\right\} \\ \Gamma_2^{-1} &:= \mathrm{diag}\left\{\gamma_{21}^{-1},\ \gamma_{22}^{-1},\ \ldots,\ \gamma_{2_N}^{-1}\right\} \end{aligned}$$

Let $(z(t),\ \tilde{\psi}(t),\ \tilde{\phi}(t))$ be the solutions to (20) and (21) for $t \geq t_0$. Then by taking the derivative of $V(\cdot)$ along the trajectories of (20) and (21), and by making use of some manipulations, it can be obtained that

$$\begin{aligned} \frac{dV(z,\tilde{\psi},\tilde{\phi})}{dt} \leq & \sum_{i=1}^{N} \Bigg\{ - z_i^\top(t)\tilde{Q}_i z_i(t) \\ & + \big(1+\mu_i^*\big)\Bigg[- \eta_i\hat{\psi}_i(t)\left\|B_i^\top P_i z_i(t)\right\|^2 \\ & + \eta_i\psi_i^*\left\|B_i^\top P_i z_i(t)\right\|^2 + 2\phi_i^*\left\|B_i^\top P_i z_i(t)\right\| \\ & - \frac{2\hat{\phi}_i^2(t)\left\|B_i^\top P_i z_i(t)\right\|^2}{\left\|B_i^\top P_i z_i(t)\right\|\hat{\phi}_i(t)\ +\ \sigma_i(t)} \\ & + \gamma_{1i}^{-1}\tilde{\psi}_i(t)\frac{d\tilde{\psi}_i(t)}{dt} + 2\gamma_{2i}^{-1}\tilde{\phi}_i(t)\frac{d\tilde{\phi}_i(t)}{dt}\Bigg]\Bigg\} \end{aligned} \tag{24}$$

where for any $i \in \{1, 2, \ldots, N\}$,

$$\tilde{Q}_i \ := \ Q_i - \alpha^{-1}(1+N)I_i \ > \ 0 \tag{25}$$

Notice that the facts that for any $i \in \{1, 2, \ldots, N\}$,

$$\hat{\psi}_i(t) \ = \ \tilde{\psi}_i(t) + \psi_i^*, \qquad \hat{\phi}_i(t) \ = \ \tilde{\phi}_i(t) + \phi_i^*$$

it follows from (21) and (25) that

$$\begin{aligned} \frac{dV(z,\tilde{\psi},\tilde{\phi})}{dt} \ \leq \ & \sum_{i=1}^{N} \Bigg\{ - \lambda_{\min}\left(\tilde{Q}_i\right)\|z_i(t)\|^2 \\ & + \big(1+\mu_i^*\big)\Bigg[\frac{2\left\|B_i^\top P_i z_i(t)\right\|\hat{\phi}_i(t)\cdot\sigma_i(t)}{\left\|B_i^\top P_i z_i(t)\right\|\hat{\phi}_i(t)\ +\ \sigma_i(t)} \\ & - \sigma_i(t)\tilde{\psi}_i^2(t) - \sigma_i(t)\tilde{\psi}_i(t)\psi_i^* \\ & - 2\sigma_i(t)\tilde{\phi}_i^2(t) - 2\sigma_i(t)\tilde{\phi}_i(t)\phi_i^*\Bigg]\Bigg\} \end{aligned} \tag{26}$$

Then, in the light of the inequality of the form

$$0 \ \leq \ \frac{ab}{a+b} \ \leq \ a, \quad \forall\, a,\, b \ > \ 0$$

from (26) we can obtain that for all $(t, z, \tilde{\psi}, \tilde{\phi}) \in R \times R^n \times R^N \times R^N$,

$$\begin{aligned} \frac{dV(z,\tilde{\psi},\tilde{\phi})}{dt} \ \leq \ & \sum_{i=1}^{N} \Bigg\{ - \lambda_{\min}\left(\tilde{Q}_i\right)\|z_i(t)\|^2 \\ & + \frac{1}{4}\big(1+\mu_i^*\big)\sigma_i(t)\Big[8 + |\psi_i^*|^2 + 2\,|\phi_i^*|^2\Big]\Bigg\} \end{aligned} \tag{27}$$

