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Integrated-Equilibrium Routing of Traffic Flows with Congestion

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Abstract: The routing of traffic flows is an important mechanism to alleviate traffic congestion. However, it faces a dilemma. For the traffic manager, it is desirable to achieve system optimum, which may discriminate against some users, and yet the user wishes to use the shortest route to maximize his/her utility, which may result in inferior system performance. Based on the conflict between the "system" and the "users", the game theory is used to study the traffic routing problem in this paper. After shortage of the traditional routing model had been analyzed, a new concept called satisfactory degree is introduced. An integrated-equilibrium model based on double-objective optimization and corresponding algorithm are respectively proposed. At last, an example is conducted to illustrate that traffic flows are guided more efficiently and rationally using the integrated-equilibrium model.

1. INTRODUCTION

Congestion is a daily occurrence on many portions of road networks in urban areas, and it is said that about 6-10% of urban congestion is caused by inefficient route choice. It can alleviate traffic congestion if there is a more efficient trip distribution over space and time, and ATIS (Advanced Traveler Information System), In-vehicle system, Roadside Information System can do it. These systems can provide drivers with the necessary information and optimal routes. Traveler information system can issue information to travelers for purposes of aiding decision-making. In-vehicle system can provide drivers with direction leading them from their current position to a desired destination. Roadside information system, for example, the VMS (Variable Message Signs), is the device located within the roadway network to provide drivers with traffic information. Now, more and more cars are equipped with in-vehicle system. Typically, these systems obtain the current position with the help of GPS (Global Positioning System) and compute the shortest route based on the digital maps. It is hoped that these systems can help to alleviate traffic congestion.

1.1 Drawbacks of current routing model

The algorithms of current in-vehicle system are simple. They only compute the shortest route with respect to travel time or distance. Some advanced systems incorporate traffic flows forecast estimated from the models into the computation [Ben-Akiva, 1995]. Many simulations show that in-vehicle system indeed decrease the total travel time, yet these simulations also predict that the benefits will be lost once the number of equipped vehicles exceeds a certain threshold. In some cases it could even happen that unequipped drivers obtain shorter travel times than those with guidance [Mahmassani, 1991]. The reason may be that current algorithms try to minimize the individual travel time of each driver separately, without taking into account the effects of their own route recommendation [Jahn and Mohring, 2005]. In fact, these problems involve the routing and assignment mechanism. The mechanism of routing and assignment, which is suite of models and algorithms, is very important for these systems. The models and algorithms seek to optimize demand-side performance objectives and supply-side network performance objectives based on best predictions of traffic flows over time and space.

Now, most of models of routing and assignment are based on the Wardrop's first and second principle [Wardrop, 1952]. The first principle is user equilibrium optimum, which states that "*the journey time on all the routes actually used are equal and less than those which would be experienced by a single vehicle on any unused route*". The second principle is system optimum, which states that "*the average journey is minimum*". The two principles have been studied extensively in the literatures such as references [Merchant (1978), Ho (1980), Wie (1990)].However, the two principles aren't suitable for the route guidance systems. While the user equilibrium should satisfy the drivers, it does not necessarily minimize the total travel time of the system [Jahn and Mohring, 2005]. Roughgarden and Tardos investigate the relation between the system optimum and the user equilibrium [Roughgarden and Tardos, 2002]. In general, the total travel time in equilibrium can be larger compared to the system optimum. The system optimum expects to minimize the total travel time, however, it may route some drivers on unacceptable routes in order to obtain the whole system benefits. So, neither user equilibrium optimum nor system optimum is practical for the routing. The challenge is to find a model which achieves an efficient routing and assignment of traffic flows. The model that actually pays attention to the system-wide performance and individual benefits is needed.

1.2 A different approach

From the viewpoint of the traffic manager, it is desirable to achieve the system optimum. Yet it is desirable to achieve the user optimum for the users. Accordingly, a conflict comes into being. Jahn and Mohring proposed a model and corresponding algorithms to resolve this conflict. The model's quintessence is system-optimal routing of traffic flows with explicit integration of user constraints [Jahn and Mohring, 2005]. In this paper we use the game theory [Fudenberg (1991), Gibbons (1992)] to analyze the conflict and apply double-objective optimization to the routing and assignment problem. After analyzing shortage of the traditional routing model in Section 2, we propose the concept of satisfactory degree and an integrated-equilibrium model based on double-objective optimization as well as corresponding algorithms in Section 3. In Section 4, we give an example which illustrates that traffic flows are guided more efficiently and rationally using the integrated-equilibrium model. At last, we give the conclusion and further work.

