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Group Behaviour Control Based on Aggregation and Dilation ⋆

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Abstract: The phenomenon of aggregation and dilation(A&D) widely exists in nature. The mechanism behind it is regarded as the effect of some kind of attraction and repulsion(A&R). A&R control becomes a popular and promising way of controlling the structure and distribution of a group composed of several and even numerous individuals. This paper presents the concepts of aggregation, dilation and group evolution criticality based on group variance. We investigate the relationship between different levels of A&D as a foundation for the introduction of A&D analysis. The applications of A&D analysis and A&R control in several researches, including population-based optimization and group behavior control about multi-agent, are given in the form of simulation experiments.

1. INTRODUCTION

The phenomenon of aggregation and dilation is ubiquitous in nature and human society. In group systems constituted by several or even a mass of matters or individuals, A&D is used to characterize their systemic evolution tendency. In the opinion of physical scientists who affirm Big Bang theory, the formation and evolution of the universe is a process during which aggregation and dilation coexist and contribute to the diversity of matters. In the biologic world, individuals in some feeblish species spontaneously congregate together in order to improve their fitness to environment and resist the attack from enemies; however, as a result of competition, the dilation of the colony occurs in the form of mutual repulsion and sometimes a war follows. In the physical or vital world, certain laws such as Newton's Law dominate the interaction among individuals so that the evolution of group systems exhibits the phenomenon of aggregation or dilation. Generally, we call such interactions attraction and repulsion which contribute to aggregation and dilation respectively.

In general, the evolution process which only exhibits aggregation or dilation is *ordinary* and *dull* since it lacks abundant *dynamic* traits, though monotone aggregation or dilation is important to further research on complex A&D processes. As it is in practice, aggregation and dilation often coexist and show relative differences in intensity through the temporal accumulation of the contest between attraction and repulsion(A&R). During the process in which attraction dominates, aggregation will finally stand out. In contrast, repulsion dominance will result in dilation. A resultant field with attraction and repulsion distributed in space and time leads to abundant and complicated dynamic behaviour of group systems. This hybrid A&R strategy was introduced by scholars in different fields into their researches, which accelerated the development of correlative disciplines. For example, in the research of global optimization, repulsion is introduced into optimization algorithms to increase the probability of breaking away from local optima and finding global optima.(Barhen et al.,1997;Riget et al., 2002). The TRUST algorithm proposed by Barhen et al.(1997) is particular in a sense as it is not a population-based optimizer. However, with local optima considered as fixed part of a population, this algorithm can also be analyzed based on A&D. In contrast, ARPSO(Riget et al., 2002) is a population-based optimizer which controls A&D by alternatively changing the sign of its two acceleration factors. In these algorithms, the space between individuals is controlled to keep a balance between exploration and exploitation in search and optimization. Another prominent application of A&R is the artificial potential field method(Reif et al., 1999;Gazi et al., 2004;Liu et al., 2004) in the formation control of overland or underwater robots, uninhabited autonomous(air) vehicles and other multi-agent systems. The introduction of A&R benefits the control of the space between individual agents so that more challenging tasks, such as encircling an object, rounding or getting over an obstacle and cooperating with each other to move a bulky object, can be accomplished. This is of great importance for achieving high-level goals such as working in a dangerous or uninhabitable environment and exploring in outer space.

As shown later, A&D analysis can help to predict the evolution tendency of group systems and construct effective control methods to achieve a desirable group structure and distribution. As a basis for such analysis, a *quantitative* characterization of A&D is required. Besides, A&R con-

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trol is a promising way of controlling the structure and distribution of group systems.

This paper focuses on the A&D analysis and A&R control in group evolution from the viewpoint of *statistics*. It is organized as follows. The following section will present the concept of aggregation, dilation, equilibration, monotone aggregation, monotone dilation and group evolution criticality based on *continuous differentiable* process. In Section 2.1, the critical evolution modes of groups composed of different numbers of individuals are analyzed. In Section 2.2, the relationship between the definitions of A&D at different levels is analyzed. Section 2.3 analyzes the effects of A&R control on A&D. Section 3 introduces the applications of A&D analysis in search and optimization. Group behaviour control about multi-agent based on A&R is presented in Section 4. Conclusions are made in Section 5.

