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Fault Detection for Networked Systems with Incomplete Measurements ⋆

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Abstract: This paper investigates the fault detection problem for a class of discrete-time networked systems. Three kinds of typically encountered incomplete measurements induced by the limited-capacity communication channel are simultaneously considered, which include 1) measurements with communication delays; 2) measurements with data missing; and 3) measurements with signal quantization. Attention is focused on the analysis and design of a full-order fault detection filter such that, for all admissible unknown inputs and incomplete measurements, the error between residual and fault is kept as small as possible. Sufficient conditions for the existence of the desired fault detection filter are first established in terms of certain linear matrix inequalities (LMIs), then the explicit expression of the desired filter is characterized when these LMIs are feasible. A simulation example is provided to illustrate the effectiveness and applicability of the proposed method.

1. INTRODUCTION

With the increasing complexity and safety demand of real time systems, fault detection and isolation (FDI) and fault tolerant control (FTC) problems have attracted persistent research attention in recent years, see Hou et al. (1998); Patton (2000); Frank et al. (2000); Kinnaert (2003). The key step of an FDI process is the generation of a residual, which can be further compared with a prescribed threshold to determine whether a fault occurs. In the case that exact decoupling can not be achieved, one of the widely adopted ways is to introduce a performance index and convert the FDI problem into an optimization problem, in which the H_∞ norm of transfer function matrix from unknown input to residual is made small, while the H_∞ norm (or the smallest nonzero singular value) of transfer function from fault to residual is made large (Ding et al. (2000)). In Zhong et al. (2003), the robust fault detection problem has been solved as an H_∞ model match problem. In Zhong et al. (2005), another approach to robust fault detection problem has been proposed to make the error between residual and weighted fault as small as possible, which can be implemented with help from the robust H_∞ filtering approach (Gao et al. (2004)).

On the other hand, rapid developments in network technologies have led to more and more control systems based on a network. This kind of networked control systems (NCSs) have many advantages, such as low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability (Zhang et al. (2001)). At the same time, the limited capacity of network cable gives rise to new interesting and challenging issues such as transmission delay, packet dropout, signal quantization, scheduling confusion, etc. Therefore, there is an urgent need to develop new FDI methodologies in the network environment. Recently, there have been some initial research interests on network-based FDI problem. In Zhang et al. (2004), the fault detection problem for systems with missing measurements has been discussed. In He et al. (2007a), the fault detection problem has been dealt with in the presence of both communication delay and measurement missing, which are taken into account in a unified way. For more results, see survey paper Fang et al. (2006) and the references therein.

In the present paper, a new network-based fault detection problem is investigated. Three kinds of typically encountered incomplete measurements induced by the finite-bit communication channel, namely, measurements with communication delays, measurements with data missing and measurements with signal quantization, are simultaneously considered. A novel measurement model including a set of Kronecker delta functions is proposed to describe the delay and missing phenomenon in a unified way. The quantization is assumed to be of the logarithmic type. By augmenting the states of the original system and the fault detection filter, the addressed fault detection problem is converted into an auxiliary H_∞ filtering problem for an uncertain parameter system. Sufficient conditions for the existence of the desired fault detection filter are established

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in terms of certain linear matrix inequalities (LMIs), and a numerical example is provided to illustrate the effectiveness and applicability of the proposed design method.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following class of discrete-time linear system with multiple delays in the state:

$$\begin{cases} x_{k+1} &= A_0 x_k + \sum_{i=1}^{q} A_i x_{k-i} + B_w w_k + B_f f_k, \\ x_k &= \varphi_k, \quad k = -q, -q+1, \ldots, 0, \end{cases} \tag{1}$$

where $x_k \in R^n$ is the state vector; $w_k \in R^p$ is the unknown input belonging to $l_2[0, \ \infty)$; $f_k \in R^l$ is the fault to be detected; $i = 1, \ldots, q$ are known constant time delays. φ_k is a given real initial sequence on $[-q, 0]$. The system matrices have appropriate dimensions and are assumed to be known real constant matrices.

The insertion of network cable between measurement node and fault detection filter causes random delay and missing phenomena. Furthermore, by taking the logarithmic quantization into consideration, the measurement can be described by

$$\begin{aligned} \tilde{y}_k &= \delta(\tau_k, 0)(I + \Delta_k) C_0 x_k \\ &\quad + \sum_{i=1}^{q} \delta(\tau_k, i)(I + \Delta_k) C_i x_{k-i} + (I + \Delta_k) D_w w_k, \end{aligned} \tag{2}$$

where $\tilde{y}_k \in R^m$ is the input signal to the fault detection filter, which may contain random communication delays, stochastic data missing, and quantization effect. $\delta(\tau_k, i)$ $(0 \le i \le q)$ are Kronecker delta functions with $E\{\delta(\tau_k, i)\} = Prob\{\tau_k = i\} = p_i$, where p_i $(0 \le i \le q)$ are known positive scalars and $\sum_{i=0}^{q} p_i \le 1$. τ_k is a stochastic variable used to determine, at time k, how large the occurred delay is and the possibility of data missing.

Remark 1. From the definition stated in Fu et al. (2005), the logarithmic effect can be then can be transformed into *sector bound uncertainties*. Defining

$$\Delta_k = \mathrm{diag}\{\Delta_k^{(1)}, \ldots, \Delta_k^{(m)}\},$$

the measurements after quantization can be expressed as $\tilde{y}_k = (I + \Delta_k) y_k$.