2. THE COMMENTARY OF THE TRADITIONAL ROUTING MODEL

The traditional routing model is based on the user equilibrium optimum or system optimum. Consider a simple route choice shown in Fig.1.

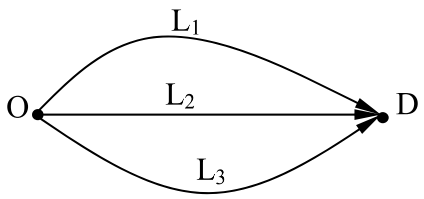

Fig.1. Simple routing

This example considered consists of three links and the travel time function uses the function of the U.S. Bureau of Public Roads shown as follows.

$$t_1(f^1) = 10(1 + 0.15(\frac{f^1}{200})^4) \quad (1)$$

$$t_2(f^2) = 15(1 + 0.15(\frac{f^2}{250})^4) \quad (2)$$

$$t_3(f^3) = 20(1 + 0.15(\frac{f^3}{300})^4) \quad (3)$$

Where, f^k is the number of drivers who choose the kth route;

$t(\ )$ is the travel time.

The OD demand is 750 vehicle trips between node O and D. The results of routing are given in the table below. Table1 is the results of user equilibrium optimum, where the travel time on all routes is equal. Table2 is the results of system optimum, where the total travel time is minimal. We analyze the results using the game theory.

Table1 Results of user equilibrium optimum

Routes	Flows	Travel time
L_1	322	20.08
L_2	306	20.08
L_3	122	20.08
Total travel time		15060

Table2 Results of system optimum

Routes	Flows	Travel time
L_1	246	13.42
L_2	255	17.42
L_3	249	21.42
Total travel time		13077

2.1 Analysis of the user equilibrium optimum

Each driver aims to find a route that minimizes his/her travel time. A driver pursues the minimum travel time when he travels from the origin to the destination, and other drivers simultaneously pursue the minimum travel time. In other words, each driver makes decision on route choices in order to optimize his/her travel time. Drivers' decisions interact with each other. Whether the minimum travel time can be achieved depends not only on his/her route choice but also on route choices of other drivers. The larger drivers who choose the same route are, the longer travel time is, as Figure 2 shown.

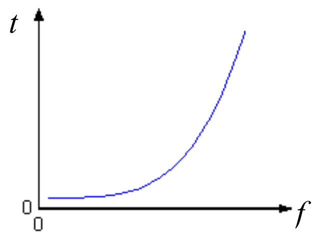

Fig.2 Link travel time *(f is number of drivers who choose the same route, t is the travel time)*

Each driver adjusts his route choice, namely his routing strategy, in order to minimize his cost. This self-optimizing mode of operation leads to a behaviour that can be modelled using non-cooperative game. We consider a set I= {1, 2, ..., 750} of drivers, who share a set L={L_1, L_2, L_3} of routes interconnecting the source node O to the destination node D. The drivers are non-cooperative, which means that each driver chooses his route in order to optimize his individual performance objective. The set L= {L_1, L_2, L_3} is called the strategy space of drivers. Let f^k denote the number of drivers who choose the kth routes. The flow configuration f= {f^1, f^2, f^3} is called a routing strategy profile.

The travel time can be expressed by

$$t_i = g(s^i, s^{-i}) \quad (4)$$

Where, s^i denotes the strategy of the driver i, s^{-i} denotes the vector of strategy of the other drivers.

The equation (4) shows that the travel time of driver i not only relates to his strategy, but also relates to other drivers' strategy. A natural problem that arises in this situation is whether there is a Nash equilibrium [Nash, 1950] or not. In other words, we are interested in finding a traffic flows configuration such that no driver can benefit by changing his strategy unilaterally. A traffic flows configuration f= {f^1, f^2, f^3} is a Nash equilibrium, if for all $i \in$ I, the following holds

$$t_i = \min g(s^i, s^{-i}) \quad i = 1,2,\cdots,750 \quad s^i \in L \qquad (5)$$

The importance of Nash equilibrium is that it is a state at which no driver has an incentive to deviate. That is, each driver is unable to obtain the benefit through changing his action strategy unilaterally. Accordingly, a kind of equilibrium state is achieved. In terms of network flows, "*a flow pattern is in Nash equilibrium if no individual decision maker on the network can change to less costly strategy, or, route*." In fact, the Nash equilibrium converges to the Wardrop equilibrium when the number of users becomes large. So, the state of the user equilibrium optimum of Table 1 can be regarded as the state of Nash equilibrium.