2. CHARACTERIZATION OF AGGREGATION AND DILATION

Since A&D is a charaterization of systemic evolution behaviour, a systematic measure - group variance is adopted to define A&D. In a group composed of m individuals, the group variance can be given by

$$s(t) = \frac{1}{m}\sum_{i=1}^{m}\sum_{j=1}^{n}(x_{ij}(t) - x_j^c(t))^2, x_j^c(t) = \frac{1}{m}\sum_{p=1}^{m} x_{pj}(t) \quad (1)$$

where $s(t)$ is group variance at time t, n is the dimension of the space in which individuals move, $x_{ij(t)}$ is the j^{th} component of the position of the i^{th} individual at time t, and $x_j^c(t)$ is the j^{th} component of the position of the group center at time t.

Definition(Aggregation & Dilation): In a group evolution process, if the initial group variance $s(t_0)$ and the final group variance $s(t_f)$ satisfy $s(t_0) > s(t_f)$, then the evolution mode of this group is aggregation. If $s(t_0) < s(t_f)$, then it is dilation. Particularly, we call the critical case $s(t_0) = s(t_f)$ equilibration.

The above definition emphasizes on the evolution result but don't care about the whole process. The following new concepts present three typical and special evolution modes.

Assume that $x_{ij}(t)$ is continuous and differential for arbitrary i and j.

Definition(Monotone Aggregation & Monotone Dilation): If a group evolution satisfies $s^{(1)}(t) \le 0$ where $s^{(1)}(t)$ is the first-order derivative of $s(t)$, then the evolution mode is called monotone aggregation. If $s^{(1)}(t) \ge 0$, the evolution mode is called monotone dilation. In the critical case $s^{(1)}(t) \equiv 0$, the property of such a process is called group evolution criticality.

For monotone dilation, if $s(\infty) \to 0$, then the process is called infinite(maximal) monotone aggregation. Otherwise, it is called finite monotone aggregation.

2.1 Group Evolution Criticality

The concept of group evolution criticality contains particular but abundant dynamic behaviours. It means a static or dynamic equilibration. In static equilibration, all individuals have no velocity. In contrast, individuals are active and take some special forms of evolution in dynamic equilibration. For example, if a group composed of two individuals in one-dimensional Euclidean space is in equilibration, then we can conclude that these two individuals have the same velocity. This is because $s^{(1)}(t) \equiv 0$ implies $x_1^{(1)}(t) \equiv x_2^{(1)}(t)$. In this sense, static equilibration is just a very special case. If we increase the number of individuals or the dimension of evolution space, then critical evolution modes will become diverse since there are more free variables for the equation $s^{(1)}(t) \equiv 0$. For a 4-individual group in one-dimensional Euclidean space, except the above isovelocity mode, a symmetrical cosine vibration, denoted by
$\{(x_1, x_2, x_3, x_4)|x_1(t) = a \cdot cos(t), x_2(t) = -a \cdot cos(t),$
$x_3(t) = b \cdot sin(t), x_4(t) = -b \cdot sin(t), a \neq 0, b \neq 0\}$,
satisfies $s^{(1)}(t) \equiv 0$. It's easy to verify that group rotation in high dimensional space also satisfies $s^{(1)}(t) \equiv 0$. In essence, neither isovelocity evolution nor group rotation evolution changes the inner structure of the group. But the above vibration evolution does have an effect on the inner structure of a group. Accordingly, the evolution modes that affect the inner structure of a group are regarded as internal dynamics while the other modes are external dynamics. Complicated critical evolution modes can be the combination of internal and external dynamics. For those group systems whose structure and distribution we can't control directly, the understanding of criticality is a foundation for A&D analysis in a statistical way.

2.2 Aggregation and Dilation at Different Levels

In a group composed of more than two individuals, A&D has different levels since the group variance based on (1) can't guarantee a systematic aggregation or dilation. For example, if most individuals gather together and fewer individuals move far away from the most, then the group variance may appear large but it usually doesn't imply a distinct dilation. Thus, a further definition of A&D is needed.

