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Design of Quadratic Estimators using Covariance Information in Linear Discrete-Time Stochastic Systems ⋆

Seiichi Nakamori * **Aurora Hermoso-Carazo** ** **Josefa Linares-Pérez** ***

* *Department of Technical Education, Kagoshima University, Kagoshima 890-0065 Japan (Tel: +81-992857866; e-mail: nakamori@edu.kagoshima-u.ac.jp).*
** *Departamento de Estadística e I.O, Universidad de Granada, Campus Fuentenueva S/N, 18071 Granada, Spain (e-mail: ahermoso@ugr.es)*
*** *Departamento de Estadística e I.O, Universidad de Granada, Campus Fuentenueva S/N, 18071 Granada, Spain (e-mail: jlinares@ugr.es)*

Abstract: This paper, apart from the polynomial estimation technique based on the state-space model, examines to develop an estimation method for the quadratic estimation problem by applying the multivariate RLS Wiener estimator to the quadratic estimation of a stochastic signal in linear discrete-time stochastic systems. The augmented signal vector includes the signal to be estimated and its quadratic quantity. The signal vector is modeled in terms of an AR model of appropriate order. A numerical simulation example for the speech signal as a practical stochastic signal is implemented and its estimation accuracy is fairly improved in comparison with the existing RLS Wiener estimators. It is advantageous that the proposed method can be applied to the quadratic estimations of wide-sense stationary stochastic signals in general.

1. INTRODUCTION

It is reported that the quadratic filter improves the estimation accuracy in comparison with the optimal linear filter De Santis et al. [1995]. The filter in De Santis et al. [1995] is designed along with the state-space model in discrete-time non-Gaussian stochastic system. In Bondon [1994], the problem of estimating a signal by taking a polynomial of the observation is considered and it is indicated that the estimate variance decreases in comparison with linear estimation technique. In Caravetta et al. [1997], the general type of polynomial filter for linear systems with multiplicative noise is proposed based on the state-space model. In Uppala and Sahr [1997], a quadratic filter is designed for edge detection in images. In Dalla Mora et al. [2001], the quadratic estimation technique in De Santis et al. [1995] is applied to the restoration of images corrupted by additive non-Gaussian noise. For the quadratic estimation approach using the state-space models, the augmented signal and observation vectors are introduced by aggregating their second-order powers to each original vector, and orthogonal projection method is applied to obtain the linear estimator of the augmented signal based on the augmented observations. So, the obtained quadratic estimator consists of a linear combination of linear and quadratic filters and, hence, it has a structure in the form of a second-order Volterra series whose input is just the set of measurements used for estimation. The specific properties of these measurements lead to a recursive structure for the estimator. The Volterra filters express general models for a various nonlinear systems and are applied to nonlinear system identification problems (Nowak [1998], Raz and Van Veen. [1998], Yamada et al. [2003]).

In Nakamori [1995], the RLS Wiener fixed-point smoother and filter using covariance information are shown in linear discrete-time stochastic systems. The RLS Wiener estimators use the information of the system matrix for the state vector, the observation matrix for the signal and the variance of the state vector. In addition, the variance of the observation noise is necessary. In Nakamori [1996], the linear multivariate RLS Wiener fixed-point smoother and filter, using covariance information, are designed. The multivariate stochastic signal is modeled in terms of the autoregressive (AR) model of appropriate order. In Nakamori et al. [2003], by using covariance information of the signal and observation noise, the recursive mean-squared error and second-order polynomial filtering and fixed-point smoothing algorithms to estimate the signal, from uncertain observations, are proposed. Here, the covariance information is expressed in the semi-degenerate kernel form. It might have been a task to develop a method applicable to the estimation of stochastic signals generally in the engineering aspect. From these respects, this paper, apart from the method adopted in De Santis et al. [1995], newly examines to develop a technique for the quadratic estimation problem by applying the multivariate RLS Wiener estimators to the quadratic estimation of

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stochastic signals in linear discrete-time systems. The augmented signal vector includes the signal to be estimated and its quadratic quantity. The augmented signal vector is modeled in terms of an AR model.