Remark 2. The measurement model in (6) can easily describe the delay and missing phenomena in a unified way. At any time instant, one of the following cases (random events) occurs: measurement missing case, no time-delay case, one-step delay case, two-step delay case, …, q-step delay case. Similar to Wang et al. (2003, 2006), the sequence of τ_k is assumed to be mutually independent, see He et al. (2007b) for detailed discussion about this measurement model.

Consider a full-order fault detection filter of the form

$$\begin{cases} \tilde{x}_{k+1} &= G \tilde{x}_k + K \tilde{y}_k, \\ r_k &= L \tilde{x}_k, \end{cases} \tag{3}$$

where $\tilde{x}_k \in R^n$ is the filter state vector, $r_k \in R^l$ is the so-called residual that is compatible with the fault vector. Our aim is to design a filter of the type (3) that makes the error between residual and fault signal as small as possible. By defining $\bar{\Delta} = diag\{\delta_1, \ldots, \delta_m\}$ and $F_k = \Delta_k \bar{\Delta}^{-1}$, we obtain an unknown real-valued time-varying matrix satisfying $F_k F_k^T \le I$. Furthermore, letting

$$\eta_k = \begin{bmatrix} x_k \\ \tilde{x}_k \end{bmatrix}, \quad v_k = \begin{bmatrix} w_k \\ f_k \end{bmatrix}, \quad \text{and } \tilde{r}_k = r_k - f_k, \tag{4}$$

we have the overall fault detection dynamics governed by

$$\begin{cases} \eta_{k+1} &= \tilde{\mathcal{A}}_0 \eta_k + [\delta(\tau_k, 0) - p_0] \bar{\mathcal{A}}_0 \eta_k + \sum_{i=1}^{q} \tilde{\mathcal{A}}_i Z \eta_{k-i} \\ &\quad + \sum_{i=1}^{q} [\delta(\tau_k, i) - p_i] \bar{\mathcal{A}}_i Z \eta_{k-i} + \tilde{\mathcal{B}} v_k, \\ \tilde{r}_k &= \tilde{\mathcal{C}} \eta_k + \tilde{\mathcal{D}} v_k, \end{cases} \tag{5}$$

where

$$\begin{aligned} &\tilde{\mathcal{A}}_0 = \begin{bmatrix} A_0 & 0 \\ p_0 K (I + \Delta_k) C_0 & G \end{bmatrix} = \tilde{A}_0 + p_0 \tilde{H} F_k \tilde{E}_0, \\ &\bar{\mathcal{A}}_0 = \begin{bmatrix} 0 & 0 \\ K (I + \Delta_k) C_0 & 0 \end{bmatrix} = \bar{A}_0 + \tilde{H} F_k \tilde{E}_0, \\ &\tilde{\mathcal{A}}_i = \begin{bmatrix} A_i \\ p_i K (I + \Delta_k) C_i \end{bmatrix} = \tilde{A}_i + p_i \tilde{H} F_k \tilde{E}_i, \\ &\bar{\mathcal{A}}_i = \begin{bmatrix} 0 \\ K (I + \Delta_k) C_i \end{bmatrix} = \bar{A}_i + \tilde{H} F_k \tilde{E}_i, \\ &\tilde{\mathcal{B}} = \begin{bmatrix} B_w & B_f \\ K (I + \Delta_k) D_w & 0 \end{bmatrix} = \tilde{B} + \tilde{H} F_k \tilde{J}, \\ &\tilde{A}_0 = \begin{bmatrix} A_0 & 0 \\ p_0 K C_0 & G \end{bmatrix}, \bar{A}_0 = \begin{bmatrix} 0 & 0 \\ K C_0 & 0 \end{bmatrix}, \tilde{A}_i = \begin{bmatrix} A_j \\ p_j K C_j \end{bmatrix}, \\ &\bar{A}_i = \begin{bmatrix} 0 \\ K C_j \end{bmatrix}, \ \bar{B} = \begin{bmatrix} B_w & B_f \\ K D_w & 0 \end{bmatrix}, \ \tilde{H} = \begin{bmatrix} 0 \\ K \end{bmatrix}, \\ &\tilde{E}_0 = \begin{bmatrix} \bar{\Delta} C_0 & 0 \end{bmatrix}, \ \tilde{E}_i = \bar{\Delta} C_i, \ \tilde{J} = \begin{bmatrix} \bar{\Delta} D_w & 0 \end{bmatrix}, \\ &\tilde{\mathcal{C}} := \begin{bmatrix} 0 & L \end{bmatrix}, \ \tilde{\mathcal{D}} := \begin{bmatrix} 0 & -I \end{bmatrix}, \ Z := \begin{bmatrix} I & 0 \end{bmatrix}. \end{aligned} \tag{6}$$

After the above manipulations, the network-based fault detection filter design (NFDF) problem can now be formulated as an H_∞ filtering problem: design an H_∞ filter of the form (3) for the system (1) and (2) such that, for all possible incomplete measurements, (i) the overall fault detection dynamics (5) is exponentially mean-square stable (Wang et al. (2006)) when $w_k = 0$, $f_k = 0$ and, (ii) under zero initial condition, the infimum of γ is made small in the feasibility of

$$\sup_{v_k \neq 0} \frac{E\{\|\tilde{r}_k\|^2\}}{\|v_k\|^2} < \gamma^2, \quad \gamma > 0 \tag{7}$$

We adopt a residual evaluation stage including an evaluation function J_k and a threshold J_{th} of the following form

$$J_k = \left\{ \sum_{s=0}^{k} r_s^T r_s \right\}^{1/2}, \quad J_{th} = \sup_{w \in l_2, f=0} E\{J_k\}. \tag{8}$$

Based on (8), the occurrence of faults can be detected by comparing J_k with J_{th} according to the following rule:

$$\begin{aligned} J_k > J_{th} &\Rightarrow \text{with faults} \Rightarrow \text{alarm}, \\ J_k \le J_{th} &\Rightarrow \text{no faults}. \end{aligned}$$

3. MAIN RESULTS

In this section, we deal with NFDF analysis and NFDF design problems for system (1) in the presence of incomplete measurements (2), and the results are given in the form of LMI.