Definition(Level-(m-n) Aggregation & Dilation): For a group composed of m individuals($m \ge 3$), if the variance of arbitrary subgroup composed of n($n \le$m) individuals satisfies $s_{i_1,\cdots,i_n}(t_0) > s_{i_1,\cdots,i_n}(t_f)$, where $i_1,\cdots,i_n$ denote n individuals arbitrarily selected from the whole group, then the evolution of this group is called level-(m-n) aggregation. Similarly, level-(m-n) dilation can be defined.

Theorem 1. For any integer denoted by l satisfying $2<l\le m$, level-(m-l)aggregation(dilation) is a necessary condition for level-(m-l+1)aggregation(dilation).

Proof: Level-(m-l+1) aggregation means that the inequality $s_{i_1,\cdots,i_{l-1}}(t_0) > s_{i_1,\cdots,i_{l-1}}(t_f)$ holds for l-1 arbitrary individuals $i_1,\cdots,i_{l-1}$ in an m-individual group.

Denote by $\mathbf{G}_l = \{i_1, i_2, \cdots, i_l\}$ a group composed of l individuals arbitrarily selected from the whole m-individual group and by $\mathbf{G}_l/\{i_k\}$ the subgroup which includes each individual except i_k in $\mathbf{G}_l$.

According to the definition of group variance, we can derive

$$s_{i_1,\cdots,i_l}(t) = \frac{(l-1)^2}{(l-2)l^2} \sum_{q=1}^{l} s_{\mathbf{G}/\{i_q\}}(t) \quad (2)$$

where $\alpha = (l-1)^2/[(l-2)l^2]$, which means level-($m$-$l$) aggregation. The conclusion for dilation can be proved in a similar way.

Theorem 1 indicates that higher-level aggregations are more intense. Level-(m-2) aggregation(dilation) is the most intense, which means that any two individuals congregate(separate). The relationship between level-0 group variance and level-(m-2) group variance can be given as follows.

$$s(t) = \frac{2}{m^2} \sum_{\forall i_p, i_q \in \mathbf{G}_m, p \neq q} s_{i_p,i_q}(t) \quad (3)$$

2.3 A&D and A&R

A&D analysis can only predict group evolution tendency but has no effect on group evolution. A&R provides a feasible way of controlling group evolution. Pure repulsion will cause dilation and finally lead to group disruption regardless of initial group state. In particular, with pure repulsion, a group in which all members have zero initial velocity will follow an evolution mode of infinite monotone dilation. In contrast, the effect of pure attraction depends on the initial group state. Usually, in the case of pure attraction without any outside interference, any individual which has a high speed and tends to escape from its group can have a potential to break away, which may bring some dilation at a certain level. Without sufficient escape velocity, all group members will be trapped in a bounded space and vibrations often occur in this case. A dissipative mechanism, such as a force which is proportional to velocity but has an opposite direction, is required to stabilize the inner structure of the group. In nature, attraction and repulsion often coexist to control group evolution. The distribution of the relative intensity between attraction and repulsion on scales of distance contributes to the diversity of evolution. Such a rule can be borrowed from physics to wide engineering applications. Constructing various A&Rs may have various control effects. The following section will give several illustrations about A&D analysis and A&R control.

3. SEARCH AND OPTIMIZATION BASED ON A&D AND A&R

A&D has a great effect on the performance of population-based optimizers(Chen et al. 2007). A&D analysis helps to indicate the diversity of the distribution of individuals which is important for a population-based optimizer to adjust its balance between exploration and exploitation so as to find global optima. When premature convergence occurs, the diversity of population is usually very poor and A&D analysis can be used to detect such a situation. Ursem(2002) proposed an efficient diversity-guided evolutionary algorithm (DGEA) which defines diversity according to population variance. The ARPSO proposed by Riget et al.(2002), which is an improved form of particle swarm optimizer, also benefits from A&D analysis. The group variance adopted in DGEA and ARPSO is level-0 type, which cannot reflect local aggregations that cause the deficiency of diversity while level-0 group variance is rather large. If an optimizer keeps in local aggregations chronically, its exploration performance will be weakened and the probability of finding global optima will be greatly reduced. A new optimizer based on PSO is proposed here according to a new group variance which is similar to level-(m-2) group variance but randomly selects only two individuals from the current population to make A&D analysis. When global or local aggregation occurs, most individuals congregate together and the selected two individuals will reflect such local aggregation in a big probability while computational complexity can be restrained. For convenience, we call this A&D-guided optimizer ADGPSO. The way of A&R control is the same with ARPSO which controls diversity by changing the sign of acceleration factors. Without loss of generality, the new algorithm is used for the minimization of functions. The algorithm in pseudocode follows.