A numerical simulation example by the proposed quadratic estimators is implemented for the speech signal as a practical stochastic signal. As expected from the literatures on the quadratic estimators, its estimation accuracy is fairly improved in comparison with the RLS Wiener estimators in Nakamori [1995].

2. QUADRATIC ESTIMATION PROBLEM

Let $z(k)$ and $y(k)$ be $m \times 1$ vectors which describe the signal that we wish to estimate and the observed value of the signal at time k, respectively. Let the observation equation be given by

$$y(k) = z(k) + v(k), \tag{1}$$

where $v(k)$ is the observation noise vector.

By defining the random vector $y^{[2]}(k) = y(k) \otimes y(k)$, where $\otimes$ denotes Kronecker product De Santis et al. [1995], and by assuming that $E[y^{[2]T}(k)y^{[2]}(k)] < \infty$, the least mean squared error (LMSE) second-order polynomial (or quadratic) estimate of $z(k)$ based on the observed values, $y(1), \cdots, y(L)$, is the orthogonal projection of $z(k)$ on the space of m-dimensional linear transformations of $y(1), \cdots, y(L)$ and their second-order powers $y^{[2]}(1), \cdots, y^{[2]}(L)$.

In order to analyze the LMSE second-order polynomial estimation problem, we assume the following hypotheses on the signal and the noise processes involved in (1).

(I) The signal process $\{z(k); k \geq 0\}$, $z(k) = Hx(k)$, has zero mean. Let the autovariance function of the state vector $x(k)$ be denoted by $K_x(k, k)$. Then the autovariance function of the signal is given by

$$E[z(k)z^T(k)] = HK_x(k, k)H^T. \tag{2}$$

(II) The noise process $\{v(k); k \geq 0\}$ is a zero-mean white sequence and the matrices

$$R_v(k) = E[v(k)v^T(k)] \tag{3}$$

$$R_{vv^{[2]}}(k) = E[v(k)v^{[2]T}(k)] \tag{4}$$

$$R_{v^{[2]}}(k) = E[(v^{[2]}(k) - E[v^{[2]}(k)])(v^{[2]}(k) - E[v^{[2]}(k)])^T] \tag{5}$$

are known. Here, $v^{[2]}(k)$ represents its Kronecker power of $v(k)$.

(III) The processes $\{z(k); k \geq 0\}$ and $\{v(k); k \geq 0\}$ are mutually independent.

Our aim is to obtain the LMSE second-order polynomial fixed-point smoothing estimate $\widehat{z}(k, L)$, at the fixed point k, of the signal $z(k)$, based on the observed values $\{y(s), 1 \leq s \leq L, L > k\}$ and the filtering estimate $\widehat{z}(k, k)$ of $z(k)$ based on the observed values $\{y(s), 1 \leq s \leq k\}$.The fixed-point smoothing estimate $\widehat{z}(k, L)$ is a linear function of $y(1), \cdots, y(L)$ and their Kronecker powers $y^{[2]}(1), \cdots, y^{[2]}(L)$.

To treat this problem, let us define the augmented signal and observation vectors by adding to the original vectors their second-order powers that is

$$\zeta(k) = \begin{bmatrix} z(k) \\ z^{[2]}(k) \end{bmatrix}, \ \psi(k) = \begin{bmatrix} y(k) \\ y^{[2]}(k) \end{bmatrix} \tag{6}$$

Here, $z^{[2]}(k)$ represents its Kronecker power of $z(k)$.Then the vector constituted by the first m entries of the LMSE linear estimate of $\zeta(k)$ based on $\psi(1), \cdots, \psi(L)$ provides the LMSE second-order polynomial fixed-point smoothing estimate of the signal $z(k)$.

In the next section, we analyze the properties of the random vectors $\zeta(k)$ and $\psi(k)$ which will be utilized to obtain the LMSE linear estimates of $\zeta(k)$.