From the Table 1, the travel time of each driver is 20.08 and each driver achieves optimum in the case of user equilibrium optimum. But the system total travel time is 15060, which is more than the 13077 in the case of system optimum. In fact, one problem with Nash equilibrium is that it is not necessarily very efficient [Dubey, 1986]. Korillis,Lazar and Orda have given the numerical examples with natural cost functions where the difference between the total cost at the system-wide optimum point and that at the Nash equilibrium could be more than 20 percent [Korillis,1997]. System benefits are damaged at the state of user equilibrium optimum. Traffic manager is unsatisfied with this kind of result. So, user equilibrium optimum model is not practical for the traffic flows routing.

2.2 Analysis of the system optimum

System optimum is the state which the traffic manager hopes. Traffic manager pursues the minimal total cost, namely, wants to minimize the total travel time. The total travel time can be expressed as follows:

$$T = f^1 t_1(f^1) + f^2 t_2(f^2) + f^3 t_3(f^3) \qquad (6)$$

The total cost of the system depends only on the route flow configuration $f = \{f^1, f^2, f^3\}$. Since cost function T is convex function, there exists a unique route flow configuration that minimizes the total cost. As Table 2 shown, the total travel time achieves the minimum 13077 at the traffic flows configuration f= {246,255,249}. For the system optimum, drivers are assumed to cooperate. Drivers are asked to cooperate in order to achieve the global optimization. So, we use the cooperative game theory to analyze. Let P(n) denote the set of drivers, which is called a coalition in the cooperative game. T(n) is worth of the coalition, which denotes income of coalition (here refers to the total travel time of system). Let t_i （$i \in$ n） denote the travel time of driver *i*. $t = (t_1, t_2, \cdots, t_n) \in R^n$ denotes a utility vector of drivers. If $t = (t_1, t_2, \cdots, t_n) \in R^n$ satisfies the following

$$t_i \le \mathrm{T}(\{i\}) \qquad i \in n \qquad (7)$$

$$\sum_{i \in n} t_i = \mathrm{T}(n) \qquad (8)$$

$t = (t_1, t_2, \cdots, t_n) \in R^n$ is called a feasible payoff.

Equation (7) is individually rational condition, which indicates each driver's income is at least equal to income which he obtains when he go alone. Equation (8) indicates the sum of the driver's income is equal to the overall coalition's income.

From the table 2, we can see

t={t_1=t_2=…=t_{246}=13.42,t_{247}=t_{248}=…=t_{501}=17.42,t_{502}=t_{503}=…=t_{750}=21.42}

T(n)＝13077

Obviously, this payoff does not satisfy individual rational condition (7). The users who use the route L_3 can't accept it and will withdraw from the cooperative coalition. So system optimum model is also not practical for the traffic flows routing.

From the above analysis, we can find out that it is very difficult to achieve system optimum or user equilibrium optimum in reality. In fact, traffic routing problem of road network is different from a classic routing of computer or telecommunication network [Eitan *et al*., 2002]. Firstly, packets in these networks blindly go where they are routed, drivers do not. Secondly, packets can be dropped, but drivers must complete their trips. Among these difference, ration, behaviors and preferences are the distinctive difference. Just because of this reason, we can reconcile traffic manager and drivers' benefits to obtain a mutually satisfactory solution through the negotiation. Each side makes certain concession and sets up a benefits balanced mechanism through the negotiation. Final solution should be acceptable for traffic manager, and at the same time drivers are satisfied with the solution. In order to achieve satisfactory solution of traffic routing, we introduce the concept of satisfactory degree and use double-objective optimization to study the feasible model of routing.

3 INTEGRATED-EQUILIBRIUM ROUTING MODEL

Traffic manager and drivers have different objectives. Each driver is mandated by the desire to minimize an individual cost function, namely, his travel time. The goal of the manager is to find a routing strategy which drivers the traffic flows to the system optimum and achieves the most efficient utilization of system resources. Is there a balanced distribution of benefits among drivers and traffic manager? What is strategy profile which can be accepted by both traffic manager and drivers? How to obtain the good strategy? Obviously, these problems involve multi-objective decision-making process of drivers and traffic manager.