Initialize population, let $dir = 1$
While termination criterion is not met, do
For i=1 to PS
if $f(\mathbf{x}_i)<f(\mathbf{p}_i)$ then $\mathbf{p}_i=\mathbf{x}_i$
$\mathbf{p}_g=arg_i min\{f(\mathbf{p}_i)\}$
For $d = 1$ to Dim
$v_{id}=w \cdot v_{id} + dir \cdot [c_1 \cdot r_1 \cdot (p_{id} - x_{id}) + c_2 \cdot r_2 \cdot (p_{gd} - x_{id})]$
$v_{id}=sign(v_{id}) \cdot min(abs(v_{id}), v_{max})$
$x_{id}=x_{id} + v_{id}$
Next d
Next i
Randomly select two individuals $\mathbf{x}_p, \mathbf{x}_q$ from the population with $p \neq q$
Compute diversity: $div = \sum_{l=1}^{Dim}(x_{pl} - x_{ql})^2/(4 \cdot L)$
if dir<0 & div>dh then dir=1
if dir>0 & div<dl then dir=-1
End While

The denotation instructions are shown as follows: $f(\mathbf{x})$ – the objective function; Dim – the number of dimensions; PS – population size; $\mathbf{x}_i$ – the position of the i^{th} particle; $\mathbf{v}_i$ – the velocity of the i^{th} particle; $\mathbf{p}_i$ – the best position found by the i^{th} particle; $\mathbf{p}_g$ – the best position found by the whole swarm; x_{id},v_{id}, p_{id},p_{gd} – the d^{th} component of $\mathbf{x}_i$,$\mathbf{v}_i$, $\mathbf{p}_i$ and $\mathbf{p}_g$ respectively; v_{max} – the maximal value of velocity in each dimension; w – inertia weight; c_1, c_2 – acceleration factors; r_1, r_2 – random numbers uniformly distributed in (0,1); div – diversity index; dh – the upper threshold of diversity; dl – the lower threshold of diversity; L – the diagonal length of search space.

Note that the A&R control in ARPSO and ADGPSO is implemented by changing the sign(i. e. the direction) of acceleration factors, so it is a kind of switching control.

In order to validate the effectiveness of A&D analysis(contained in both ARPSO and ADGPSO) on the improvement of PSO, we use the basic PSO(bPSO), ARPSO, ADGPSO, CLPSO(Liang et al., 2006) and three new variants of CLPSO(CLPSO-AR, CLPSO-ADG and CLPSO-b) to minimize several Benchmark functions which include Rastrigin, Rosenbrock, Griewank and Schwefel functions(Riget et al., 2002; Sutton et al., 2006). Besides, relatively speaking, Schwefel function is regarded as a difficult one for many PSOs(Sutton et al., 2006; Liang et al., 2006). Note that CLPSO is one of the state-of-the-art PSOs. CLPSO-AR is a combination of CLPSO and ARPSO which run independently with the share of the information about the up-to-date best position of the whole swarm. Similarly, CLPSO-ADG is a combination of CLPSO and ADGPSO, and CLPSO-b is composed of CLSPO and bPSO. For ARPSO, ADGPSO and bPSO, w is set to 0.729, c_1 and c_2 are both set to 1.494(The setting of these parameters was suggested by Clerc et al.(2002) since PSO performs very well with such a setting). In each dimension, the maximal absolute value of velocity(v_{max}) is set to one fifth of the search scope in this dimension. The population size for any optimizer is set to 20. In CLPSO-AR(CLPSO-ADG, CLPSO-b), the whole population is halved by CLPSO and ARPSO(ADGPSO, bPSO). The setting of the other parameters for CLPSO and the embedded CLPSO in CLPSO-AR, CLPSO-ADG and CLPSO-b is the same as that in Liang et al.(2006). For all optimizers, if the total number of function evaluations reaches its maximum which is set to 10^5, then optimization process will be terminated. The diversity-control parameter dl is empirically set to two typical values 5e-6 and 5e-8, as the optimization performances of correlative optimizers corresponding to these two settings approach their best. Another diversity-control parameter dh is fixed at 0.25(Riget et al., 2002). For each test function and each optimizer, 100 tests are implemented. The mean and standard deviation of the discovered optimal value are compared. In each case, the first column corresponds to the mean and the second to the standard deviation. The experimental results are shown in Table 1-4.

