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# Fast Iterative Learning Control for Delay Systems: A Predictive Approach ⋆

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**Abstract:** In a previous paper (Li et al. (2005)), an iterative learning control (ILC) law, proposed for linear continuous systems with a single time delay, has the ability to drive the output tracking error to zero only after one learning iteration. The convergence result is quite attractive; however, it requires unavailable system state. The aim of this paper is to provide a predictive approach to not only reach this result, but also extend it to linear continuous systems with multiple time delays. To this end, that unavailable system state is predicted and its corresponding equivalent form is obtained, based on which new ILC laws with fully available information are determined, ensuring the zero output tracking only after one learning iteration. The numerical simulation shows that this kind of ILC design is available, and furthermore the extension of the results from systems with a single time delay to those with multiple time delays is feasible.

## 1. INTRODUCTION

Consider the linear system with a single time delay

$$\begin{aligned}\frac{\partial x(t,k)}{\partial t} &= Ax(t,k)+A_0x(t-t_0,k)+Bu(t,k)\\ y(t,k) &= Cx(t,k)\end{aligned}\quad (1)$$

where $t\in[0,T]$ is the continuous-time index, $k\in\mathbb{Z}_+$ is the iteration number, $x(\cdot,\cdot)\in\mathbb{R}^n$ is the state, $u(\cdot,\cdot)\in\mathbb{R}^m$ is the input, and $y(\cdot,\cdot)\in\mathbb{R}^l$ is the output. The delay parameter $t_0$ is time-invariant, and the system matrices $A$, $A_0$, $B$ and $C$ are real constant matrices of appropriate dimensions. Define a general ILC law as

$$u(t,k+1) = u(t,k)+\Delta u(t,k) \quad (2)$$

then boundary conditions for the iterative learning control system (ILCS) (1) and (2) are given by, i.e., (Li et al., 2005, (3))

$$\begin{aligned} x(t,k) &= x_0(t), \text{ for } t\in[-t_0,0] \text{ and } k\in\mathbb{Z}_+\\ u(t,0) &= u_0(t), \text{ for } t\in[0,T]\end{aligned}\quad (3)$$

where $x_0(\cdot)$ and $u_0(\cdot)$ can be arbitrarily chosen. Assume that the desired output trajectory is denoted by $y_d(t)$ for $t\in[0,T]$ which satisfies $y_d(0)=Cx_0(0)$. From Li et al. (2005), it follows that there exists an ILC law, i.e., (Li et al., 2005, (15))

$$\begin{aligned}u(t,k+1) =& u(t,k)+K_1[x(t,k+1)-x(t,k)]\\ &+K_2[x(t-t_0,k+1)-x(t-t_0,k)]\\ &+K_3\left[\frac{dy_d(t)}{dt}-\frac{\partial y(t,k)}{\partial t}\right]\end{aligned}\quad (4)$$

where

$$\begin{aligned}K_1 &= -(CB)^T\left[CB(CB)^T\right]^{-1}CA\\ K_2 &= -(CB)^T\left[CB(CB)^T\right]^{-1}CA_0\\ K_3 &= (CB)^T\left[CB(CB)^T\right]^{-1}\end{aligned}$$

driving the output tracking error to zero for the whole desired trajectory over the interval $t\in[0,T]$ only after one learning iteration if and only if (iff) the matrix $CB$ has full-row rank. Here, ILC with such fast learning speed is called **FILC**. However, the information of $x(\cdot,k+1)$ is not available in (4) [1], since it is yielded by the new control sequence which is required to be determined. In practice, $x(\cdot,k+1)-x(\cdot,k)$ may be totally replaced by $x(\cdot,k)-x(\cdot,k-1)$, but this estimation is not acceptable since $x(\cdot,k)-x(\cdot,k-1)$ is an estimate, and thus, even the convergence of (4) can not be ensured.

The key idea behind FILC is to improve learning efficiency of the ILCS under consideration. By introducing the information on system states into a conventional learning law, the fastest convergence rate is achieved, which is quite attractive. The type of ILC has been proposed for linear discrete-time systems in Kurek & Zaremba (1993) and Fang & Chow (1998), and extended to linear continuous-time systems in Chow & Fang (1998). Furthermore, a learning law introducing the state feedback controller is obtained in Fang & Chow (1998), which not only exhibits the fastest convergence rate but also uses available information. In contrast to (4), this type of FILC is called the **available FILC**.

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[1] It has been pointed out that the current system state $x(\cdot,k+1)$ is not available for ILC laws (Kurek & Zaremba (1993); Fang & Chow (1998); Chow & Fang (1998); Li et al. (2005)). In fact, the essence of ILC is a type of learned open-loop control strategy (Bristow et al. (2006)), and in this sense, $x(\cdot,k+1)$ as well as $u(\cdot,k+1)$ in (4) is required to be determined.