3. AUGMENTED OBSERVATION EQUATION

To study the properties of the vector $\psi(k)$ we need to obtain an appropriate expression for $y^{[2]}(k)$. By employing the Kronecker product properties Nakamori et al. [2003], it can be shown that

$$y^{[2]}(k) = z^{[2]}(k) + f(k) \tag{7}$$

with

$$f(k) = (I_{m^2} + K_{m^2})(z(k) \otimes v(k)) + v^{[2]}(k), \tag{8}$$

where I_{m^2} is the $m^2 \times m^2$ identity matrix and K_{m^2} is the $m^2 \times m^2$ commutation matrix satisfying $K_{m^2}(z(k) \otimes v(k)) = v(k) \otimes z(k)$. Then, by denoting

$$\upsilon_o(k) = \begin{bmatrix} v(k) \\ f(k) \end{bmatrix}, \tag{9}$$

it is clear that $\psi(k)$ satisfies the following equation

$$\psi(k) = \zeta(k) + \upsilon_o(k). \tag{10}$$

It should be noted that the signal and noise in this equation, $\zeta(k)$ and $\upsilon_o(k)$ respectively, have non-zero mean. Anyway, we can subtract the mean values and we obtain that $Y(k) = \psi(k) - E[\psi(k)]$ satisfies the following equation

$$Y(k) = Z(k) + V(k), \tag{11}$$

where $Z(k) = \zeta(k) - E[\zeta(k)]$ and $V(k) = \upsilon_o(k) - E[\upsilon_o(k)]$.

In the following proposition the statistical properties of the process $\{V(k); k \geq 0\}$ involved in (11) are established.

Proposition. Under hypotheses (I)-(III), the noise of (11) is a sequence of zero-mean mutually uncorrelated random vectors with covariance matrices

$$R_V(k) = \begin{bmatrix} R_v(k) & R_{vv^{[2]}}(k) \\ R^T_{vv^{[2]}}(k) & \overline{R}_{22}(k) \end{bmatrix} \tag{12}$$

being

$$\overline{R}_{22}(k) = (I_{m^2} + K_{m^2})(HK_x(k, k)H^T \otimes R_v(k)) \times (I_{m^2} + K_{m^2}) + R_{v^{[2]}}(k). \tag{13}$$

Moreover, $\{V(k); k \geq 0\}$ is uncorrelated with the process $\{Z(k); k \geq 0\}$.

Proof. Clearly, $\forall k \geq 0$, $E[V(k)] = 0$, and, $\forall k, s \geq 0$,

$$E[V(k)V^T(s)] = E[(\upsilon_o(k) - E[\upsilon_o(k)])(\upsilon_o(s) - E[\upsilon_o(s)])^T]. \tag{14}$$

Using the independence hypotheses on the model we have that, for $k \neq s$, $E[V(k)V^T(s)] = 0$, and, for $k = s$,

$$R_V(k) = E[(\upsilon_o(k) - E[\upsilon_o(k)])(\upsilon_o(k) - E[\upsilon_o(k)])^T]. \tag{15}$$

Using the independence hypotheses on the model and expression (8), the Kronecker product properties lead to

$$E[(v_o(k) - E[v_o(k)])(v_o(k) - E[v_o(k)])^T] = \begin{bmatrix} R_v(k) & R_{vv^{[2]}}(k) \\ R^T_{vv^{[2]}}(k) & \overline{R}_{22}(k) \end{bmatrix}, \quad (16)$$

where

$$\overline{R}_{22}(k) = (I_{m^2} + K_{m^2})E[z(k)z^T(k)] \otimes E[v(k)v^T(k)] \times (I_{m^2} + K_{m^2}) + R_{v^{[2]}}(k). \quad (17)$$

¿From hypotheses (I), $E[z(k)z^T(k)] = HK_x(k,k)H^T$, and from (II), $R_v(k) = E[v(k)v^T(k)]$, so, expression (13) is obtained.

Finally, the uncorrelation between $\{V(k); k \geq 0\}$ and the process $\{Z(k); k \geq 0\}$ is derived in a similar way, by using hypotheses (I) and (III) (Q.E.D.).