3.1 NFDF Analysis

The following Lemmas will be useful in deriving our main results in the sequel.

Lemma 3. (Wang et al. (2006)) Let $M = M^T$, H and E be real matrices of appropriate dimensions, with F satisfying $F_k F_k^T \leq I$, then

$$M + HFE + E^T F^T H^T < 0, \tag{9}$$

if and only if, there exists a positive scalar $\varepsilon > 0$ such that

$$M + \frac{1}{\varepsilon} HH^T + \varepsilon E^T E < 0, \tag{10}$$

or equivalently

$$\begin{bmatrix} M & H & \varepsilon E^T \\ H^T & -\varepsilon I & 0 \\ \varepsilon E & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{11}$$

Lemma 4. Consider system (1) and (2). For a given fault detection filter (3) and a scalar $\gamma > 0$, the fault detection dynamics (5) is exponentially mean-square stable when $v_k = 0$ and, under zero initial conditions, satisfies (7) if there exist matrices $0 < P^T = P \in R^{2n\times 2n}$, $0 < Q_i^T = Q_i \in R^{n\times n}$ such that the following LMIs

$$\begin{bmatrix} -I & 0 & \Omega_{13} \\ * & \Omega_{22} & \Omega_{23} \\ * & * & \Omega_{33} \end{bmatrix} < 0 \tag{12}$$

hold, where

$$\Omega_{13} = \begin{bmatrix} \tilde{\mathcal{C}} & 0 & \tilde{\mathcal{D}} \end{bmatrix}, \quad \Omega_{22} = -\mathrm{diag}_{q+2}\{P\},$$

$$\Omega_{23} = \begin{bmatrix} 0 & \rho_d P_d \bar{\mathcal{A}}_d & 0 \\ \rho_0 P \bar{\mathcal{A}}_0 & 0 & 0 \\ P\tilde{\mathcal{A}}_0 & P\tilde{\mathcal{A}}_c & P\tilde{\mathcal{B}} \end{bmatrix}, \quad \Psi = \sum_{i=1}^{q} Z^T Q_i Z,$$

$$\Omega_{33} = \begin{bmatrix} -P+\Psi & 0 & 0 \\ * & -Q_d & 0 \\ * & * & -\gamma^2 I \end{bmatrix}, \quad P_d = \mathrm{diag}_q\{P\},$$

$$\mathrm{diag}_q\{\star\} = \mathrm{diag}\{\underbrace{\star \cdots \star}_{q}\}, \quad \bar{\mathcal{A}}_d = \mathrm{diag}\{\bar{\mathcal{A}}_1, \cdots, \bar{\mathcal{A}}_q\},$$

$$\tilde{\mathcal{A}}_c := \begin{bmatrix}\tilde{\mathcal{A}}_1 & \cdots & \tilde{\mathcal{A}}_q\end{bmatrix}, \quad Q_d = \mathrm{diag}\{Q_1, \cdots, Q_q\},$$
$$\rho_i = \sqrt{p_i} \ (0 \leq i \leq q), \quad \rho_d = \mathrm{diag}\{\rho_1 I_{2n}, \cdots, \rho_q I_{2n}\},$$

and $\tilde{\mathcal{A}}_0$, $\bar{\mathcal{A}}_0$ $\tilde{\mathcal{A}}_i$, $\bar{\mathcal{A}}_i$, $\tilde{\mathcal{B}}$, $\tilde{\mathcal{C}}$, $\tilde{\mathcal{D}}$, Z are defined in (6).

Proof. Consider the following Lyapunov functional:

$$V_k = \eta_k^T P \eta_k + \sum_{i=1}^{q} \sum_{j=k-i}^{k-1} \eta_j^T Z^T Q_i Z \eta_j, \tag{13}$$

where $P > 0$, $Q_i > 0$ $(i = 1, \ldots, q)$, and Z is given in (6). Define $\Delta V_k := E\{V_{k+1}(\Theta_{k+1})|\Theta_k\} - V_k(\Theta_k)$. Note that for $0 \leq i \leq q$ and $0 \leq j \leq q$,

$$E\{(\delta(\tau_k, i) - p_i)(\delta(\tau_k, j) - p_j)\} = \begin{cases} p_i(1-p_i), & i = j \\ -p_i p_j, & i \neq j \end{cases}$$