In this paper, by combining main points of the way used in Fang & Chow (1998) with an early result of Li et al. (2005), a new approach is proposed to directly reach available FILC for system (1). The salient feature of this approach is its predictive results derived for unavailable system state. Furthermore, these obtained results for linear continuous systems with a single time delay are extended to a class of linear continuous systems with multiple time delays, which widens the application of FILC and, hence, can be viewed as a significant extension of those results in Li et al. (2005) to general linear time-delay systems.

## 2. DESIGN OF AVAILABLE FILC

In this section, we first show how to use a state feedback controller to predict the FILC law (4) by replacing the information of the $(k+1)$th iteration with its equivalent form which is available for the $k$th iteration. Based on the predictive results, an FILC algorithm only using information from the previous iteration is then proposed for system (1).

*Prediction of (4):* Note that system (1) is the 2-D description at the $k$th iteration of the following system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_0 x(t-t_0) + Bu(t) \\ y(t) &= Cx(t). \end{aligned} \tag{5}$$

Introducing a state feedback controller to system (5), we get

$$\begin{aligned} \dot{\hat{x}}(t) &= (A+BK_1)\hat{x}(t) + (A_0+BK_2)\hat{x}(t-t_0) + B\hat{u}(t) \\ \hat{y}(t) &= C\hat{x}(t) \end{aligned} \tag{6}$$

where $\hat{x}(t) = x_0(t)$ for $t \in [-t_0, 0]$ holds, and $\hat{u}(\cdot)$ is defined by

$$\hat{u}(t) = (I - K_3CB)u(t) + K_3 \frac{dy_d(t)}{dt}. \tag{7}$$

In what follows, let $\hat{x}(\cdot,k)$, $\hat{u}(\cdot,k)$ and $\hat{y}(\cdot,k)$ denote the state, input and ouput of system (6) at the $k$th iteration, respectively.

According to the previous development, a predictive result is summarized by the following lemma.

*Lemma 1.* For systems (5) and (6) under the updating law (4), the following two results hold iff the matrix $CB$ has full-row rank.

1) The state of system (5) at the $(k+1)$th iteration is identical to that of system (6) at the $k$th iteration, i.e., $x(t,k+1) = \hat{x}(t,k)$ for $t \in [0,T]$ and $k \in \mathbb{Z}_+$.
2) The output of system (6) at each iteration is equal to the desired output, i.e., $\hat{y}(t) = y_d(t)$ for $t \in [0,T]$.

**Proof.** *Proof of 1):* Let $\check{x}(t,k) = \hat{x}(t,k) - x(t,k)$. After some algebraic manipulations, it follows from (5), (6), and (7) that

$$\begin{aligned} \frac{\partial \check{x}(t,k)}{\partial t} &= (A+BK_1)\,\hat{x}(t,k) + (A_0+BK_2)\,\hat{x}(t-t_0,k) \\ &\quad + B\hat{u}(t,k) - [Ax(t,k) + A_0x(t-t_0,k) + Bu(t,k)] \\ &= (A+BK_1)\,\check{x}(t,k) + (A_0+BK_2)\,\check{x}(t-t_0,k) \\ &\quad + BK_3\left[\frac{dy_d(t)}{dt} - \frac{\partial y(t,k)}{\partial t}\right]. \end{aligned} \tag{8}$$

Let $\bar{x}(t,k) = x(t,k+1) - x(t,k)$. Iterating system (5) from $k$ to $k+1$, we have

$$\begin{aligned} \frac{\partial \bar{x}(t,k)}{\partial t} &= A\bar{x}(t,k) + A_0\bar{x}(t-t_0,k) + B\Delta u(t,k) \\ &= (A+BK_1)\bar{x}(t,k) + (A_0+BK_2)\bar{x}(t-t_0,k) \\ &\quad + BK_3\left[\frac{dy_d(t)}{dt} - \frac{\partial y(t,k)}{\partial t}\right]. \end{aligned} \tag{9}$$

Subtracting (8) from (9), we obtain

$$\begin{aligned} \frac{\partial[\bar{x}(t,k) - \check{x}(t,k)]}{\partial t} =& (A+BK_1)\,[\bar{x}(t,k) - \check{x}(t,k)] \\ & + (A_0+BK_2)\,[\bar{x}(t-t_0,k) - \check{x}(t-t_0,k)]. \end{aligned} \tag{10}$$

Note that $\bar{x}(t,\cdot) - \check{x}(t,\cdot) = 0$ for $t \in [-t_0, 0]$ holds. It follows from (10) that $\bar{x}(t,\cdot) = \check{x}(t,\cdot)$ for any $t$ according to the theory of functional differential equations (see Hale (1977)). Hence, $x(t,k+1) = \hat{x}(t,k)$ for $t \in [0,T]$ and $k \in \mathbb{Z}_+$ is immediate.