As shown in the experimental results, ARPSO performs very well in the optimization of single-funnel functions including Rastrigin, Rosenbrock, and Griewank functions. This is mainly due to the level-0 A&D analysis and A&R control applied in ARPSO, since the only difference between ARPSO and bPSO is the introduction of A&D-based diversity control. In particular, ARPSO performs the best in the optimization of Rosenbrock function.

Table 1. Optimizing Rastrigin Function

Dimension:10, Optimal value:0, $\mathbf{x} \in [-5.12, 5.12]^{10}$				
Optimizer	dl=5e-6		dl=5e-8	
bPSO	1.0e+1	3.4e-1	1.0e+1	3.4e-1
ARPSO	9.1e-1	9.0e-2	1.4e+0	7.7e-2
ADGPSO	3.2e+0	5.2e-1	4.0e+0	4.4e-1
CLPSO	2.3e+0	1.3e-1	2.3e+0	1.3e-1
CLPSO-AR	*5.8e-6*	*3.1e-6*	*6.1e-6*	*2.5e-6*
CLPSO-ADG	*1.6e-4*	*5.1e-5*	**4.5e-6**	**1.3e-6**
CLPSO-b	2.7e+0	1.7e-1	2.7e+0	1.7e-1

Table 2. Optimizing Rosenbrock Function

Dimension:10, Optimal value:0, $\mathbf{x} \in [-100, 100]^{10}$				
Optimizer	dl=5e-6		dl=5e-8	
bPSO	*1.1e+0*	*1.8e-1*	1.1e+0	1.8e-1
ARPSO	*1.3e-1*	*9.4e-2*	**9.0e-2**	**2.1e-2**
ADGPSO	7.4e+0	4.0e+0	10102	10050
CLPSO	2.1e+1	1.7e+0	2.1e+1	1.7e+0
CLPSO-AR	*1.1e+0*	*1.7e-1*	*1.7e+0*	*2.0e-1*
CLPSO-ADG	2.1e+0	1.8e-1	2.9e+0	3.4e-1
CLPSO-b	4.8e+0	1.0e+0	4.8e+0	1.0e+0

Table 3. Optimizing Griewank Function

Dimension:10, Optimal value:0, $\mathbf{x} \in [-700, 500]^{10}$				
Optimizer	dl=5e-6		dl=5e-8	
bPSO	5.9e-2	3.0e-3	5.9e-2	3.0e-3
ARPSO	6.0e-2	2.0e-3	6.0e-2	2.0e-3
ADGPSO	6.4e-2	2.6e-3	7.0e-2	2.5e-3
CLPSO	1.3e-1	4.9e-3	1.3e-1	4.9e-3
CLPSO-AR	*2.0e-2*	*1.4e-3*	**1.6e-2**	**1.4e-3**
CLPSO-ADG	*2.2e-2*	*1.7e-3*	*2.8e-2*	*1.6e-3*
CLPSO-b	8.4e-2	4.4e-3	8.4e-2	4.4e-3