4. EXPRESSION FOR AUTO-COVARIANCE FUNCTION OF THE AUGMENTED SIGNAL

In Nakamori [1995], the linear RLS Wiener estimators use the information of the system matrix for the state vector, the observation vector and the variance of the state vector for the observation equation (1) particularly for $m = 1$. In Nakamori [1996], the linear multivariate RLS Wiener estimators are designed for multi-channel observation equations, where the signal process is expressed in terms of an AR model. From the AR model, the state equations for the state vector and the observation matrix are obtained. The system matrix is included in the state equation. The variance of the state vector is expressed in terms of the auto-covariance function of the signal. Since the stochastic signal process is fitted to the AR model, this kind of approach might be applicable to the estimation of stochastic signals generally.

Now, in Nakamori et al. [2003], by using covariance information of the signal and observation noise, the recursive LMSE and second-order polynomial filtering and fixed-point smoothing algorithms to estimate the signal, from uncertain observations, are proposed. Here, the covariance information is expressed in the semi-degenerate kernel form. In Nakamori et al. [2003], it is shown that the quadratic estimators improve the estimation accuracy in comparison with the linear estimators using covariance information for non-Gaussian observation noise.

From these respects, this paper derives the quadratic RLS Wiener estimators. For this purpose, let us consider the augmented observation equation (11) expressed in the following form:

$$Y(k) = Z(k) + V(k) = \begin{bmatrix} z(k) \\ z^{[2]}(k) - E[z^{[2]}(k)] \end{bmatrix} + \begin{bmatrix} v(k) \\ f(k) - E[f(k)] \end{bmatrix}$$

$$= \begin{bmatrix} I_{m(m+1)} & 0_{m(m+1)\times m(m+1)(n-1)} \end{bmatrix} \begin{bmatrix} \chi_1(k) \\ \chi_2(k) \\ \vdots \\ \chi_{n-1}(k) \\ \chi_n(k) \end{bmatrix} + \begin{bmatrix} v(k) \\ f(k) - E[f(k)] \end{bmatrix},$$

$$Z(k) = H\chi(k), \quad H = \begin{bmatrix} I_{m(m+1)} & 0_{m(m+1)\times m(m+1)(n-1)} \end{bmatrix}, \quad (18)$$

where $\chi(k) = [\, \chi_1(k) \; \chi_2(k) \; \cdots \; \chi_n(k) \,]^T$ is a $m(m+1)n \times 1$ state vector for both the signal $z(k)$ and the quadratic quantity $z^{[2]}(k) - E[z^{[2]}(k)]$. H represents the $m(m+1) \times mn(m+1)$ observation matrix for $Z(k)$.

The expressions of the state vector and the observation matrix in (18) are obtained by an extension of the treatment for the multivariate signal Nakamori [1996] to the case of the quadratic estimation problems. Namely, we assume that the augmented signal $Z(k)$ is generated by the AR model of order n:

$$Z(k) = -a_1 Z(k-1) - a_2 Z(k-2) - \cdots a_n Z(k-n) + e_1(k), \; e_1(k) = u(k-n). \quad (19)$$

Hence, as in Nakamori [1996], it is seen that the processes $\chi_i(k)$, $i = 1, 2, \cdots, n$, are generated by the stochastic system of order $m(m+1)n$

$$\begin{bmatrix} \chi_1(k+1) \\ \chi_2(k+1) \\ \vdots \\ \vdots \\ \chi_n(k+1) \end{bmatrix} = \begin{bmatrix} 0 & I_{m(m+1)} & 0 & \cdots & 0 & 0 \\ 0 & 0 & I_{m(m+1)} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & I_{m(m+1)} \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_2 & -a_1 \end{bmatrix} \times \begin{bmatrix} \chi_1(k) \\ \chi_2(k) \\ \vdots \\ \vdots \\ \chi_n(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \vdots \\ I_{m(m+1)} \end{bmatrix} u(k),$$

$$E[u(k)u^T(s)] = \sigma^2 I_{m(m+1)} \delta_K(k-s). \quad (20)$$

Let the auto-covariance function of the augmented signal $Z(k)$ be expressed by

$$K_Z(k,s) = E[Z(k)Z^T(s)] = H\Phi^{k-s} K_\chi(s,s) H^T, \quad Z(k) = H\chi(k), \; 0 \leq s \leq k, \quad (21)$$

where Φ represents a system matrix for $\chi(k)$ and $K_\chi(s,s)$ represents the variance matrix of $\chi(s)$.