From (5), the difference of the Lyapunov functional (13) with $v_k = 0$ can be calculated as follows:

$$\begin{aligned}\Delta V_k = {} & \zeta_k^T \begin{bmatrix} M_1 & M_2 \\ * & M_3 \end{bmatrix} \zeta_k - \left(p_0 \bar{\mathcal{A}}_0 \eta_k + \sum_{i=1}^{q} p_i \bar{\mathcal{A}}_i Z \eta_{k-i} \right)^T \\ & \times P \left(p_0 \bar{\mathcal{A}}_0 \eta_k + \sum_{i=1}^{q} p_i \bar{\mathcal{A}}_i Z \eta_{k-i} \right) \\ \leq {} & \zeta_k^T \begin{bmatrix} M_1 & M_2 \\ * & M_3 \end{bmatrix} \zeta_k \end{aligned}$$

where $\zeta_k = [\eta_k^T \ \eta_{k-1}^T Z^T \cdots \eta_{k-q}^T Z^T]^T$, $M_1 = \tilde{\mathcal{A}}_0^T P \tilde{\mathcal{A}}_0 + \rho_0^2 \bar{\mathcal{A}}_0^T P \bar{\mathcal{A}}_0 - P + \Psi$, $M_2 = \tilde{\mathcal{A}}_0^T P \tilde{\mathcal{A}}_c$, $M_3 = \tilde{\mathcal{A}}_c^T P \tilde{\mathcal{A}}_c + \rho_d^2 \bar{\mathcal{A}}_d^T P_d \bar{\mathcal{A}}_d - Q_d$.

By the Schur Complement (Boyd et al. (1994)), LMI (12) implies $\Delta V_k < 0$ for all nonzero ζ_k, so we can always find a positive scalar $\vartheta > 0$ such that$\Delta V_k < -\vartheta \|\eta_k\|^2$. Furthermore, from Lemma 1 of Wang et al. (2006), we can confirm that the filtering error system (5) is exponentially mean-square stable.

Next, for any nonzero w_k, it follows from (5) and (13) that

$$\Xi(\Theta_k) \leq E\left\{ \chi_k^T \begin{bmatrix} \Gamma_1 & \Gamma_2 & \Gamma_3 \\ * & \Gamma_4 & \Gamma_5 \\ * & * & \Gamma_6 \end{bmatrix} \chi_k \right\}, \tag{14}$$

where $\Xi(\Theta_k) = E\{V_{k+1}(\Theta_{k+1})|\Theta_k\} - V_k(\Theta_k) + E\{\tilde{r}_k^T \tilde{r}_k\} - \gamma^2 E\{v_k^T v_k\}$, $\chi_k = [\eta_k^T \ \eta_{k-1}^T Z^T \ \cdots \ \eta_{k-q}^T Z^T \ v_k^T]^T$, $\Gamma_1 = \tilde{\mathcal{A}}_0^T P \tilde{\mathcal{A}}_0 + \rho_0^2 \bar{\mathcal{A}}_0^T P \bar{\mathcal{A}}_0 - P + \Psi + \tilde{\mathcal{C}}^T \tilde{\mathcal{C}}$, $\Gamma_2 = \tilde{\mathcal{A}}_0^T P \tilde{\mathcal{A}}_c$, $\Gamma_3 = \tilde{\mathcal{C}}^T \tilde{\mathcal{D}} + \tilde{\mathcal{A}}_0^T P \tilde{\mathcal{B}}$, $\Gamma_4 = \tilde{\mathcal{A}}_c^T P \tilde{\mathcal{A}}_c + \rho_d^2 \bar{\mathcal{A}}_d^T P_d \bar{\mathcal{A}}_d - Q_d$, $\Gamma_5 = \tilde{\mathcal{A}}_c^T P \tilde{\mathcal{B}}$, $\Gamma_6 = \tilde{\mathcal{D}}^T \tilde{\mathcal{D}} + \tilde{\mathcal{B}}^T P \tilde{\mathcal{B}} - \gamma^2 I$.

Again, using Schur complement, it can be observed from (12) and (14) that, for any χ_k and v_k that are not all zero, $\Xi(\Theta_k) < 0$. Summing up this inequality from 0 to ∞ with respect to k yields

$$\sum_{k=0}^{\infty} E\{\|\tilde{r}_k\|^2\} < \gamma^2 \sum_{k=0}^{\infty} E\{\|v_k\|^2\} + E\{V_0\} - E\{V_\infty\}. \tag{15}$$

Since the system (5) is exponentially mean-square stable, it is straightforward to see that (7) holds under the zero initial condition. This concludes the proof.

Based on Lemma 4, we further give the analysis result of NFDF.

Theorem 5. Consider the matrices defined in (6), inequality (12) is equivalent to

$$\begin{bmatrix} -I & 0 & \Omega_{13} & 0 & 0 \\ * & \Omega_{22} & \Gamma_{23} & \Gamma_{24} & 0 \\ * & * & \Omega_{33} & 0 & \Gamma_{35} \\ * & * & * & \Gamma_{44} & 0 \\ * & * & * & 0 & \Gamma_{55} \end{bmatrix} < 0, \tag{16}$$

where

$$\Gamma_{23} = \begin{bmatrix} 0 & \rho_d P_d \bar{A}_d & 0 \\ \rho_0 P \bar{A}_0 & 0 & 0 \\ P\tilde{A}_0 & P\tilde{A}_c & P\tilde{B} \end{bmatrix},$$

$$\Gamma_{35} = \begin{bmatrix} 0 & \varepsilon_a \tilde{E}_0^T & \varepsilon_b p_0 \tilde{E}_0^T \\ \bar{\Sigma}_d \tilde{E}_d^T & 0 & \varepsilon_b p_d \tilde{E}_c^T \\ 0 & 0 & \varepsilon_b \tilde{J}^T \end{bmatrix},$$