Though traffic manager wishes to minimize system total travel time, it is acceptable that only very little time is beyond the minimum travel time. The different is only the degree of satisfaction. Let T_{SO} be the system total travel time when achieving system optimum. (T－T_{SO})/ T_{SO} denotes that more time spent on travel constitutes the percentage of minimum system total travel time T_{SO}. Let $S_S = 1 - \dfrac{T - T_{SO}}{T_{SO}}$ denote the system satisfactory degree. If system total travel time achieves the minimum, system satisfactory degree is 100%. If system total travel time is a% more than minimum travel time, then system satisfactory degree is 1－a%. We stipulate that S_S＝0 when T⩾2T_{SO}, 0⩽S_S ⩽1, as shown in Figure 3.

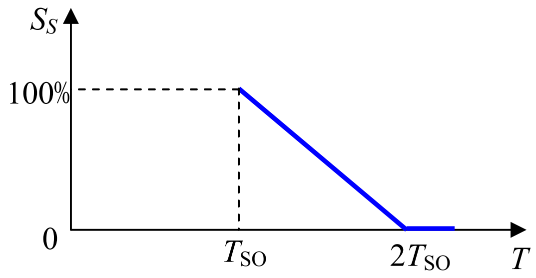

Fig.3 System satisfactory degree

Similarly, we can define user satisfactory degree

$$S_U = 1 - \frac{t - t_{UE}}{t_{UE}} \tag{9}$$

Where, t_{UE} is the travel time from origin to destination when user equilibrium optimum is achieved. Stipulate that S_U＝100% when t⩽t_{UE} and S_U＝0 when t⩾2t_{UE}, 0⩽S_U⩽1. According to these definitions, for Table1, system satisfactory degree is 84.8%, and user satisfactory degree is 100%. For Table2, system satisfactory degree is 100%, and the satisfactory degree of users who use the route L_1 and L_2 is 100%, while the satisfactory degree of users who use the route L_3 is 93.33 %(there are 249 drivers).

3.1 Model formulation

In this section, let us proceed with the modelling. Traffic network is made up of different links and nodes. One OD pair can have different routes. Let a directed graph G (N, A) denote the road network. Let

N be the set of nodes in the network;

A be the set of links in the network;

R be the set of source nodes;

S be the set of sink nodes;

$R_{r,s}$ be the set of the routes between origin-destination pair r-s, $R_{r,s}$＝{ $r_{rs}^1, r_{rs}^2, \cdots, r_{rs}^l$ },it also is the strategy space of drivers;

I be the set of drivers,I＝{1,2,…….n};

x_a be the flow on the link a;

$t_a(x_a)$be the travel time on link a described as a function of link flow x_a;

f_{rs}^k be the number of drivers who choose the *k*th route r_{rs}^k, f＝{ $f_{rs}^1, f_{rs}^2, \cdots, f_{rs}^l$ } is called a routing strategy profile.

The traffic manager aims at optimizing the overall system performance, namely, minimizing the total travel time, which can be stated as the sum of the time on all links for all vehicles. Beckman stated the mathematical programming formulation for Wardrop's second principle (SO). We use the formulation in the paper.

$$\begin{cases} Z_1 = \min \sum_{a \in A} t_a(x_a) x_a \\ s.t. \quad \sum_k f_{rs}^k = n \\ \quad f_{rs}^k \ge 0 \quad \forall\, k, r, s \\ \quad x_a = \sum_{r,s} \sum_k f_{rs}^k \delta_{ak}^{rs} \quad \forall\, a \end{cases} \tag{10}$$

Where, δ_{ak}^{rs} is 1 if route k between O-D pair r-s includes link a, and 0 otherwise.