For the signal process with the wide-sense stationary property $K_Z(k,k) = K_Z(k-k) = K_Z(0)$, the variance of the state vector $\chi(k)$ is expressed as Nakamori [1996]

$$K_\chi(k,k) = E[\chi(k)\chi^T(k)] = \begin{bmatrix} K_Z(0) & K_Z^T(1) & \cdots & K_Z^T(n-1) \\ K_Z(1) & K_Z(0) & \cdots & K_Z^T(n-2) \\ \vdots & \vdots & \ddots & \vdots \\ K_Z(n-2) & K_Z(n-3) & \cdots & K_Z^T(1) \\ K_Z(n-1) & K_Z(n-2) & \cdots & K_Z(0) \end{bmatrix}. \quad (22)$$

Then the AR parameters, $a_1, a_2, \cdots, a_n$, are calculated by the Yule-Walker equations Nakamori [1996],

$$\begin{bmatrix} K_Z(0) & K_Z^T(1) & \cdots & K_Z^T(n-1) \\ K_Z(-1) & K_Z(0) & \cdots & K_Z^T(n-2) \\ \vdots & \vdots & \ddots & \vdots \\ K_Z(-(n-2)) & K_Z(-(n-3)) & \cdots & K_Z^T(1) \\ K_Z(-(n-1)) & K_Z(-(n-2)) & \cdots & K_Z(0) \end{bmatrix} \cdot \begin{bmatrix} a_1^T \\ a_2^T \\ \vdots \\ a_{n-1}^T \\ a_n^T \end{bmatrix} = \begin{bmatrix} -K_Z(-1) \\ -K_Z(-2) \\ \vdots \\ -K_Z(-(n-1)) \\ -K_Z(-n) \end{bmatrix}. \quad (23)$$

Let the system matrix in (20) for the signal $Z(k)$ be Φ

$$\Phi = \begin{bmatrix} 0 & I_{m(m+1)} & 0 & \cdots & 0 & 0 \\ 0 & 0 & I_{m(m+1)} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & I_{m(m+1)} \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_2 & -a_1 \end{bmatrix}. \quad (24)$$

¿From (23), Φ is calculated in terms of the auto-covariance data $K_Z(i)$, $i = -n, -(n-1), \cdots, n-1$.

Henceforth, the observation matrix H, the system matrix Φ and the variance matrix $K_\chi(k,k)$ of the state vector $\chi(k)$ concerned with the augmented signal $Z(k) = \left[\, z(k)\ z^{[2]}(k) - E[z^{[2]}(k)] \,\right]^T$ are given by (18), (24) and (22), respectively. H, Φ and $K_\chi(k,k)$ suffice to express the auto-covariance function of the augmented signal $Z(k)$ in (21).

5. FIXED-POINT SMOOTHING AND FILTERING ALGORITHMS FOR AUGMENTED SIGNAL $Z(K)$

Using the properties of the processes involved in (11), we derive recursive algorithms for the fixed-point smoothing estimate $\widehat{Z}(k,L)$, $L > k$, based on the observations $Y(1), \cdots Y(L)$, and the filtering estimate $\widehat{Z}(k,k)$ of the signal $Z(k)$. These estimation equations are presented in [**Theorem 1**] and allow us to obtain the required quadratic RLS Wiener fixed-point smoothing and filtering estimates of the signal $z(k)$.

[**Theorem 1**].Let the auto-covariance function of the augmented signal $Z(k)$ be given by (21), then the quadratic RLS Wiener algorithms for the fixed-point smoothing estimate $\widehat{Z}(k,L)$, $L > k$, and the filtering estimate $\widehat{Z}(k,k)$ of the augmented signal $Z(k)$ consist of (25)-(33).