Letting

$$\begin{aligned} \tilde{z}(t) &:= \left[\, z^\top(t)\ \tilde{\psi}^\top(t)\ \tilde{\phi}^\top(t)\,\right]^\top \\ \bar{\varepsilon}_i &:= \frac{1}{4}\big(1+\mu_i^*\big)\Big[8 + |\psi_i^*|^2 + 2\,|\phi_i^*|^2\Big] \\ \bar{\eta}_{\min} &:= \min\Big\{\ \lambda_{\min}\left(\tilde{Q}_i\right),\ \ i = 1, 2, \ldots, N\ \Big\} \end{aligned}$$

we can obtain from (27) that for any $t \geq t_0$,

$$\frac{dV(\tilde{z}(t))}{dt} \ \leq \ -\bar{\eta}_{\min}\|z(t)\|^2 + \sum_{i=1}^{N}\bar{\varepsilon}_i\sigma_i(t) \tag{28}$$

On the other hand, in the light of the definition, given in (23), of the Lyapunov–Krasovskii functional, there always exist two positive constants $\delta_{\min}$ and $\delta_{\max}$ such that for any $t \geq t_0$,

$$\tilde{\gamma}_1(\|\tilde{z}(t)\|) \ \leq \ V(\tilde{z}(t)) \ \leq \ \tilde{\gamma}_2(\|\tilde{z}(t)\|) \tag{29}$$

Then, in the light of (28) and (29), by employing the method which has been used in Wu [2004], we can show that the solutions $\tilde{z}(t)$ of (20) and (21) are uniformly bounded, and that the auxiliary state $z(t)$ converges uniformly asymptotically to zero. $\nabla\nabla\nabla$

Thus, from Theorem 3.1 we can obtain the following theorem which shows that by employing the decentralized local state feedback controllers described in (10) with (18) and (19), one can guarantee the zero–tracking errors between each time–delay subsystem and the local reference model without time–delay.

Theorem 3.2. Consider the model following problem of the uncertain large scale time–delay system described by (1) and (5), which satisfies *Assumptions 2.1* to *2.3*. Then, by using the decentralized local state feedback controllers $u_i(t)$ described in (10) with (18) and (19), one can guarantee that the tracking error $e_i(t)$, $i \in \{1, 2, \ldots, N\}$, between each subsystem and local reference model, decreases uniformly asymptotically to zero.

Proof: From *Theorem 3.1*, we have shown that each adaptive closed–loop auxiliary time–delay subsystem described by (20) and (21) is uniformly asymptotically stable. That is, for the auxiliary state $z_i(t)$ of each subsystem, we can obtain that

$$\lim_{t\to\infty} \|z_i(t)\| = 0, \quad i \in \{1, 2, \ldots, N\}$$

Then, it can easily be obtained from the relationship between $e_i(t)$ and $z_i(t)$, i.e. $e_i(t) = C_i z_i(t)$, that each local tracking error $e_i(t)$, $i \in \{1, 2, \ldots, N\}$, also decreases uniformly asymptotically to zero. $\nabla\nabla\nabla$

Remark 3.1. It is well known that though the adaptive controllers resulting from the adaptation laws with σ–modification can guarantee the uniform ultimate boundedness of the resulting adaptive closed–loop systems, we cannot obtain the results that states of the adaptive closed–loop systems converge always uniformly asymptotically to zero. In a recent paper Wu [2004], the adaptation laws with σ–modification have been improved to guarantee an asymptotic stability results. In this paper, the improved adaptation laws, described by (19), are extended to the problem of decentralized robust tracking model following for uncertain large scale time–delay systems to develop a class of decentralized memoryless adaptive robust tracking controllers.

Remark 3.2. In order to illustrate the validity of the results obtained in the paper, a numerical example is also given, and the simulation is carried out. It is known from the results of the simulation that the proposed decentralized local robust tracking controllers can indeed guarantee that the tracking errors between each subsystem and the corresponding local reference model decrease asymptotically to zero in the presence of delayed state perturbations, uncertain parameters, and external disturbances. (The details of the illustrative numerical example and the figures of the simulation will be displayed in the presentation.)

4. CONCLUDING REMARKS

The problem of the decentralized robust tracking and model following has been considered for a class of uncertain large scale systems including delayed state perturbations in the interconnections. Here, the upper bounds of the delayed state perturbations, uncertainties, and external disturbances are assumed to be unknown. For such a class of uncertain large scale time–delay systems, we have proposed a class of decentralized memoryless adaptive robust state feedback controllers for robust tracking of dynamical signals. We have shown that by employing the proposed decentralized adaptive robust tracking controllers, one can guarantee that the tracking errors between each time–delay subsystem and the corresponding local reference model without time–delay decrease uniformly asymptotically to zero.

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