and $\Gamma_{24} = diag\left\{\rho_d P_d \tilde{H}_d,\ \rho_0 P\tilde{H},\ P\tilde{H}\right\}$, $\Gamma_{44} = \Gamma_{55} = diag\{-\Sigma_d, -\varepsilon_a I_m, -\varepsilon_b I_m\}$, $\bar{A}_d = \text{diag}\{\bar{A}_1, \cdots, \bar{A}_q\}$, $\tilde{A}_c = [\tilde{A}_1\ \cdots\ \tilde{A}_q]$, $\tilde{H}_d = diag_q\{\tilde{H}\}$, $\tilde{E}_d = diag\{\tilde{E}_1, \cdots, \tilde{E}_q\}$, $\tilde{E}_c = [\tilde{E}_1, \cdots, \tilde{E}_q]$, $p_d = \text{diag}\{p_1 I_n, \cdots, p_q I_n\}$, $\Sigma_d = \text{diag}\{\varepsilon_1 I_m, \cdots, \varepsilon_q I_m\}$, $\bar{\Sigma}_d = \text{diag}\{\varepsilon_1 I_n, \cdots, \varepsilon_q I_n\}$.

Proof. Considering the parameters defined in (6) and using Lemma 3, we can easily obtain the equivalence between (12) and (16) after some appropriate row and column exchanges.

3.2 NFDF Design

Theorem 5 provides a sufficient condition for the filtering error system (5) to be exponentially mean-square stable and also achieve the H_∞-norm constraint (7). From now on, we focus on the NFDF design problems for system (1) with incomplete measurement of the form (2).

Theorem 6. Consider the networked system (1) with incomplete measurement (2) and let $\gamma > 0$ be a given scalar. An admissible full-order fault detection filter of the form (3) exists if there exist matrices $V \in R^{n\times n}$, $F \in R^{n\times n}$, $\bar{G} \in R^{n\times n}$, $\bar{K} \in R^{n\times m}$, $\bar{L} \in R^{l\times n}$ and $0 < Q_i^T = Q_i \in R^{n\times n}$ $(i = 1, \ldots, q)$ such that the following LMI

$$\begin{bmatrix} -I & 0 & \Lambda_{13} & 0 & 0 \\ * & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} & 0 \\ * & * & \Lambda_{33} & 0 & \Lambda_{35} \\ * & * & * & \Lambda_{44} & 0 \\ * & * & * & * & \Lambda_{55} \end{bmatrix} < 0, \tag{17}$$

holds, where

$$\Lambda_{13} = \left[\ [0\ \bar{L}]\quad 0\quad [0\ -I]\ \right],\ \Lambda_{23} = \begin{bmatrix} 0 & \Phi_2 & 0 \\ \Phi_3 & 0 & 0 \\ \Phi_4 & \Phi_5 & \Phi_6 \end{bmatrix},$$

$$\Lambda_{22} = -\text{diag}\left\{\begin{bmatrix} V_d & F_d \\ * & F_d \end{bmatrix}, \begin{bmatrix} V & F \\ * & F \end{bmatrix}, \begin{bmatrix} V & F \\ * & F \end{bmatrix}\right\},$$

$$\Lambda_{24} = \text{diag}\left\{\begin{bmatrix} \bar{\rho}_d \bar{K}_d \\ 0 \end{bmatrix}, \begin{bmatrix} \rho_0 \bar{K} \\ 0 \end{bmatrix}, \begin{bmatrix} \bar{K} \\ 0 \end{bmatrix}\right\},$$

$$\Lambda_{33} = \text{diag}\left\{-\begin{bmatrix} V & F \\ * & F \end{bmatrix} + \Phi_8,\ -Q_d,\ -\gamma^2 I_{p+l}\right\},$$

$$\Lambda_{35} = \begin{bmatrix} 0 & \Phi_9 & \Phi_{10} \\ \bar{\Sigma}_d C_d^T \bar{\Delta}_d & 0 & \varepsilon_b p_d C_d^T \bar{\Delta}_c^T \\ 0 & 0 & \Phi_{11} \end{bmatrix},$$

$$\Lambda_{44} = \Lambda_{55} = diag\{-\Sigma_d,\ -\varepsilon_a I,\ -\varepsilon_b I\},$$

$$\Phi_2 = \begin{bmatrix} \bar{\rho}_d \bar{K}_d C_d \\ 0 \end{bmatrix},\ \Phi_3 = \begin{bmatrix} \rho_0 \bar{K} C_0 & \rho_0 \bar{K} C_0 \\ 0 & 0 \end{bmatrix},$$

$$\Phi_4 = \begin{bmatrix} VA_0 + p_0 \bar{K} C_0 & VA_0 + p_0 \bar{K} C_0 + \bar{G} \\ FA_0 & FA_0 \end{bmatrix},$$

$$\Phi_5 = \begin{bmatrix} VA_c + \bar{K} C_c p_d \\ FA_c \end{bmatrix}, \Phi_6 = \begin{bmatrix} VB_w + \bar{K} D_w & VB_f \\ FB_w & FB_f \end{bmatrix},$$

$$\Phi_8 = \sum_{i=1}^{q} \begin{bmatrix} Q_i & Q_i \\ * & Q_i \end{bmatrix},\ \Phi_9 = \varepsilon_a \begin{bmatrix} C_0^T \bar{\Delta} \\ C_0^T \bar{\Delta} \end{bmatrix},$$