Table 4. Optimizing Schwefel Function

Dimension:5, $\mathbf{x} \in [-500, 500]^5$, Approximate Optimal Value:-2094.9144				
Optimizer	dl=5e-6		dl=5e-8	
bPSO	-1409	1.2e+1	-1409	1.2e+1
ARPSO	-1944	8.9e+0	-1910	1.5e+1
ADGPSO	-1990	7.0e+0	-2001	1.3e+1
CLPSO	**-2094**	**2e-13**	-2094	2e-13
CLPSO-AR	*-2092*	*1.7e+0*	**-2094**	**2e-13**
CLPSO-ADG	**-2094**	**2e-13**	**-2094**	**2e-13**
CLPSO-b	-2093	1.2e+0	-2093	1.2e+0

On the whole, ADGPSO is unsuccessful since it is even defeated by bPSO in the optimization of the Rastrigin function and especially, the Rosenbrock function. However, ADGPSO performs better in the optimization of Schwefel function than bPSO and ARPSO. The level-(m-2) A&D analysis benefits ADGPSO in the aspect of detecting local aggregation. But local aggregation may contribute to the discovery of global optima of single-funnel functions, when partial particles keep away from the majority of the population(Sutton et al., 2006). In this case, repulsion may slow and even destroy the convergence into global optima, which results in the poor performance of ADGPSO. In contrast, the detection of local aggregation in case of premature convergence takes effect in the optimization of the multi-funnel Schwefel function. In fact, even the state-of-the-art CLPSO is, in a way, inferior to bPSO in the optimization of Griewank and Rosenbrock functions. Note that Rosenbrock function is unimodal and Griewank function becomes easier as its dimension increases. However, CLPSO obviously outperforms the others in the optimization of Schwefel function.

Among three hybrid optimizers(CLPSO-b, CLPSO-ADG, CLPSO-AR), CLPSO-ADG and CLPSO-AR outperform nonhybrid PSOs in almost all cases. Particularly, CLPSO-AR performs the best in the optimization of Griewank and Schwefel functions and CLPSO-ADG is the best in the optimization of Rastrigin and Schwefel functions. Compared with CLPSO-ADG, it's a little more sensitive to the diversity parameter dl when optimizing Schwefel function. Although CLPSO-b performs better in the optimization of Rosenbrock function, it is no better than CLPSO since it is inferior to CLPSO in the optimization of Schwefel function. Generally speaking, the hybridization of CLPSO and ARPSO(ADGPSO) which is based on A&D analysis and A&R control produces a more efficient variant of PSO.

As the experimental results indicate, no optimizer overwhelms the others over all problems though some of them have obvious advantages. Such a performance difference among these optimizers can be explained by the NFL theorem proposed by Wolpert et al.(1997). Different problems may require different tradeoffs between exploration and exploitation.

4. GROUP BEHAVIOUR CONTROL ABOUT MULTI-AGENT

A&R in muti-agent formation control was originally proposed by Reif et al.(1999). Attraction and repulsion forces can be imposed between individuals or between any individual and its goals or obstacles. In practice, the desired group behaviour of multi-agent is complicated and any group member is required to have certain self-determination. In other words, the A&D of multi-agent group is a high-level issue which usually depends on the mission of the group and detailed assignment of group members. In many cases, group members act independently, which doesn't need attraction among group members. In contrast, repulsion is always necessary in order to avoid collisions. As mentioned by Gazi et al. (2004), repulsion only takes effect within a small distance, which means repulsion is local. In the following experiment, a virtual group with 4 members, which can be regarded as some robots, is required to collect some objects distributed in a square work area. The objective is to collect as many objects as possible with a predefined time restriction and avoid collisions. Many interesting phenomena, such as competition and deadlock, can be observed in this experiment. All members are supposed to hold sufficient objects. Repulsion is used to avoid collisions against other members in the group or obstacles. Attraction is used to make group members approach scanned objects. Thus, attraction is active and optional but repulsion is passive and indispensable. The instructions about this experiment are made as follows.

• The work area is divided into a 500×500 grid. All objects and obstacles lie at the centre of a cell and any object or obstacle occupies only one cell.

• At any point, any member has a scanning radius of 20 units. One unit is the length of one side of a cell. Obstacles have no effects on the scanning of any member but group members should round the obstacles on their way.