*Proof of 2):* Computing $\dot{\hat{y}}(t)$ and using (6) and (7), we get

$$\begin{aligned} \dot{\hat{y}}(t) &= C(A+BK_1)\hat{x}(t) + C(A_0+BK_2)\hat{x}(t-t_0) + CB\hat{u}(t) \\ &= \dot{y}_d(t). \end{aligned} \tag{11}$$

Since $\hat{y}(0) = Cx_0(0) = y_d(0)$ holds, it immediately follows from (11) that $\hat{y}(t) = y_d(t)$ for $t \in [0,T]$. This completes the proof.□

*Remark 1.* Lemma 1 implies that the state information of (5) at the $(k+1)$th iteration can be predicted by that of (6) at the $k$th iteration, and the output of the closed-loop system (6) is always identical with the desired output trajectory $y_d(\cdot)$. This makes it possible to design FILC with available information as described in the following.

*Remark 2.* In particular, it can be noticed from the previous proof that Lemma 1 does not depend on the control input $u(\cdot)$ used in $\hat{u}(\cdot)$, which implies that $u(\cdot)$ has no effect upon the results shown in Lemma 1. Without loss of generality, the input $u(\cdot)$ in $\hat{u}(\cdot)$ is chosen as the input of system (5).

*Available FILC:* As soon as the previous predictive results are obtained, an FILC law both with available information and the fastest convergence rate can be designed for system (5).

*Theorem 1.* Consider the linear time-delay system (5) that satisfies (3). Let $\hat{u}(\cdot)$ be given by (7), and $\hat{x}(\cdot)$ be the state of system (6). Then, there exists an updating law

$$u(t) \Leftarrow \hat{u}(t) + K_1\hat{x}(t) + K_2\hat{x}(t-t_0) \tag{12}$$

such that the output tracking error is driven to zero for the whole desired trajectory only after one learning iteration over $t \in [0,T]$ iff the matrix $CB$ has full-row rank. Moreover, (12) is equivalent to (4), and therefore can totally replace this FILC law.

**Proof.** According to the result 2) of Lemma 1, the output of system (6) is identical to the desired trajectory, i.e., $\hat{y}(t) = y_d(t)$ for $t \in [0,T]$. Using this fact, the output tracking error can be expressed by

$$\begin{aligned} e(t) &= y_d(t) - y(t) \\ &= C\,[\hat{x}(t) - x(t)]. \end{aligned} \tag{13}$$

Applying the law (12) to system (5) yields that $x(\cdot)$ satisfies

$$\begin{aligned} \dot{x}(t) &= Ax(t) + A_0x(t-t_0) + B\,[\hat{u}(t) + K_1\hat{x}(t) + K_2\hat{x}(t-t_0)] \\ y(t) &= Cx(t). \end{aligned} \tag{14}$$

Subtracting (14) from (6), we get

$$\dot{\hat{x}}(t) - \dot{x}(t) = A\,[\hat{x}(t) - x(t)] + A_0\,[\hat{x}(t-t_0) - x(t-t_0)]. \tag{15}$$

Since $\hat{x}(t) = x(t) = x_0(t)$ for $t \in [-t_0, 0]$, a consequence of (15) is that $\hat{x}(t) = x(t)$ for $t \in [0,T]$ (see Hale (1977)). Thus, $e(t) = 0$ for $t \in [0,T]$ holds according to (13). That is, the zero output tracking is achieved only after one learning iteration.

Furthermore, using the result 1) of Lemma 1 and replacing $x(\cdot,k+1)$ used in (4) by $\hat{x}(\cdot,k)$, we have

$$\Delta u(t,k) = K_1\left[\hat{x}(t,k)-x(t,k)\right]+K_2\left[\hat{x}(t-t_0,k)-x(t-t_0,k)\right] + K_3\left[\frac{dy_d(t)}{dt}-\frac{\partial y(t,k)}{\partial t}\right]. \tag{16}$$

Substituting (1) and (7) into (16), we obtain

$$u(t,k+1)=\hat{u}(t,k)+K_1\hat{x}(t,k)+K_2\hat{x}(t-t_0,k) \tag{17}$$

which implies that the learning law (12) is an equivalent form of (4). Then, the learning law (4) can be replaced by (12). This completes the proof. □

*Remark 3.* Obviously, the previous theorem provides available FILC for system (1) in comparison with (Li et al., 2005, Theorem 2). This results from that the present predictive approach establishes a connection between systems (5) and (6), and furthermore exactly produces the desired output trajectory $y_d(t)$ for $t\in[0,T]$. Moreover, by taking advantage of Lemma 1, the previous proof shows that the learning law (12) not only exhibits the same learning efficiency with (4), but also can totally replace (4).

*Remark 4.* In Fang & Chow (1998), a type of available FILC is derived based on a similar result to the 2) of Lemma 1. In contrast to this, Theorem 1 not only obtains available FILC (12) but also points out the equivalence between it and the FILC (4) by developing the predictive result 1) of Lemma 1. In fact, (4) is a kind of closed-loop ILC, whereas (12) is an open-loop ILC in essence. In this sense, a equivalent relationship between the closed-loop and open-loop ILC have been established by our proposed predictive lemma. And, to the best of our knowledge, this point has not been pointed out by any reference in the ILC literature.