As previous mentioned, it is acceptable that only very little time is beyond the minimum travel time.

$$Z_1 \le T_{SO} + \varepsilon_S \tag{11}$$

Where, $\varepsilon_S = (1 - S_S) T_{SO}$. So, the decision model of traffic manager with system satisfactory degree is:

$$\begin{cases} Z_1 = \min_f \sum_{a \in A} t_a(x_a) x_a \\ s.t \quad Z_1 \le T_{SO} + \varepsilon_S \\ \quad \varepsilon_S = (1 - S_S) T_{SO} \\ \quad \sum_k f_{rs}^k = n \\ \quad f_{rs}^k \ge 0 \quad \forall\, k, r, s \\ \quad x_a = \sum_{r,s} \sum_k f_{rs}^k \delta_{ak}^{rs} \quad \forall\, a \end{cases} \tag{12}$$

The drivers aim at optimizing their own performance, namely, minimizing their own travel time. Beckman stated the mathematical programming formulation for Wardrop's first principle (UE).

$$\begin{cases} Z_2 = \min \sum_{a \in A} \int_0^{x_a} t_a(w) dw \\ s.t. \quad \sum_k f_{rs}^k = n \\ \quad f_{rs}^k \ge 0 \quad \forall\, k, r, s \\ \quad x_a = \sum_{r,s} \sum_k f_{rs}^k \delta_{ak}^{rs} \quad \forall\, a \end{cases} \tag{13}$$

Similarly, the decision model of drivers with user satisfactory degree is:

$$\begin{cases} Z_2 = \min \sum_{a \in A} \int_0^{x_a} t_a(w) dw \\ s.t. \quad Z_2 \le t_{UE} + \varepsilon_i \\ \quad \varepsilon_i = (1 - S_{Ui}) t_{UE} \quad i = 1, 2, \cdots, n \\ \quad \sum_k f_{rs}^k = n \\ \quad f_{rs}^k \ge 0 \quad \forall\, k, r, s \\ \quad x_a = \sum_{r,s} \sum_k f_{rs}^k \delta_{ak}^{rs} \quad \forall\, a \end{cases} \tag{14}$$

In order to achieve a balanced distribution of benefits among drivers and traffic manager, double-objective optimization is used to model. The integrated-equilibrium model with satisfactory degree is as follows,

Objective A $$Z_1 = \min \sum_{a \in A} t_a(x_a) x_a$$

Objective B $$Z_2 = \min \sum_{a \in A} \int_0^{x_a} t_a(w) dw$$

$$\begin{aligned} s.t \quad & Z_1 \le T_{SO} + \varepsilon_S \\ & \varepsilon_S = (1 - S_S) T_{SO} \\ & \sum_k f_{rs}^k = n \\ & f_{rs}^k \ge 0 \quad \forall\, k, r, s \\ & x_a = \sum_{r,s} \sum_k f_{rs}^k \delta_{ak}^{rs} \quad \forall\, a \\ & Z_2 \le t_{UE} + \varepsilon_i \\ & \varepsilon_i = (1 - S_{Ui}) t_{UE} \quad i = 1, 2, \cdots, n \end{aligned}$$

Satisfactory degree applied, the degree of user satisfactory is improved through sacrificing a little system benefits. The model balances system and user's benefits and the users are satisfied on the basis of system optimum.

3.2 Solution algorithm

A solution, which is acceptable for the traffic manager, can be obtained through solving the decision problem of objective A. If the drivers are satisfied with the solution, then stop. Otherwise, user satisfactory degree S_U is reduced, and the routing strategy profile $f^{(k)}$ is obtained under this user satisfactory degree. Repeat above process until obtain the solution with which both traffic manager and drivers are satisfied. Therefore, an algorithm of the model is the following:

Step0: Set S_S =100%, S_{Ui}=100 %(i=1,2,…,l),and set k=1;

Step1: Solve the system optimum and user optimum using the Frank-Wolfe algorithm, T_{SO}, t_{UE} and initial routing strategy profile $f^{(0)}$ are obtained;

Step2: If the $f^{(0)}$ satisfies objective B, then stop. Otherwise go to Step3;

Step3: The user satisfactory degree S_{Ui} is reduced by k *1%, and obtain the routing strategy profile $f^{(k)}$ under this user satisfactory degree S_{Ui};

Step4: The system satisfactory degree S_S is reduced by k *1%, if the $f^{(k)}$ satisfies objective A, then stop; Otherwise, k=k+1, go to Step3.

For Figure1, the results of the integrated-equilibrium model are as shown in Table 3.

Compared with the results of system optimum, user satisfactory degree of L_3 rises to 96.96% from 93.33% though system total travel time is more. Traffic manager can accept the result, and users are satisfied too. Compared with the results of user optimum, system satisfactory degree rise to 98.5% from 84.8% though travel time of drivers on route L_3 rises to 20.69 from 20.08. In addition, vehicles do not congregate on the shortest route L_1 and do not make L_1 overload seriously. We can find out that the solution obtained by the integrated-equilibrium model is acceptable for traffic manager, and at the same time drivers are satisfied whit the solution.