• All scanned objects will be recorded into the memories of the members who find them. If an object is collected, its corresponding record will be cleared.

• Collisions mean that more than one member occupies the same cell. The number of collisions is recorded.

• The A&R control strategy can be described by the following expressions.

$$g_a(\alpha) = [H(\alpha) - H(\alpha - 100)] \cdot min\{\frac{5(1-e^{-0.223\alpha})}{\alpha}, 0.6\},$$

$$g_r(\alpha) = -10[H(\alpha) - H(\alpha - 50)]/\alpha^2,$$

$$\mathbf{v}_j(k+1) = g_a(\|\mathbf{x}_j(k) - \mathbf{x}_s\|)[\mathbf{x}_s - \mathbf{x}_j(k)] + \sum_i g_r(\|\mathbf{x}_j(k) - \mathbf{x}_{obs,i}\|)[\mathbf{x}_{obs,i} - \mathbf{x}_j(k)] + \sum_{p(\neq j)} g_r(\|\mathbf{x}_j(k) - \mathbf{x}_p\|)[\mathbf{x}_p - \mathbf{x}_j(k)]$$

$$\mathbf{x}_j(k+1) = \mathbf{x}_j(k) + [[\mathbf{v}_j(k+1)]],$$

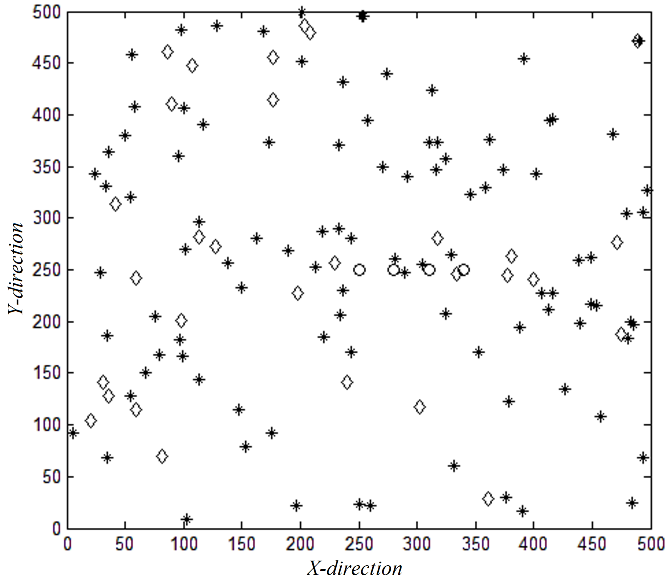

Fig. 1. The distribution of objects, obstacles and multi-agent group('$*$':objects;'$\diamond$':obstacles;'$\circ$':initial positions of group members)

$\mathbf{v}_j(k)$ and $\mathbf{x}_j(k)$ are the velocity vector and the position vector of the j^{th} member at time k respectively. g_a and g_r are the attraction and repulsion functions respectively. $[[\cdot]]$ rounds the value of velocity in each dimension to an integer. $\mathbf{x}_s$ and $\mathbf{x}_{obs,i}$ are the position vector of an object and that of the i^{th} obstacle respectively. $\|\cdot\|$ denotes the Euclidean norm. H($\cdot$) is the Heaviside function which is equal to 1 for positive values of the argument and zero otherwise.

• If a member doesn't detect any objects within its current scanning scope, it will take a random walk with its step in each dimension selected from $\{-5U, +5U\}$(U: unit).

• The state of each member is examined to avoid potential competitions for one object among group members. If a member discovers it's hunting for one object with other members or it continuously doesn't move twice, it will discard its current object and turn to another object or take a random walk.

The distributions of objects, obstacles and multi-agent group in an experiment are shown in Fig.1. There are 4 members, 100 objects and 30 obstacles on the map. The instructions about symbols used in the following figures are the same.

The remaining objects and the distribution of group members after 1000 iterations are shown in Fig.2. After 1000 iterations, this group obtained 34 objects without any collisions. So A&R control is effective in the control of group behaviour.