## 3. EXTENSION OF AVAILABLE FILC

In this section, FILC is developed for a class of linear continuous multivariable systems with multiple time delays as follows

$$\begin{aligned}\frac{\partial x(t,k)}{\partial t}&=Ax(t,k)+\sum_{i=0}^{p}A_ix(t-t_i,k)+Bu(t,k)\\ y(t,k)&=Cx(t,k).\end{aligned} \tag{18}$$

Let $t_0=\max_{0\le i\le p}t_i$, then boundary conditions for system (18) take the form of (3). It is also assumed that the desired output $y_d(\cdot)$ satisfies $y_d(0)=Cx_0(0)$. Now, define two variables as

$$\begin{aligned}e(t,k)&=y_d(t)-y(t,k)\\ \eta(t,k)&=\int_0^t[x(\tau,k+1)-x(\tau,k)]d\tau.\end{aligned} \tag{19}$$

Noting that $x(t,k+1)-x(t,k)=0$ for $t\in[-t_0,0]$ and $k\in\mathbb{Z}_+$ holds, we have

$$\frac{\partial\eta(t,k)}{\partial t}=A\eta(t,k)+\sum_{i=0}^{p}A_i\eta(t-t_i,k)+B\int_0^t\Delta u(\tau,k)d\tau \tag{20}$$

$$e(t,k+1)-e(t,k)=-CA\eta(t,k)-C\sum_{i=0}^{p}A_i\eta(t-t_i,k) - CB\int_0^t\Delta u(\tau,k)d\tau. \tag{21}$$

Consider the following updating law for system (18)

$$u(t,k+1)=u(t,k)+R^1\frac{\partial\eta(t,k)}{\partial t}+\sum_{i=0}^{p}R_i\frac{\partial\eta(t-t_i,k)}{\partial t} + R^2\frac{\partial e(t,k)}{\partial t}. \tag{22}$$

Inserting (22) into (20) and (21), we get

$$\frac{\partial\eta(t,k)}{\partial t}=(A+BR^1)\eta(t,k)+\sum_{i=0}^{p}(A_i+BR_i)\eta(t-t_i,k) + BR^2e(t,k) \tag{23}$$

$$e(t,k+1)=-(CA+CBR^1)\eta(t,k) - \sum_{i=0}^{p}(CA_i+CBR_i)\eta(t-t_i,k)+(I-CBR^2)e(t,k). \tag{24}$$

To deal with delay parameters $t_i$ for $0\le i\le p$, super vectors are introduced (like Li et al. (2005)), based on which (23) and (24) can be formulated into a 2-D Roesser model.

Let $\omega_i$ $(0\le i\le p)$ be nonnegative integers. Then, for any $t\in[0,T]$, the number of $t-\sum_{i=0}^{p}\omega_it_i$, which satisfies $t-\sum_{i=0}^{p}\omega_it_i\ge 0$, is finite and denoted by $q$. Without loss of generality, let $t^1$, $t^2$, $\cdots$, $t^q$ represent all the finite values of $t-\sum_{i=0}^{p}\omega_it_i$ from large to small. Obviously, $t^1$ is obtained iff $\omega_i=0,\forall i\in\{0,1,\cdots,p\}$, i.e., $t^1=t$. Now, define a new column vector $\tilde{\eta}(t,k)$ as

$$\tilde{\eta}(t,k)=\left[\,\eta^T(t,k)\;\;\eta^T(t^2,k)\;\;\cdots\;\;\eta^T(t^q,k)\,\right]^T$$

and let $\tilde{e}(t,k)$ be defined in the same way. Considering $\eta(t,k)=0$ for $t\in[-t_0,0]$ and $k\in\mathbb{Z}_+$, and reformulating (23) and (24), we obtain a 2-D continuous-discrete Roesser's type model as

$$\begin{bmatrix}\dfrac{\partial\tilde{\eta}(t,k)}{\partial t}\\ \tilde{e}(t,k+1)\end{bmatrix}=\begin{bmatrix}\tilde{A} & \tilde{B}\\ -\tilde{C}\tilde{A} & I-\tilde{C}\tilde{B}\end{bmatrix}\begin{bmatrix}\tilde{\eta}(t,k)\\ \tilde{e}(t,k)\end{bmatrix} \tag{25}$$

with boundary conditions: $\tilde{\eta}(0,k)=0$ for $k\in\mathbb{Z}_+$ and finite $\tilde{e}(t,0)$ for $t\in[0,T]$, and system matrices:

$$\tilde{A}=\begin{bmatrix}A+BR^1 & * & * & \cdots & *\\ 0 & A+BR^1 & * & \cdots & *\\ 0 & 0 & A+BR^1 & \cdots & *\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & 0 & A+BR^1\end{bmatrix}$$