$$\Phi_{10} = \varepsilon_b p_0 \begin{bmatrix} C_0^T \bar{\Delta} \\ C_0^T \bar{\Delta} \end{bmatrix},\quad \Phi_{11} = \varepsilon_b \begin{bmatrix} D_w^T \bar{\Delta} \\ 0 \end{bmatrix},$$

and $\bar{\rho}_d = \text{diag}\{\rho_1 I_n, \cdots, \rho_q I_n\}$, $V_d = \text{diag}_q\{V\}$, $F_d = \text{diag}_q\{F\}$, $\bar{K}_d = \text{diag}_q\{\bar{K}\}$, $C_d = \text{diag}\{C_1, \ldots, C_q\}$, $A_c = [A_1 \ldots A_q]$, $C_c = [C_1 \ldots C_q]$, $\bar{\Delta}_d = \text{diag}_q\{\bar{\Delta}\}$, $\bar{\Delta}_c = [\bar{\Delta}, \ldots, \bar{\Delta}]$. Moreover, if (17) is true, the desired filter parameters can be given by

$$G = (F - V)^{-1}\bar{G},\quad K = (F - V)^{-1}\bar{K},\quad L = \bar{L}. \tag{18}$$

Proof. Considering that P is positive definite, we partition P and P^{-1} as:

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix},\quad P^{-1} = \begin{bmatrix} S_1 & S_2 \\ S_2^T & S_3 \end{bmatrix}, \tag{19}$$

where the partitioning of the above matrices is compatible with that of $\tilde{A}_0$ defined in (6). Introducing the following matrices,

$$T_1 = \begin{bmatrix} P_1 & I \\ P_2^T & 0 \end{bmatrix},\quad T_2 = \begin{bmatrix} I & S_1 \\ 0 & S_2^T \end{bmatrix}, \tag{20}$$

and performing the congruence transformation to (16) by $\text{diag}\{I, \text{diag}_q\{T_2\}, T_2, T_2, T_2, I, I, I, I, I, I, I, I\}$, we can obtain

$$\begin{bmatrix} -I & 0 & \hat{\Lambda}_{13} & 0 & 0 \\ * & \hat{\Lambda}_{22} & \hat{\Lambda}_{23} & \hat{\Lambda}_{24} & 0 \\ * & * & \hat{\Lambda}_{33} & 0 & \hat{\Lambda}_{35} \\ * & * & * & \Lambda_{44} & 0 \\ * & * & * & * & \Lambda_{55} \end{bmatrix} < 0, \tag{21}$$

where

$$\hat{\Lambda}_{13} = \left[\ [0\ LS_2^T]\quad 0\quad [0\ -I]\ \right],\ \hat{\Lambda}_{22} = diag_{q+2}\left\{\hat{\Phi}_1\right\},$$

$$\hat{\Phi}_1 = \begin{bmatrix} -P_1 & -I \\ * & -S_1 \end{bmatrix},\ \hat{\Lambda}_{23} = \begin{bmatrix} 0 & \hat{\Phi}_2 & 0 \\ \hat{\Phi}_3 & 0 & 0 \\ \hat{\Phi}_4 & \hat{\Phi}_5 & \hat{\Phi}_6 \end{bmatrix},$$

$$\hat{\Lambda}_{24} = \text{diag}\left\{\rho_d \text{diag}_q\left\{\hat{\Phi}_7\right\},\ \rho_0\hat{\Phi}_7,\ \hat{\Phi}_7\right\},$$

$$\hat{\Lambda}_{33} = diag\left\{\hat{\Phi}_1 + \hat{\Phi}_8,\ -Q_d,\ -\gamma^2 I_{p+l}\right\},$$

$$\hat{\Lambda}_{35} = \begin{bmatrix} 0 & \hat{\Phi}_9 & \hat{\Phi}_{10} \\ \bar{\Sigma}_d C_d^T \bar{\Delta}_d & 0 & \varepsilon_b p_d C_d^T \bar{\Delta}_c^T \\ 0 & 0 & \Phi_{11} \end{bmatrix},$$

$$\hat{\Phi}_2 = diag\left\{\begin{bmatrix} \rho_1 P_2 K C_1 \\ 0 \end{bmatrix}, \cdots, \begin{bmatrix} \rho_q P_2 K C_q \\ 0 \end{bmatrix}\right\},$$

$$\hat{\Phi}_3 = \rho_0 \begin{bmatrix} P_2 K C_0 & P_2 K C_0 S_1 \\ 0 & 0 \end{bmatrix},\ \hat{\Phi}_9 = \varepsilon_a \begin{bmatrix} C_0^T \bar{\Delta} \\ S_1^T C_0^T \bar{\Delta} \end{bmatrix},$$

$$\hat{\Phi}_4 = \begin{bmatrix} P_1 A_0 + p_0 P_2 K C_0 & \hat{\Phi}_{12} \\ A_0 & A_0 S_1 \end{bmatrix},$$

$$\hat{\Phi}_5 = \begin{bmatrix} P_1 A_c + P_2 K C_c p_d \\ A_c \end{bmatrix},\ \hat{\Phi}_{10} = \varepsilon_b p_0 \begin{bmatrix} C_0^T \bar{\Delta} \\ S_1^T C_0^T \bar{\Delta} \end{bmatrix},$$

$$\hat{\Phi}_6 = \begin{bmatrix} P_1 B_w + P_2 K D_w & P_1 B_f \\ B_w & B_f \end{bmatrix},$$