For any member, when its attraction from an object and its repulsions from obstacles or other members get into equilibrium, it will not move unless some countermeasures are taken. If the last rule for A&R control is off, some interesting phenomena such as competition and deadlock can be observed. Competition occurs when several members try to take the same object and it's due to A&R equilibrium. In the case of competition, the members

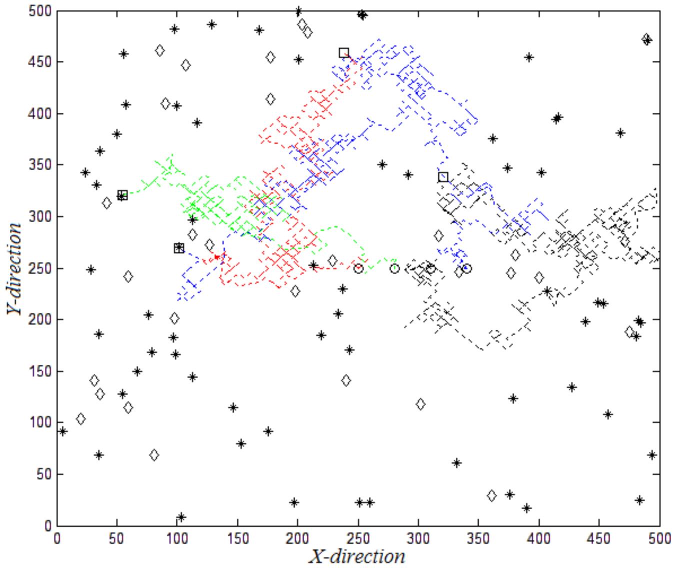

Fig. 2. The distribution of the remaining objects and group members after 1000 iterations('□':final positions of group members;dashed lines denote the tracks of group members)

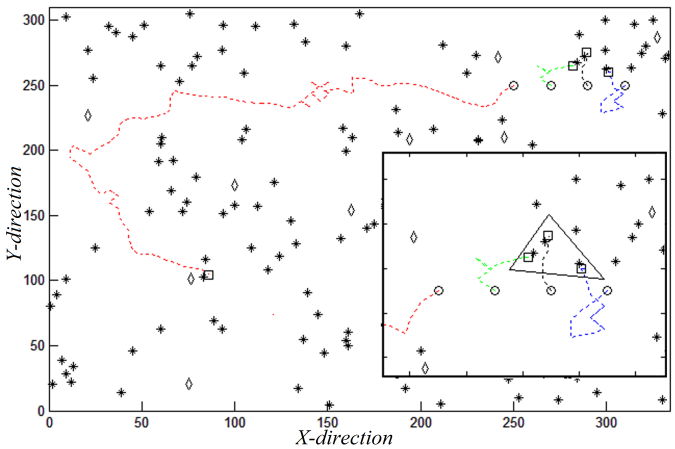

Fig. 3. Deadlock formed by three members aiming at different objects

trapped in it will be stagnated and the object they hunt for can't be acquired. As a more complicated case, deadlock occurs when group members have different aims which are adjacent to each other. A deadlock formed by three members during an experiment without state examination is shown in Fig.3 and the embedded figure in it is the magnification of the location where deadlock occurs. When the leftmost member has covered a long distance to find many objects, the other three members are trapped in a triangular deadlock. Another special case is obstacle umbrella. An object adjacent to an obstacle is under the umbrella of the repulsion of this obstacle which prevents a member from getting the object. Particularly, a cluster of obstacles forms a strong repulsion field to resist members who want to approach the objects around these obstacles. For fear of these troubles, reasonable rules are expected to be introduced into A&R control.

5. CONCLUSION

A&D analysis helps to evaluate the present state of a group, predict its future states and provide information for the A&D control of the group. For a group composed of many members whose states can be observed but cannot be controlled, A&D analysis at different levels can be used to estimate the inner structure of the group. A&R control is a promising way of implementing A&D control. A&D analysis and A&R control can be combined to achieve improved performances in search and optimization. It has been verified that A&D analysis and A&R control are promising to further improve the performance of state-of-the-art PSOs. The experiment in Section 4 can be used to test the effect of A&R control. More complicated behaviours, such as cooperation and organization, can be investigated through A&R control in this experiment.

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