$$\tilde{B}=\begin{bmatrix}BR^2 & 0 & 0 & \cdots & 0\\ 0 & BR^2 & 0 & \cdots & 0\\ 0 & 0 & BR^2 & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & \cdots & BR^2\end{bmatrix}$$

$$\tilde{C}=\begin{bmatrix}C & 0 & 0 & \cdots & 0\\ 0 & C & 0 & \cdots & 0\\ 0 & 0 & C & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots\\ 0 & 0 & 0 & \cdots & C\end{bmatrix}$$

where $*$ represents either a certain element selected among matrices $A_i+BR_i$ for $0\le i\le p$, or a zero matrix of appropriate dimensions.

According to the previous development, one can prove the following result related to ILC system (18) and (22) by the 2-D system theory.

*Theorem 2.* Given system (18), let $t_0=\max_{0\le i\le p}t_i$, and boundary conditions in (3) be satisfied. Then, there exists a convergent updating law (22) for system (18) iff the matrix $CB$ has full-row rank. Moreover, the updating law (22) can be designed such that

$$\begin{bmatrix}\eta(t,k)\\ e(t,k)\end{bmatrix}=0\;\;\text{for}\;\;t\in[0,T]\;\;\text{and}\;\;k\ge 1 \tag{26}$$

iff its gain matrices are defined by

$$R^1 = -(CB)^T \left[CB(CB)^T\right]^{-1} CA$$
$$R^2 = (CB)^T \left[CB(CB)^T\right]^{-1} \quad (27)$$
$$R_i = -(CB)^T \left[CB(CB)^T\right]^{-1} CA_i, 0 \le i \le p.$$

**Proof.** The proof is provided in Appendix A. □

*Remark 5.* Following the same steps of the previous proof, one can show that if $R^1 = 0$ and $R_i = 0$ for $0 \le i \le p$ in the updating law (22), the convergence condition provided in Theorem 2 still holds for this law, which can also be verified by the 2-D system theory since the 2-D model (25) is satisfied. In fact, this is just the result of (Li et al., 2005, Theorem 3), and hence, Theorem 2 can be viewed as an extension of this result, which moreover proposes an FILC law (22)−iff its gain matrices are defined in (27))−such that the zero output tracking of system (18) can be achieved only after one learning iteration.

An attractive FILC law (22) has been obtained for system (18) in Theorem 2; however, the shortcomings suffered by (4) also exist in (22). Similarly, we use a predictive approach to remove its employed current system state. Let matrices $R^1$, $R^2$ and $R_i$ for $0 \le i \le p$ be defined by (27). Then, for the following system

$$\dot{x}(t) = Ax(t) + \sum_{i=0}^{p} A_i x(t-t_i) + Bu(t) \quad (28)$$
$$y(t) = Cx(t)$$

a predictive system is given by

$$\dot{\breve{x}}(t) = \left(A + BR^1\right)\breve{x}(t) + \sum_{i=0}^{p}(A_i + BR_i)\breve{x}(t-t_i) + B\breve{u}(t) \quad (29)$$
$$\breve{y}(t) = C\breve{x}(t)$$

where the control $\breve{u}(\cdot)$ is defined by

$$\breve{u}(t) = \left(I - R^2 CB\right)u(t) + R^2 \frac{dy_d(t)}{dt}. \quad (30)$$

Similar to Lemma 1, one can see that the state of system (28) at the $(k+1)$th iteration is equivalent to that of system (29) at the $k$th iteration, and the output of system (29) is always identical to the desired output trajectory $y_d(t)$ for $t \in [0,T]$.

According to the previous development, one can prove the following result related to available FILC for system (28) in the same way as in the proof of Theorem 1.

*Theorem 3.* For system (28), let $t_0 = \max_{0 \le i \le p} t_i$, and boundary conditions in (3) be satisfied. Then, there exists an updating law

$$u(t) \Leftarrow \breve{u}(t) + R^1\breve{x}(t) + \sum_{i=0}^{p} R_i \breve{x}(t-t_i) \quad (31)$$

such that the output tracking error is driven to zero for the whole desired output trajectory only after one learning iteration over $t \in [0,T]$ iff the matrix $CB$ has full-row rank, where $\breve{x}(\cdot)$ is the state of system (29), $\breve{u}(\cdot)$ is given by (30), and gain matrices $R^1$ and $R_i$ for $0 \le i \le p$ are defined by (27).

*Remark 6.* From the above analysis, it is obvious that the FILC law (22) is obtained by following a similar designing procedure to (4) which inevitably uses the unavailable current system state. By exploiting further results of (22), the updating law (31) is proposed by removing its used unavailable information. This results from the fact that the present predictive approach makes it possible to replace $x(\cdot,k+1)$ with its equivalent but available form $\breve{x}(\cdot,k)$. Therefore, FILC is not only proposed to reach the key idea of (Li et al., 2005, Theorem 2) for linear continuous systems with multiple time delays, but also improved by combining it with a predictive approach. However, it should be pointed out that the FILC drives the output tracking error to zero only after one learning iteration when the accurate information on the ILCS can be obtained; otherwise, it becomes a conventional ILC method.