$$\hat{\Phi}_7 = \begin{bmatrix} P_2 K \\ 0 \end{bmatrix},\ \hat{\Phi}_8 = \begin{bmatrix} \sum_{i=1}^{q} Q_i & \sum_{i=1}^{q} Q_i S_1 \\ * & \sum_{i=1}^{q} S_1^T Q_i S_1 \end{bmatrix},$$

$$\hat{\Phi}_{12} = P_1 A_0 S_1 + p_0 P_2 K C_0 S_1 + P_2 G S_2^T.$$

Define a new matrix $\mathcal{I} \in R^{2qn\times 2qn}$ with its entries being $\mathcal{I}_{\alpha\beta,\ (2\alpha-1)\beta} = \mathcal{I}_{(\alpha+q)\beta,\ 2\alpha\beta} = 1$ for all $1 \le \alpha \le q$

and $1 \leq \beta \leq n$, and other entries being all zero. Once again, performing congruence transformation to (21) by $\text{diag}\{I,\ \text{diag}\{\mathcal{I},\ I,\ I\},\ I,\ I,\ I\}$, it can be inferred that (21) is equivalent to:

$$\begin{bmatrix} -I & 0 & \hat{\Lambda}_{13} & 0 & 0 \\ * & \tilde{\Lambda}_{22} & \tilde{\Lambda}_{23} & \tilde{\Lambda}_{24} & 0 \\ * & * & \hat{\Lambda}_{33} & 0 & \hat{\Lambda}_{35} \\ * & * & * & \hat{\Lambda}_{44} & 0 \\ * & * & * & * & \hat{\Lambda}_{55} \end{bmatrix} < 0, \tag{22}$$

where

$$\tilde{\Lambda}_{22} = diag\left\{ -\begin{bmatrix} \text{diag}_q\{P_1\} & I \\ * & \text{diag}_q\{S_1\} \end{bmatrix}, \hat{\Phi}_1, \hat{\Phi}_1 \right\},$$

$$\tilde{\Lambda}_{23} = \begin{bmatrix} 0 & \tilde{\Phi}_2 & 0 \\ \hat{\Phi}_3 & 0 & 0 \\ \hat{\Phi}_4 & \hat{\Phi}_5 & \hat{\Phi}_6 \end{bmatrix}, \quad \tilde{\Phi}_2 = \begin{bmatrix} \bar{\rho}_d P_{2d} K_d C_d \\ 0 \end{bmatrix},$$

$$\tilde{\Lambda}_{24} = diag\left\{ \begin{bmatrix} \bar{\rho}_d \text{diag}_q\{P_2\} \text{diag}_q\{K\} \\ 0 \end{bmatrix}, \ \rho_0 \hat{\Phi}_7, \ \hat{\Phi}_7 \right\}.$$

Define $\mathcal{S}_1 = \text{diag}\{\text{diag}_q\{I\}, \text{diag}_q\{S_1^{-1}\}, I, S_1^{-1}, I, S_1^{-1}\}$ and $\mathcal{S}_2 = \text{diag}\{I, S_1^{-1}, I, I\}$. Performing congruence transformation to (22) by $\text{diag}\{I, \mathcal{S}_1, \mathcal{S}_2, I, I\}$ and defining the following matrix variables: $V = P_1$, $F = S^{-1}$, $\bar{G} = P_2 G S_2 S_1^{-1}$, $\bar{K} = P_2 K$, $\bar{L} = L S_2^T S_1^{-1}$, we can arrive at the result (17) in Theorem 6. Noting that every step in our derivation is an equivalent transformation, it then follows from Lemma 4 that (17) is a sufficient condition guaranteeing that the system (5) is exponentially mean-square stable and the H_∞ norm constraint (7) is achieved.

Furthermore, we can know from (17) that F and $F - V$ are nonsingular matrices, so we can always find square and nonsingular matrices P_2 and S_2 satisfying $P_2 S_2^T = I - VF^{-1}$. Therefore, a set of fault detection filter parameters can be given as: $G_0 = P_2^{-1} \bar{G} F^{-1} S_2^{-T}$, $K_0 = P_2^{-1} \bar{K}$, $L_0 = \bar{L} F^{-1} S_2^{-T}$. By substituting these parameters into the transfer function of the filter and considering the relationship $F - V = P_2 S_2^T F$, we obtain $T(z) = \bar{L} F^{-1} S_2^{-T} (zI - P_2^{-1} \bar{G} F^{-1} S_2^{-T})^{-1} P_2^{-1} \bar{K} = \bar{L}[zI - (F - V)^{-1} \bar{G}]^{-1} (F - V)^{-1} \bar{K}$, which means that the desired filter parameters can also be given by (18). This ends the proof.

Remark 7. In most cases, one is more interested in the fault signal of a certain frequency interval, and it is not necessary to estimate f_k. We can introduce a weighting matrix to let r_k approach the weighted fault $\hat{f}_k$. Such a manipulation has been used in Zhong et al. (2005); He et al. (2007a), and can also provide us with a way of considering the fault isolation problem.

To sum up, the fault detection problem for networked systems (1) with incomplete measurements (2) can be solved by the following steps:

1) Design a fault detection filter using Theorem 6.
2) Use the result in Step 1 to determine a threshold J_{th}, which can be obtained using Monte Carlo simulation without fault.
3) Compare the evaluation function of residual signal from fault detection filter with the threshold to determine whether a fault occurs.