Fixed-point smoothing estimate of $Z(k)$: $\widehat{Z}(k,L)$

$$\widehat{Z}(k,L) = H\widehat{\chi}(k,L) \quad (25)$$

Fixed-point smoothing estimate of $\chi(k)$: $\widehat{\chi}(k,L)$

$$\widehat{\chi}(k,L) = \widehat{\chi}(k,L-1) + h(k,L,L) \times (Y(L) - H\Phi\widehat{\chi}(L-1,L-1)) \quad (26)$$

$$h(k,L,L) = (K_\chi(k,k)(\Phi^T)^{L-k}H^T R_V^{-1}(L) - q(k,L-1)\Phi^T H^T R_V^{-1}(L)) \times (I + HK_\chi(k,k)H^T R_V^{-1}(L) - H\Phi S(L-1)\Phi^T H^T R_V^{-1}(L))^{-1} \quad (27)$$

$$q(k,L) = q(k,L-1)\Phi^T + h(k,L,L)H \times (K_\chi(L,L) - \Phi S(L-1)\Phi^T), \quad (28)$$

$$q(k,k) = S(k) \quad (29)$$

Filtering estimate of $Z(k)$: $\widehat{Z}(k,k)$

$$\widehat{Z}(k,k) = H\widehat{\chi}(k,k) \quad (30)$$

Filtering estimate of $\chi(k)$: $\widehat{\chi}(k,k)$

$$\widehat{\chi}(k,k) = \Phi\widehat{\chi}(k-1,k-1) + G(k) \times (Y(k) - H\Phi\widehat{\chi}(k-1,k-1)),\ \widehat{\chi}(0,0) = 0 \quad (31)$$

$$S(k) = \Phi S(k-1)\Phi^T + G(k)H(K_\chi(k,k) - \Phi S(k-1)\Phi^T),\ S(0) = 0 \quad (32)$$

Filter gain $G(k)$

$$G(k) = (K_\chi(k,k)H^T - \Phi S(k-1)\Phi^T H^T) \times (R_V(k) + HK_\chi(k,k)H^T - H\Phi S(k-1)\Phi^T H^T)^{-1} \quad (33)$$

Proof. [**Theorem 1**] can be proved, for the augmented observation equation (18) and the auto-covariance function (21) of the augmented signal $Z(k)$, by referring to the derivation technique of the RLS Wiener fixed-point smoothing and filtering equations in Nakamori [1995].

6. A NUMERICAL SIMULATION EXAMPLE

Let a scalar observation equation be given by

$$y(k) = z(k) + v(k), \quad (34)$$

and let $\{v(k); k \geq 0\}$ be a sequence of independent random variables with

$$P[v(k) = -8] = \frac{1}{8},\ P[v(k) = \frac{8}{7}] = \frac{7}{8} \quad (35)$$

hence

$$E[v(k)] = 0,\ R_v(k) = 9.1428571,$$
$$R_{vv^2}(k) = -62.693878,\ R_{v^2}(k) = 513.49271. \quad (36)$$

Let us consider the problem of estimating a vowel signal spoken by one of the authors. Its phonetic symbol is written as "/i:/". The sampling frequency of the voice signal $z(k)$ is $11.025(kHz)$. The auto-covariance data of the signal are calculated in terms of the $N(= 5000)$ sampled signal data. Let the order of the AR model in (19) be $n = 10$. The AR parameters a_i, $i = 1, \cdots, n$, are calculated by the Yule-Walker equations (23). Here, the sampled auto-covariance data of the augmented signal are calculated by $\widehat{K}_Z(k) = \sum_{i=k}^{N} Z(i)Z^T(i-k)/N$. $\widehat{K}_Z(k)$ is also evaluated in terms of the augmented observed values $Y(\cdot)$ for $k \neq 0$.