## 4. SIMULATION RESULTS

*Example 1.* Consider the system with two sate delays, i.e., (Li et al., 2005, Example 3):

$$\dot{x}(t) = \begin{bmatrix} -2 & 0.7 & -1 \\ -1 & 0 & 1 \\ 0 & 1 & -0.5 \end{bmatrix} x(t) + \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -0.5 \\ 0.5 & 0 & 1.8 \end{bmatrix} x(t-0.5)$$
$$+ \begin{bmatrix} 0.1 & 1 & 0 \\ 0 & 1.2 & -0.3 \\ -1 & 0.2 & 0.5 \end{bmatrix} x(t-0.2) + \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ -0.8 & 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1.2 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} x(t). \quad (32)$$

The desired output for system (32) is given by

$$y_d(t) = \begin{bmatrix} y_{d1}(t) \\ y_{d2}(t) \end{bmatrix} = \begin{bmatrix} 12t^2(1-t) \\ 1.5t \end{bmatrix}, \ t \in [0,1].$$

It is assumed that boundary conditions for ILC system (32) and (31) are: $x_0(t) = \left[\, t \ t \ t \,\right]^T$ for $t \in [-0.5, 0]$ and $u_0(t) = \left[\, 0 \ 0 \,\right]^T$ for $t \in [0,1]$. Moreover, the tracking accuracy of the ILCS is evaluated by the total square error described by

$$S_i(k) = \int_0^1 [y_{di}(\tau) - y(\tau,k)]^2 d\tau, \text{for } i = 1,2 \text{ and } k \in \mathbb{Z}_+.$$

Fig. 1 shows the desired output trajectory and the output of system (32) at the first iteration, while Fig. 2 shows the total square error for the first 8 iterations. As shown in Figs. 1 and 2, the updating law (31) ensures that the desired output trajectory is accurately tracked for $t \in [0,1]$ only after one learning iteration. However, the accurate system knowledge can not be exactly obtained in practice. Here, provided that the accurate information on system parameters $A$, $A_0$, $A_1$, $B$ and $C$ is not available, and only estimation is given by

$$\hat{A} = \begin{bmatrix} -2.11 & 0.61 & -0.91 \\ -1.2 & -0.2 & 0.9 \\ 0.13 & 1.12 & -0.61 \end{bmatrix}, \hat{A}_0 = \begin{bmatrix} 0.85 & -0.95 & 0.1 \\ 0.12 & 0.91 & -0.65 \\ 0.44 & 0.1 & 1.65 \end{bmatrix}$$
$$\hat{A}_1 = \begin{bmatrix} 0.12 & 0.88 & 0.15 \\ 0.12 & 1.16 & -0.21 \\ -0.85 & 0.14 & 0.54 \end{bmatrix}, \quad \hat{B} = \begin{bmatrix} 0.08 & 0.95 \\ -1.1 & -0.13 \\ -0.92 & 0.9 \end{bmatrix}$$
$$\hat{C} = \begin{bmatrix} 1.12 & 0.02 & -0.9 \\ 1.15 & 0.12 & -0.15 \end{bmatrix}.$$

Figs. 3 and 4 show the tracking performance of the updating law (31) when perturbations are encountered by system (32). It can be seen that the outputs approach the desired output trajectories accurately within few iterations. Compared (Li et al., 2005, Fig. 12) with Fig. 4, we can conclude that, even if FILC is designed according to the estimations of system parameters, it still exhibits much faster convergence rate than the conventional ILC method, which is obtained based on the accurate system information.

## 5. CONCLUSIONS

In this paper, by summarizing main points of Fang & Chow (1998) and incorporating them into an early result Li et al. (2005), predictive results are established, based on which the

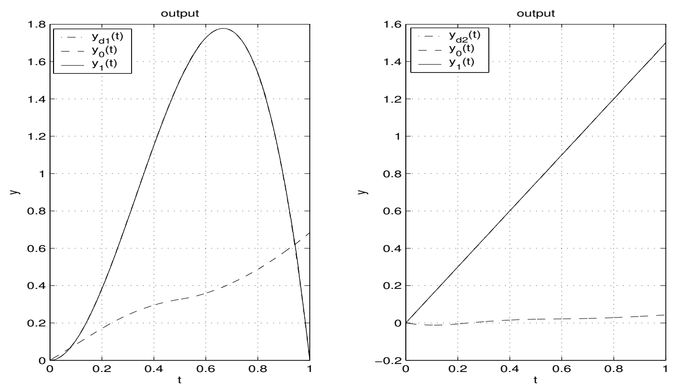


Fig. 1. The tracking performance of the ILCS (32) and (31) under accurate system knowledge for $y_1(t)$ to $y_{d1}(t)$ and $y_2(t)$ to $y_{d2}(t)$, respectively.