4. A NUMERICAL EXAMPLE

In this section, a numerical example is employed to illustrate the proposed method. For simplicity, let $q = 2$ and $\varphi_{-2} = \varphi_{-1} = \varphi_0 = 0$. The parameters of the discrete-time networked system (1) are as follows:

$$A_0 = \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0.3 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0.1 \\ 0.3 & 0.1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0.1 & 0.4 \\ 0 & 0.2 \end{bmatrix}, \quad B_w = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad B_f = \begin{bmatrix} -0.1 \\ 0.2 \end{bmatrix},$$
$$C_0 = C_1 = C_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad D_w = \begin{bmatrix} 0.2 & 0 \\ 0 & -0.1 \end{bmatrix}.$$

Consider the parameters of the logarithmic quantizer as $u_0^1 = u_0^2 = 1$ and $\rho_1 = \rho_2 = 0.9$. For $k = 0, 1, \ldots, 300$, the unknown input w_k is taken as $\exp(-k/50) \times n_k$, where n_k is uniformly distributed over $[-0.5, 0.5]$. The fault signal $f_k = 1$ when $k = 100, 101, \ldots, 200$ and $f_k = 0$ otherwise. τ_k is assumed to obey the following distribution law:

τ_k	0	1	2	others
Probability	0.6	0.2	0.1	0.1

which means that the measurements can be ideally transmitted over network with probability 0.6, one-step delay happens with probability 0.2, two-step delay occurs with probability 0.1, and the measurements are missing with probability 0.1.

With above parameters and from Theorem 6, we can solve the NFDF design problem by using Matlab LMI toolbox (Boyd et al. (1994)). The H_∞ attenuation level is minimized to achieve a bound $\gamma_{opt} = 1.0041$, and the parameters of the sub-optimal fault detection filter are given by

$$G = \begin{bmatrix} 0.2154 & 0.2441 \\ 0.3371 & 0.2644 \end{bmatrix}, \quad K = \begin{bmatrix} -0.0535 & 0.0509 \\ -0.0177 & 0.0803 \end{bmatrix},$$
$$L = [\ 0.0031 \quad 0.0032\ ].$$

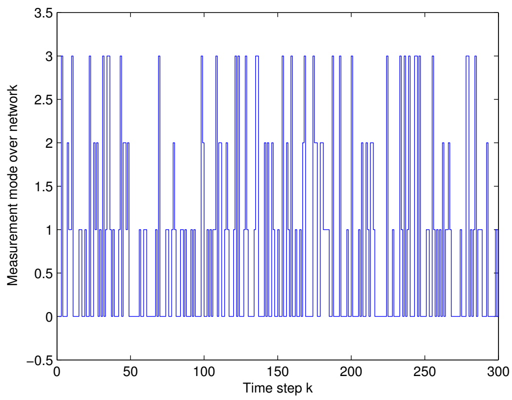

Fig. 1. Measurement mode over network

A time domain simulation is given in Figs. 1-3. Fig. 1 describes the measurement mode. The value 0, 1, 2, and 3 corresponds to the case that measurements transmitted over the network ideally, with one-step delay, with two-step delay and with missing packets, respectively.

Fig. 2 shows the measurements without (dotted line) and with (solid line) quantization, and the latter is actually used by the fault detection filter.

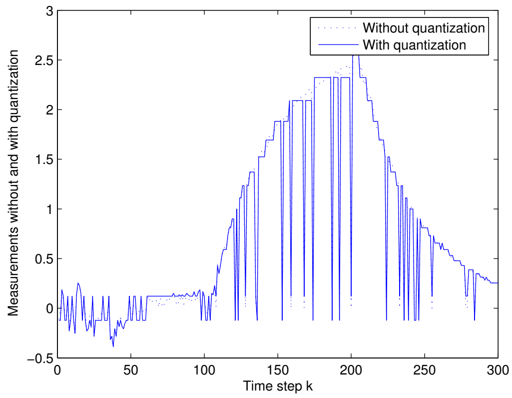

Fig. 2. Measurement with and without quantization

The evolution of the residual evaluation function is given in Fig. 3. We select a threshold as $J_{th} = 1.4429 \times 10^{-5}$ after 400 simulations with no faults. From Fig. 3, it can be shown that $1.27 \times 10^{-5} = J(106) < J_{th} < J(107) = 1.51 \times 10^{-5}$, which means the fault can be detected in 7 time steps after its occurrence.

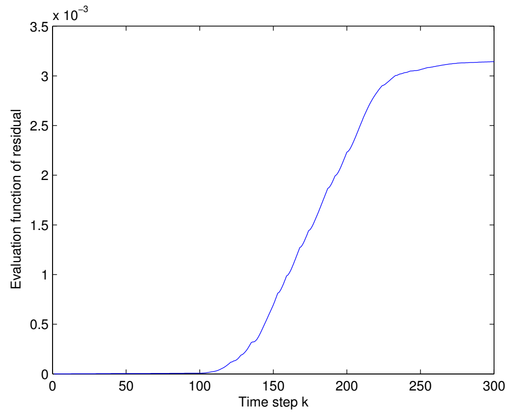

Fig. 3. Residual evaluation function

5. CONCLUSION

In this paper, the fault detection problem for a class of discrete-time networked systems with multiple state-delays and unknown input has been studied. Measurements with random delay, stochastic missing and signal logarithmic quantization have been simultaneously considered. After formulating the fault detection filter design problem into an auxiliary H_∞ filtering problem, we have proposed a sufficient condition in terms of a certain LMI. It is worth pointing out that our main results can be easily extended to the delay-probability-dependent case and parameter-dependent case for polytopic uncertain system.

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