Table3 Results of integrated-equilibrium

Routes	Flows	Travel time	Satisfactory degree
L_1	260	14.28	100%
L_2	282	18.64	100%
L_3	208	20.69	96.96%
Total travel time	13272.8		98.50%

4. NUMERICAL EXAMPLE

In order to strengthen the understanding of the problem, a simple example is given now. Consider the following simple road network as Figure 4 shown. The demand is 2000 trips from 1 to 4.

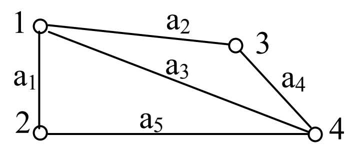

Fig.4 A simple road network

The data of road network is as Table 4 shown.

Table4 The data of road network

Links	Nodes	Distance (km)	Levels of Road	Speed (km/h)
a_1	1-2	3	3	60
a_2	1-3	4	2	80
a_3	1-4	6	3	60
a_4	3-4	3	1	100
a_5	2-4	5	2	80

Traffic routing is carried on using user equilibrium optimum, system optimum and integrated-equilibrium respectively, the results are as shown in Table 5.

Table 5 The results

Model	Routes	Flows	Travel time	Satisfactory degree
user optimum	a_1 a_5	41	6.750	100%
	a_3	764	6.750	100%
	a_2 a_4	1195	6.750	100%
	Total travel time	13500.00		88.27%
system optimum	a_1 a_5	526	6.899	97.79%
	a_3	608	6.299	100%
	a_2 a_4	866	5.339	100%
	Total travel time	12082.18		100%
Integrated-equilibrium	a_1 a_5	431	6.818	98.99%
	a_3	659	6.414	100%
	a_2 a_4	910	5.456	100%
	Total travel time	12129.91		99.60%

The satisfactory degrees are shown in Fig. 5.

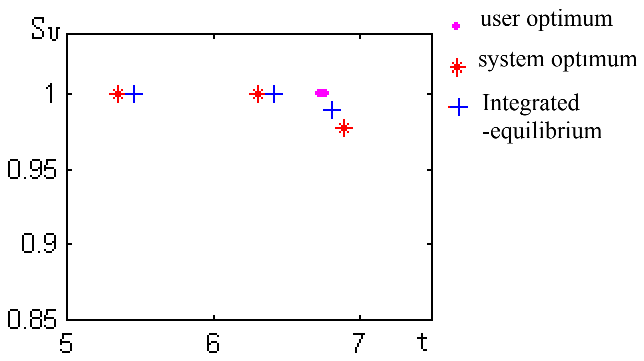

Fig.5-a User Satisfactory degree

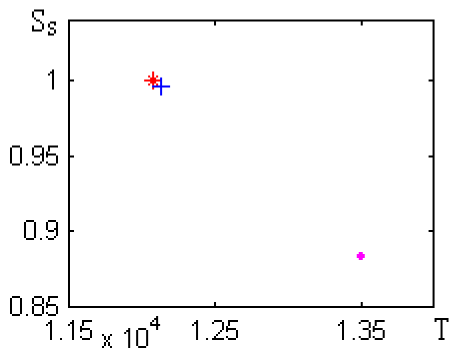

Fig.5-b System satisfactory degree

We can find out that integrated-equilibrium model achieves user satisfaction on the basis of system optimization and traffic flows are guided more efficiently and rationally. The solutions to the traffic routing problem are acceptable for individual driver and the network as a whole.

5. CONCLUSIONS AND FUTURE WORK

This paper applied the game theory to study the traffic routing and proposed a concept of satisfactory degree. The approach seeks to achieve a more optimal traffic routing. The Integrated-Equilibrium model based on double-objective optimization is proposed. This model has certain practical value and offers the new thought and method for solving the conflict between user optimum and system optimum. And the model in this paper can also be applied in the other routing problem, for example, telecommunication network. In addition, game theory also can explain congestion mechanism, behaviour characteristic of travelers to a certain extent.

It should be noted that our analysis depends on some assumes such as that a driver does not change the original route on midway. The extent to which these assumes can be generalized is important subject for further work. The fields of dynamic games should provide some insights into this problem. Only static traffic routing is studied in this paper, dynamic traffic routing and many-OD of the Integrated-Equilibrium model should be deeply analyzed and discussed.

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