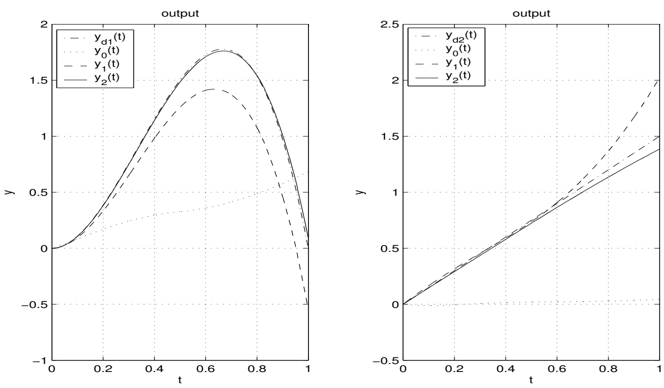


Fig. 3. The tracking performance of the ILCS (32) and (31) under estimated system knowledge for $y_1(t)$ to $y_{d1}(t)$ and $y_2(t)$ to $y_{d2}(t)$, respectively.

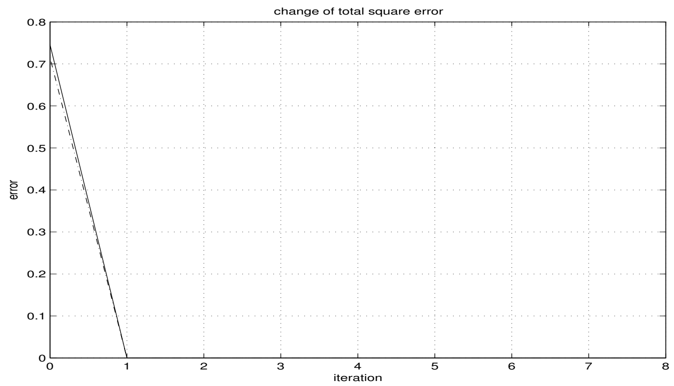


Fig. 2. The trends of the total square error under accurate system knowledge. The solid line and the dashed-dotted line represent the total square error curves of $y_1(t)$ to $y_{d1}(t)$ and $y_2(t)$ to $y_{d2}(t)$, respectively.

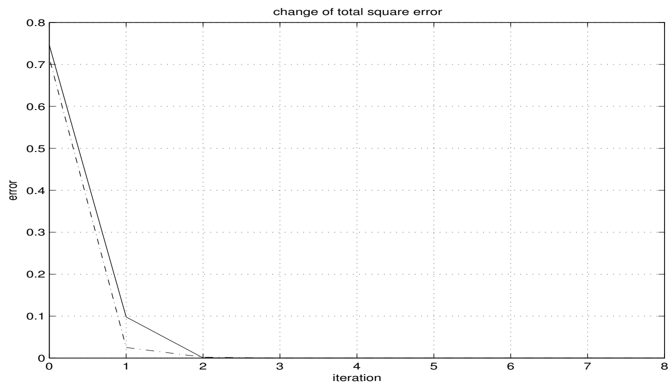


Fig. 4. The trends of the total square error under estimated system knowledge. The solid line and the dashed-dotted line represent the total square error curves of $y_1(t)$ to $y_{d1}(t)$ and $y_2(t)$ to $y_{d2}(t)$, respectively.

learning law both with the fastest convergence rate and available information is proposed for linear continuous systems with a single time delay. Moreover, these results can be extended to deal with more complex control systems in which multiple time delays exist. The simulation results show that FILC can still exhibit fast convergence rate even if it is designed according to estimated system parameters. Thus, the results of this paper not only provide a new route to design learning laws both with available knowledge and the fastest convergence rate, but also widen the application of this desirable type of ILC approach. It should be pointed out, however, that FILC needs the accurate information of the system states, and thus how to remove this restriction is open to further investigation.

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## Appendix A. PROOF OF THEOREM 2

In order to achieve the conclusion in Theorem 2, the following preliminary is useful. Let us denote

$$\phi(t,k) = \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix}$$

then, by extending the definition of $y_d(t)$ to the interval $[-t_0, 0)$ such that $y_d(t) = Cx_0(t)$, it leads to

$$\phi(t,k) = 0 \;\text{ for }\; t \in [-t_0, 0] \;\text{ and }\; k \in \mathbb{Z}_+. \qquad \text{(A.1)}$$

Consequently, let $\Phi(t,k) = \left[\, \tilde{\eta}^T(t,k)\ \tilde{e}^T(t,k) \,\right]^T$, then boundary condition (A.1) results in

$$\Phi(0,k) = 0, \text{ for } k \in \mathbb{Z}_+. \tag{A.2}$$

On the other hand, it is obvious from (A.1) that $\phi(t,k) = 0$ for $t \in [0,T]$ and $k \in \mathbb{Z}_+$ is equivalent to $\Phi(t,k) = 0$ for $t \in [0,T]$ and $k \in \mathbb{Z}_+$, and $\lim_{k\to\infty} \phi(t,k) = 0$ for $t \in [0,T]$ is equivalent to $\lim_{k\to\infty} \Phi(t,k) = 0$ for $t \in [0,T]$.