Substituting the observation matrix H in (18), the system matrix Φ in (24) and the variance matrix $K_\chi(k,k)$, for $n = 10$, into the estimation algorithms of [**Theorem 1**], the fixed-point smoothing and filtering estimates are calculated. **Fig.1** illustrates the signal $z(k)$, the filtering estimate $\widehat{z}(k,k)$ and the fixed-point smoothing estimate $\widehat{z}(k,k+5)$ calculated by the quadratic estimation algorithm in [**Theorem 1**] vs. k. Here, the variance of the signal process is 1.0873. **Fig.2** illustrates the signal $z(k)$, the filtering estimate $\widehat{z}(k,k)$ and the fixed-point smoothing estimate $\widehat{z}(k,k+5)$ calculated by the linear RLS Wiener algorithm in Nakamori [1995] vs. k. **Fig.3** illustrates the mean-square values (MSVs) of the estimation errors $z(k)-\widehat{z}(k,k+lag)$, $0 \leq lag \leq 10$, by the quadratic estimation algorithms and the linear RLS Wiener algorithms. Here, the MSVs are calculated by $\sum_{k=1}^{500}(z(k)-\widehat{z}(k,k))^2/500$ for the filter and $\sum_{k=1}^{500}\sum_{j=1}^{5}(z(k)-\widehat{z}(k,k+j))^2/2500$ for the fixed-point smoother. For $lag=0$, the MSV of the filtering error $z(k)-\widehat{z}(k,k)$ is depicted. The fixed-point smoothing estimate and the filtering estimate in **Fig.1** show the time-lead property in comparison the estimates in **Fig.2**. From **Fig1**, **Fig.2** and **Fig.3**, the proposed quadratic fixed-point smoother and filter significantly improve the estimation accuracy in comparison with the RLS Wiener estimators.

7. CONCLUSION

In this paper, the multivariate filter and fixed-point smoother using covariance information are applied to the quadratic estimation problem of a stochastic signal in linear discrete-time systems.

From the numerical simulation results, the proposed quadratic estimation technique has shown superior accuracy for both the fixed-point smoother and the filter in comparison with the linear RLS Wiener estimators. In the filtering algorithm in [**Theorem 1**], $N(N+1)/2+N$, $N=m(m+1)n$ difference equations should be simultaneously calculated. Hence, as m becomes large, the computational manupurations of the estimation algorithms increase. For the estimation characteristics for $m>1$, it might be left as a future task.

Also, the estimation of the fourth moment of the signal process from available data might be left as a future task

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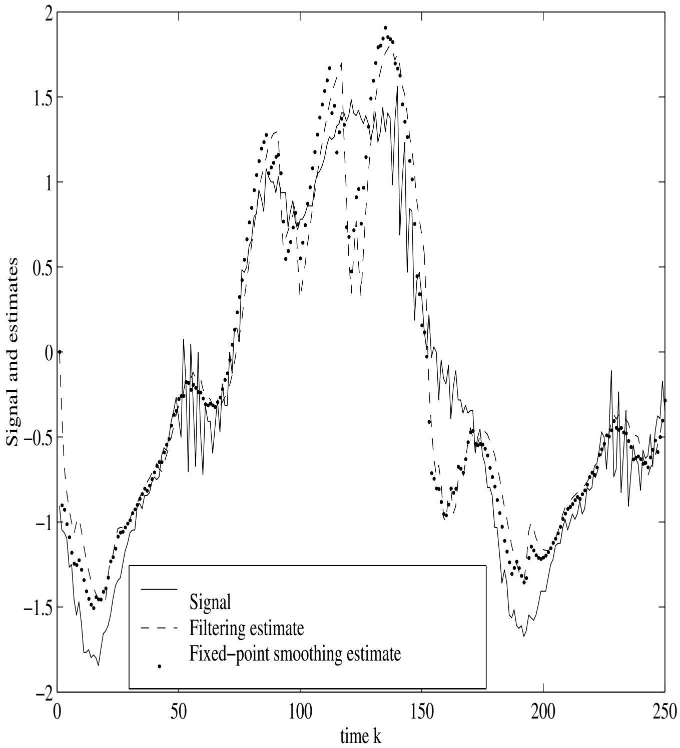

Fig. 1. The signal $z(k)$, the filtering estimate $\hat{z}(k,k)$ and the fixed-point smoothing estimate $\hat{z}(k,k+5)$ by the quadratic RLS Wiener estimators in [**Theorem 1**] vs. k.