**Proof of Theorem 2.** (i) According to (Li et al., 2005, Lemma 1), it follows from the 2-D model (25) that $\lim_{k\to\infty} \Phi(t,k) = 0$ for $t \in [0,T]$ iff $I - \tilde{C}\tilde{B}$ is stable. That is, a matrix $R^2$ exists that stabilizes $I - \tilde{C}\tilde{B}$. Considering the diagonal form of $\tilde{B}$ and $\tilde{C}$, it is easy to show that such a matrix $R^2$ exists iff the matrix $CB$ has full-row rank. Hence, the anterior part of Theorem 2 is proved.

(ii) According to the 2-D system theory (e.g., refer to (Chow & Fang, 1998, eqs. (14)-(16))), the solution of the 2-D system (25) can be expressed by

$$\Phi(t,k) = \sum_{i=1}^{\infty} T_{ik} \int_0^t \frac{(t-\tau)^{i-1}}{(i-1)!} \begin{bmatrix} 0 \\ \tilde{e}(\tau,0) \end{bmatrix} d\tau + T_{0k} \begin{bmatrix} 0 \\ \tilde{e}(t,0) \end{bmatrix} \tag{A.3}$$

where the state transition matrix $T_{ij}$ is defined by

$$T_{ij} = \begin{cases} I \text{ (the identity matrix)}, & \text{for i=j=0;} \\ T_{10}T_{i-1,j} + T_{01}T_{i,j-1}, & \text{for i}\geq 0\text{, j}\geq 0 \text{ (i+j}\neq 0\text{);} \\ 0 \text{ (the zero matrix)}, & \text{for i<0, or/and j<0.} \end{cases} \tag{A.4}$$

$$T_{10} = \begin{bmatrix} \tilde{A} & \tilde{B} \\ 0 & 0 \end{bmatrix}, \text{ and } T_{01} = \begin{bmatrix} 0 & 0 \\ -\tilde{C}\tilde{A} & I - \tilde{C}\tilde{B} \end{bmatrix}.$$

Computing the derivative of the both sides of (A.3) with respect to $t$, we get

$$\frac{\partial \Phi(t,k)}{\partial t} = \sum_{i=2}^{\infty} T_{ik} \int_0^t \frac{(t-\tau)^{i-2}}{(i-2)!} \begin{bmatrix} 0 \\ \tilde{e}(\tau,0) \end{bmatrix} d\tau + T_{1k} \begin{bmatrix} 0 \\ \tilde{e}(t,0) \end{bmatrix} + T_{0k} \begin{bmatrix} 0 \\ \dfrac{\partial \tilde{e}(t,0)}{\partial t} \end{bmatrix}. \tag{A.5}$$

Inserting (A.4) into (A.5), we have

$$T_{01} \frac{\partial \Phi(t,k-1)}{\partial t} + T_{10}\Phi(t,k) = \frac{\partial \Phi(t,k)}{\partial t} \quad \text{for } t \in [0,T] \text{ and } k \geq 1. \tag{A.6}$$

*Necessity:* If $\phi(t,k) = 0$ for $t \in [0,T]$ and $k \geq 1$, then

$$\Phi(t,k) = 0, \text{ for } t \in [0,T] \text{ and } k \geq 1. \tag{A.7}$$

Inserting this into (A.6) and considering (A.2), we obtain

$$\Phi(t,k) = T_{01}\Phi(t,k-1), \text{ for } t \in [0,T] \text{ and } k \geq 1.$$

Combining this with (A.7), we get $T_{01}\Phi(t,0) = 0$ for $t \in [0,T]$ whatever $\Phi(t,0)$ is, which implies $T_{01} = 0$. From the denotation of $T_{01}$, it follows that gain matrices of the updating law (22) should be defined in (27).

*Sufficiency:* If gain matrices of the updating law (22) are defined in (27), then it follows from the denotations of $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ that

$$-\tilde{C}\tilde{A} = 0 \text{ and } I - \tilde{C}\tilde{B} = 0. \tag{A.8}$$

That is, $T_{01} = 0$, and therefore, (A.6) becomes

$$\frac{\partial \Phi(t,k)}{\partial t} = T_{10}\Phi(t,k), \text{ for } t \in [0,T] \text{ and } k \geq 1. \tag{A.9}$$

Combining (A.2) with (A.9) yields (A.7). Consider the established preliminary, then from (A.7), (26) follows. This proof is completed. □