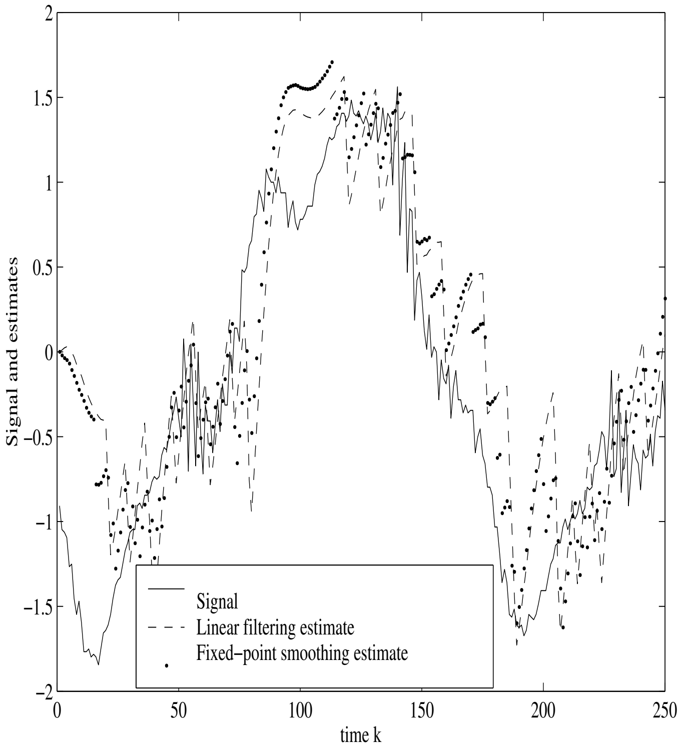

Fig. 2. The signal $z(k)$, the filtering estimate $\hat{z}(k,k)$ and the fixed-point smoothing estimate $\hat{z}(k,k+5)$ by the RLS Wiener estimators in Nakamori [1995] vs. k.

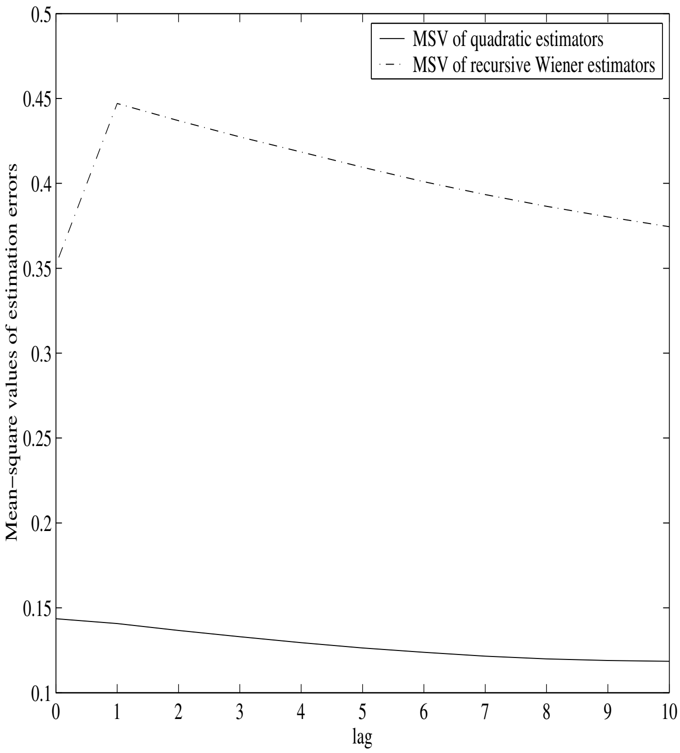

Fig. 3. MSVs of the estimation errors $z(k) - \hat{z}(k, k + lag)$, $0 \leq lag \leq 10$, by the current quadratic RLS Wiener estimators and the RLS Wiener estimators in Nakamori [1995] vs. lag.

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